Tutorial 1 - Defective Navier-Stokes Problem

Exercise 1. Consider the simplified geometry of a bifurcation Ω in the Figure below.

In Ω we solve the incompressible Navier-Stokes Equations. We mark four different portions of the boundary $\partial\Omega$. Γ_1 is the inflow section. $\Gamma_{2,u}$ is the up outflow section and $\Gamma_{2,b}$ is the bottom outflow section. The remainder is Γ_w, the wall of the artery.

We consider two boundary value problems.

BVP1

On Γ_w we set the velocity $\boldsymbol{u} = 0$.
On Γ_1 we set the pressure $P = 10\sin(\pi t)$. On $\Gamma_{2,\star}$ we set the pressure $P = 0$.
1) Solve the problem with the do-nothing approach.
2) Solve the problem after moving the position of the most distal section from $x = 15$ to $x = 20$.
3) Solve the problem after moving $\Gamma_{2,u}$ to $x = 10$.
4) Solve the problem with and without the complete strain rate tensor $\nabla\boldsymbol{u} + \nabla\boldsymbol{u}^T$. Compare the results.
5) When using the complete strain rate tensor, try also the “modified” do-nothing approach, where the tangential velocity on $\Gamma_{2,\star}$ is set to be zero.

BVP2

With respect to the previous case, we want to prescribe a flow rate of $Q(t) = 0.5\sin(\pi t)$ on Γ_1.
1) Fit a Poiseuille parabolic profile $g(y) = C(2 - y)y$ such that $\int_{\Gamma_1} g(y)dy = Q(t)$ and solve the problem.
2) Solve the flow rate problem with a Lagrange multiplier approach (fantasy with FreeFem++ is warmly welcome)

Comment on the comparison of the two approaches. What about adding a flow extension?