Departamento de Matemática da Universidade de Aveiro

ANÁLISE MATEMÁTICA III **2007/08**

Folha 5: *soluções*

2.(a) $\frac{-3\sqrt{2}}{2}$

2.(b) $\frac{-\sqrt{3}}{3}$

2 (c) 0, se $\vec{v} = e_1$ ou e_2; não existe nos restantes casos.

3.(a) $\frac{\partial f}{\partial x} = 2x\sin(x+y) + x^2\cos(x+y)$;
$\frac{\partial f}{\partial y} = x^2\cos(x+y)$;

3.(b) $\frac{\partial f}{\partial x} = \frac{1}{x+y^2}$;
$\frac{\partial f}{\partial y} = \frac{2y}{x+y^2}$;
$\frac{\partial f}{\partial z} = 2z$;

3.(c) $\frac{\partial f}{\partial x} = 2xy - y\sin(xy)$;
$\frac{\partial f}{\partial y} = x^2 - x\sin(xy) + z^3$;
$\frac{\partial f}{\partial z} = 3yz^2$;

3.(d) $\frac{\partial f}{\partial x} = 2xze^{x^2+y^2}\sin(x+y) + ze^{x^2+y^2}\cos(x+y)$;
$\frac{\partial f}{\partial y} = 2yze^{x^2+y^2}\sin(x+y) + ze^{x^2+y^2}\cos(x+y)$;
$\frac{\partial f}{\partial z} = e^{x^2+y^2}\sin(x+y)$;

3.(e) $\frac{\partial f}{\partial x} = \frac{y^2+z}{x}x^{y^2+z}$;
$\frac{\partial f}{\partial y} = 2yx^{y^2+z}\ln x$;
$\frac{\partial f}{\partial z} = x^{y^2+z}\ln x$;

3.(f) $\frac{\partial \vec{f}}{\partial x} = (0, yz^2\cos(xy), z^3)$;
$\frac{\partial \vec{f}}{\partial y} = (2y, xz^2\cos(xy), 0)$;
$\frac{\partial \vec{f}}{\partial z} = (0, 2z\sin(xy), 3xz^2)$;

4.(a) $D_{\vec{v}}f(1,1,1) = -\frac{\sqrt{3}}{6}$;

4.(b) $\vec{v} = \frac{\sqrt{21}}{21}(1,2,4)$;

5.(a) $k = 500$, e $D_{\vec{v}}T(P) = -14\sqrt{2}$;

5.(b) $\hat{v} = (\frac{4}{5}, -\frac{3}{5})$;

6.(a) $\frac{\partial^2 f}{\partial y\partial x} = \frac{\partial^2 f}{\partial x\partial y} = 6xy^2z^4 + z\cos(xyz) - xyz^2\sin(xyz)$;
$\frac{\partial^2 f}{\partial z\partial x} = \frac{\partial^2 f}{\partial x\partial z} = 8xy^3z^3 + y\cos(xyz) - xy^2z\sin(xyz)$;
$\frac{\partial^2 f}{\partial z\partial y} = \frac{\partial^2 f}{\partial y\partial z} = 12x^2y^2z^3 + x\cos(xyz) - x^2yz\sin(xyz)$;

6.(b) $\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = -\frac{4xyz}{(x^2+y^2+1)^2}$;

$\frac{\partial^2 f}{\partial z \partial x} = \frac{\partial^2 f}{\partial x \partial z} = \frac{2x}{x^2+y^2+1}$;

$\frac{\partial^2 f}{\partial z \partial y} = \frac{2y}{x^2+y^2+1}$;

6.(c) $\frac{\partial^2 f}{\partial y \partial x} = 4xye^{x^2+y^2+z^2}$;

$\frac{\partial^2 f}{\partial z \partial x} = \frac{\partial^2 f}{\partial x \partial z} = 4xze^{x^2+y^2+z^2}$;

$\frac{\partial^2 f}{\partial z \partial y} = 4yze^{x^2+y^2+z^2}$;

9.

$$\begin{cases} a = \frac{(\sum x_i t_i)(\sum t_i^2) - (\sum x_i)(\sum t_i)}{n(\sum t_i^2) - (\sum t_i)^2} \\ b = \frac{(\sum t_i^2)(\sum x_i) - (\sum t_i)(\sum x_i t_i)}{n(\sum t_i^2) - (\sum t_i)^2} \end{cases}$$