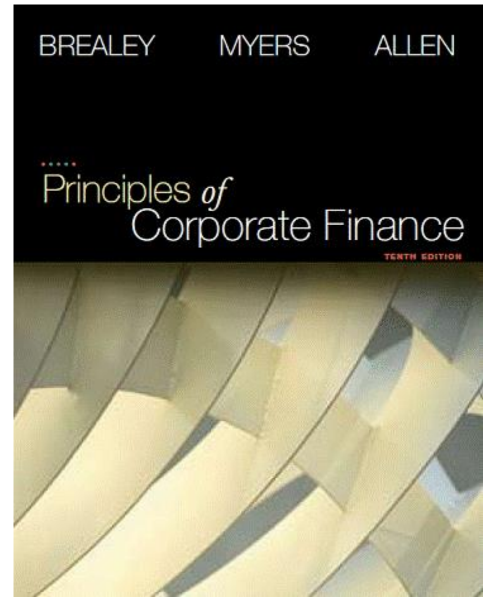
Chapter 2

How to Calculate Present Values

Part II

Principles of Corporate Finance

Tenth Edition



Slides by Matthew Will

Cash flows and assets

Question: What is an "Asset"?

- Business entity
- Property, plant, and equipment
- Patents, R&D
- Stocks, bonds, …
- Knowledge, reputation, ...

From a business perspective, an asset is a sequence of cash-flows:

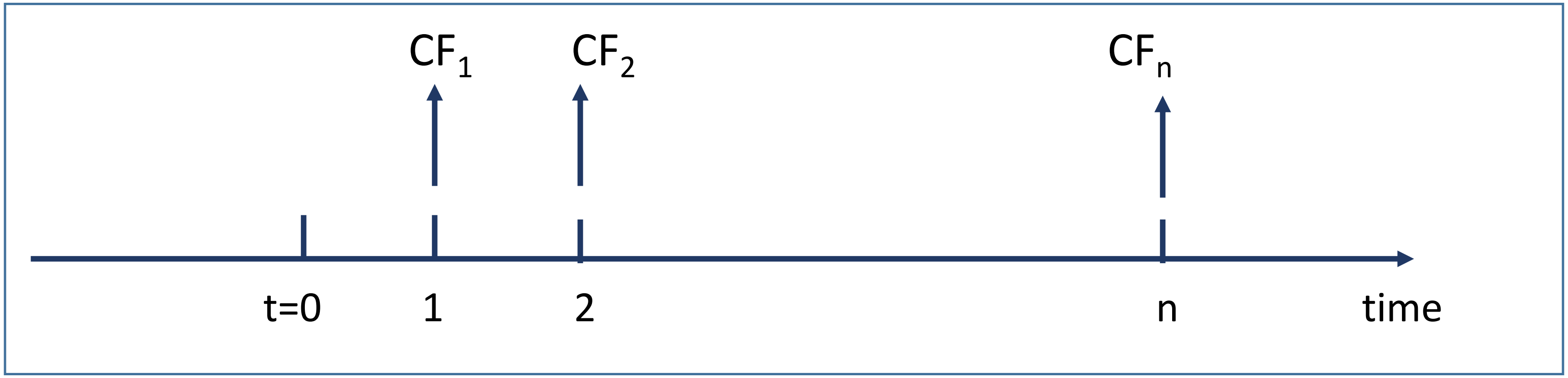
$$Asset_t = \{CF_t, CF_{t+1}, CF_{t+2}, \ldots\}$$

Cash-flows and assets

Valuing an asset requires valuing a sequence of cash-flows

$$\text{Value of Asset}_t = V_t\{CF_t, CF_{t+1}, CF_{t+2}, \ldots\}$$

Always draw a timeline to visualize the timing of cash-flows:



Present value operator

Cash-flows at different dates are different “currencies”.

Consider manipulating foreign currencies:

?

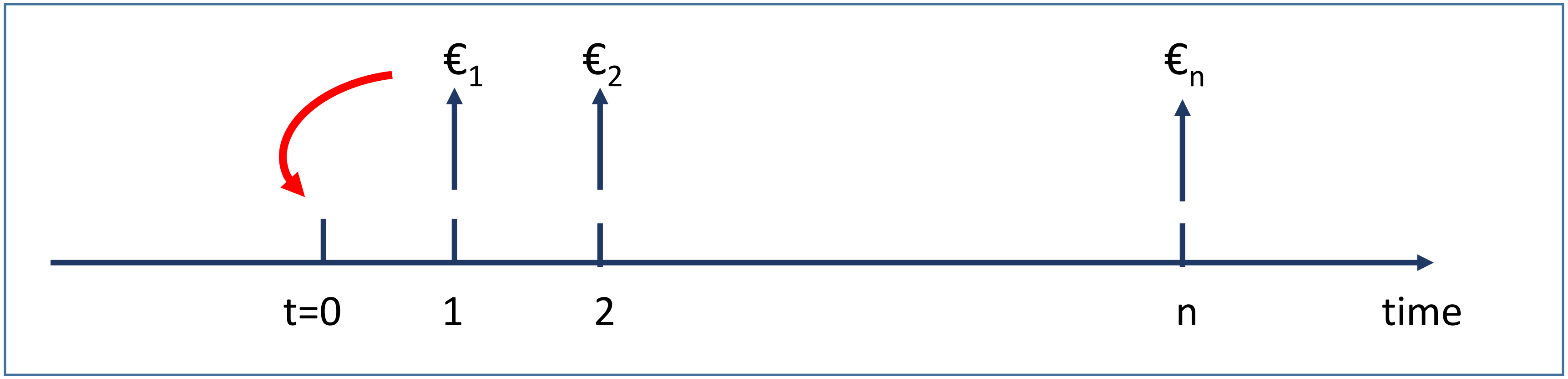
€150 + £300 = ??450

Conclusion: cannot add currencies without first converting into common currency

Same idea for cash-flows of different dates

Present value operator

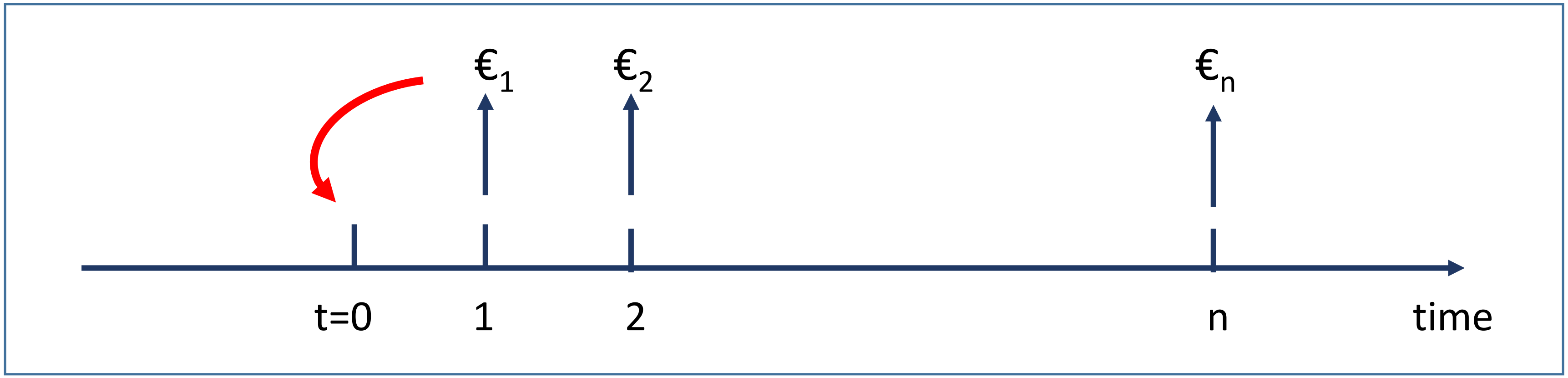
- Cash-flows at different dates are different 'currencies'
- Past and future cannot be combined without first converting them



- Once 'exchange rates' are given, combining cash-flows is easy
- A date should be picked, typically t=0 or 'today'

Cash-flows can then be converted to present value

Present value operator



Cash-flows can then be converted to present value:

$$V_0\ (CF_1, CF_2, CF_3, \ldots) = \left(\frac{€1}{€0}\right) * CF_1 + \left(\frac{€2}{€0}\right) * CF_2 + \left(\frac{€3}{€0}\right) * CF_3 + \ldots$$

Net Present Value (NPV = VAL, Valor Actualizado Líquido)

$$V_0\,(CF_0, CF_1, CF_2, \ \dots) = CF_0 + \left(\frac{€1}{€0}\right)*CF_1 + \left(\frac{€2}{€0}\right)*CF_2 + \dots$$

- Initial Cost or Investment, captured by cash-flow CF_0
- If there is an initial investment, then $CF_0 < 0$
- Note that any CF_t can be negative (future costs)
- Net Present Value: 'Net' of Initial Cost or Investment: including positive cash-flows (revenues) and negative cash-flows (costs)

Example: NPV

Suppose we have the following 'exchange rates':

$$\left(\frac{€1}{€0}\right) = 0{,}9 \qquad \left(\frac{€2}{€0}\right) = 0{,}8$$

What is the net present value of a project requiring a current investment of 10M€ with cash-flows of 5M€ in Year 1 and 7M€ in Year 2?

NPV = VAL = -10 + (0,9)*5 + (0,8)*7 = 0.10M€

Time value of money

What determines the growth of €1 over n years?

- €1 today should be worth more than €1 in the future (why?)

- Opportunity cost of capital r

$$€1 \text{ in Year } 0 = €1 \times (1+r) \text{ in Year } 1$$
$$€1 \text{ in Year } 0 = €1 \times (1+r)^2 \text{ in Year } 2$$
$$\vdots$$
$$€1 \text{ in Year } 0 = €1 \times (1+r)^n \text{ in Year } n$$

Time value of money

What determines the value today of €1 in Year n?

- €1 in Year n should be worth less than €1 today (why?)
- Opportunity cost of capital r

$$€1/(1+r) \text{ in Year 0} = €1 \text{ in Year 1}$$
$$€1/(1+r)^2 \text{ in Year 0} = €1 \text{ in Year 2}$$
$$\vdots$$
$$€1/(1+r)^n \text{ in Year 0} = €1 \text{ in Year n}$$

- These are our 'exchange rates' (€t/€0) or discount factors

Net Present Value

We now have an explicit expression for V_0:

$$V_0 = CF_0 + \left(\frac{1}{1+r}\right)*CF_1 + \left(\frac{1}{(1+r)^2}\right)*CF_2 + \left(\frac{1}{(1+r)^3}\right)*CF_3 + \ldots$$

$$V_0 = CF_0 + \left(\frac{CF_1}{1+r}\right) + \left(\frac{CF_2}{(1+r)^2}\right) + \left(\frac{CF_3}{(1+r)^3}\right) + \ldots$$

- Using this expression, any cash-flow can be valued!

Take positive NPV (VAL>0) projects, reject negative NPV (VAL<0) projects

Net Present Value (VAL) and Internal Rate of Return (IRR)/Taxa Interna de Rentabilidade (TIR)

$$VAL = \sum_{k=0}^{n} \frac{CF_k}{(1+r)^k}$$

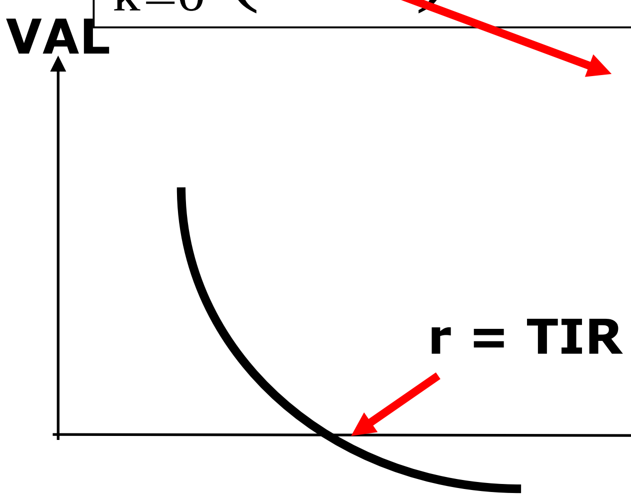
Accept investment if VAL > 0

or

TIR > r (opportunity cost of capital)

$$\sum_{k=0}^{n} \frac{CF_k}{(1+r)^k} = 0$$

How to find TIR? TIR is the rate for which VAL = 0



Net Present Value

For NPV calculations, treat inflation consistently:

- Discount real cash-flows using real interest rates
- Discount nominal cash-flows using nominal interest rates

Nominal cash-flows are expressed in actual-euros cash-flows
Real cash-flows are expressed in constant purchasing power

Example: Net Present Value

Decision: Buy a building?

Step 1: Forecast cash-flows

Cost of building = C_0 = 370,000

Sale price in Year 1 = C_1 = 420,000

Step 2: Estimate opportunity cost of capital

If equally risky investments in the capital market offer a return of 5%, then

Cost of capital = r = 5%

Net Present Value

Step 3: Discount future cash flows

$$PV = \frac{CF1}{1,05} = \frac{420\ 000}{1,05} = 400\ 000$$

Step 4: Go ahead if PV of payoff exceeds investment

Higher risk projects require a higher rate of return
Higher required rates of return cause lower PVs

Rate of return

Com 1.000 euros, o que preferiam?

a) um depósito a prazo em vosso nome, a levantar daqui a um ano, com uma taxa de juro de 2%;
b) idêntico valor em ações, para venderem daqui a um ano, com um rendimento esperado também de 2%?
c) E se em b) o rendimento esperado fosse de 10%?

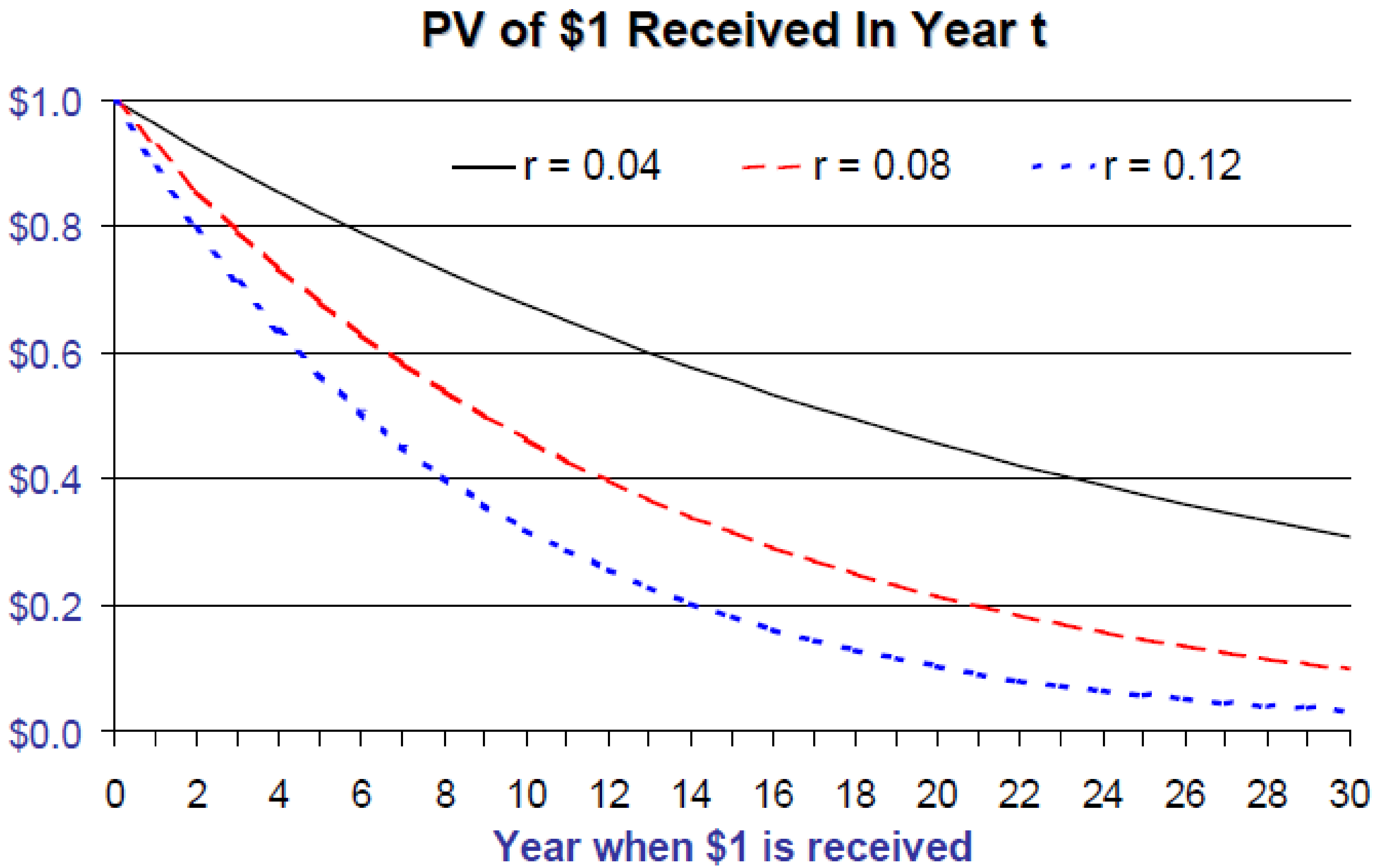
Net Present Value

Accept investments that offer rates of return in excess of their opportunity cost of capital

Example:
In the project “Cost of building=C0= 370,00; Sale price in Year 0 = C0 = 420,000”, the foregone investment opportunity is 5%. Should we do the project?

$$\text{Return} = \frac{\text{profit}}{\text{investment}} = \frac{420{,}000 - 370{,}000}{370{,}000} = .135 \text{ or } 13.5\%$$

Time value of money



Exercício

A sua empresa gasta 800 000€ ao ano em electricidade. Há um equipamento à venda no mercado que lhe vai permitir poupar electricidade em 90 000€ em cada um dos próximos 3 anos e que custa 230 000€. Deverá comprar este equipamento? Assuma que o custo do capital são 4%.

Ano	**0**	**1**	**2**	**3**
CFs	-230 000	90 000	90 000	90 000

Exercício

Ano	**0**	**1**	**2**	**3**
CFs	-230 000	90 000	90 000	90 000
÷		(1.04)	$(1.04)^2$	$(1.04)^3$
PV	-230 000	86 538	83 210	80 010

NPV = VAL = -230000 + 86538 + 83210 + 80010 = 19 758

Special cash-flows: Annuities and perpetuities

A stream of cash-flows corresponding to a fixed sum each period or having a pre-specified growth rate.
in which:

Annuity – the number of flows is finite.

Perpetuity – the stream of cash-flows is perpetual, and initial capital is not returned to investors.

Special cash-flows: Annuities and perpetuities

Annuity examples:

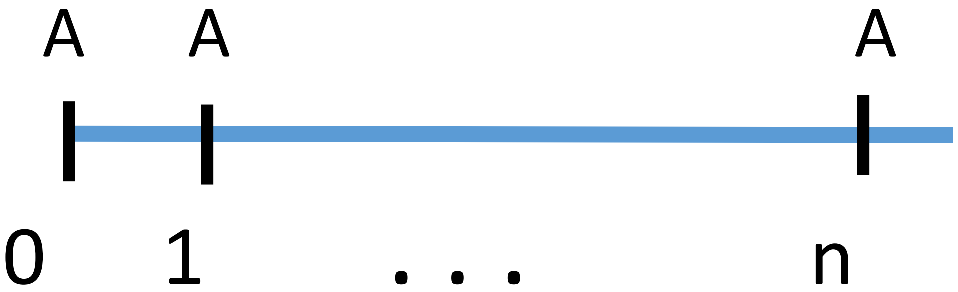
Housing or car loan with constant monthly payments.

Perpetuity examples:

Perpetual income payment; perpetual pension payment; perpetual subsidy.

Annuity

T	1	2	...	n
Discounted Cash Flow	$A/(1+r)^1$	$A/(1+r)^2$		$A/(1+r)^n$



$$Sn = 1º\ termo * \frac{1 - (razão)^n}{1 - razão}$$

Annuity

$$VA = A\frac{\frac{1}{1+r} - \frac{1}{(1+r)^n} \times \frac{1}{1+r}}{1 - \frac{1}{1+r}} = A\frac{\frac{(1+r)^n - 1}{(1+r)^{n+1}}}{\frac{1+r-1}{1+r}} =$$

$$= A\frac{(1+r)^n - 1}{(1+r)^{n+1}} \times \frac{1+r}{r} = A\frac{(1+r)^n - 1}{(1+r)^n \times r}$$

Exercício

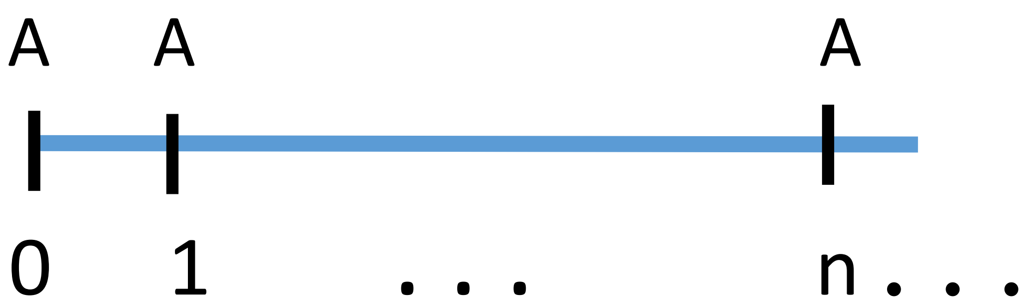
Imagine que lhe propõem um aluguer de longa duração de um carro: 4 anos a pagar 300€ por mês. Não tem de dar nenhum valor de entrada e o seu custo de oportunidade do capital é de 0,5% ao mês. Qual o valor actual do aluguer total?

$$VA = 300 \times \left[\frac{(1+0.005)^{48} - 1}{.005(1+.005)^{48}} \right]$$

$$\text{Cost} = €12774.10$$

Perpetuity

T	1	2	…	N→∞
Discounted Cash Flow	$A/(1+r)^1$	$A/(1+r)^2$		$A/(1+r)^n$



Perpetual and constant cash-flows

Perpetuity

$$VA = A\lim_{\to\infty}\frac{\frac{1}{1+r}-\frac{1}{(1+r)^n}\times\frac{1}{1+r}}{1-\frac{1}{1+r}} = A\frac{\frac{1}{1+r}}{\frac{1+r-1}{1+r}} = \frac{A}{r}$$

Constant Growth Annuity (growth rate = g < r)

T	1	2	…	n
Discounted Cash Flow	$A/(1+r)^1$	$A(1+g)/(1+r)^2$		$A(1+g)^{n-1}/(1+r)^n$

$$VA = \frac{A}{r-g} * \left[1 - \frac{(1+g)^n}{(1+r)^n} \right]$$

Constant Growth Perpetuity (growth rate = g < r)

T	1	2	…	n
Discounted Cash Flow	$A/(1+r)^1$	$A(1+g)/(1+r)^2$		$A(1+g)^{n-1}/(1+r)^n$

$$VA = \frac{A}{r-g}$$

Exercício

Qual o valor actual de 1 milhão€ por ano, para toda a eternidade? Admitindo um custo do capital de 10%.

$$VA = \frac{1}{0.10} = 10\,M€$$

E se o investimento só começa a render daqui a 3 anos?

$$VA = \frac{1}{0.10} * \left(\frac{1}{1.1^2} \right) = 8.2645\,M€$$

Exercício 1

A empresa Gama investiu um capital em regime de juro composto, à taxa anual efectiva de 21%, durante 4 anos. A aplicação vence juros semestrais.

a) Calcule a taxa semestral efectiva deste investimento.
b) Sabendo que o juro vencido no final do 2º semestre do 2º ano foi de 931,70 euros, calcule o valor inicialmente investido pela empresa.

Exercício 2

Uma pessoa adquire certa publicação nas seguintes condições: no acto da compra 25% do seu valor; o restante, em 11 prestações mensais de 25 euros cada, durante um ano. Sabendo que a taxa de juro anual nominal acordada foi de 10%, calcule o preço certo da publicação se a primeira prestação se vencer 1 mês após a compra.

Exercício 3

Como vencedor de um concurso, pode optar por um dos seguintes prémios:

i) 100.000 € já;
ii) 150.000 € ao fim de 5 anos;
iii) 10.500 € todos os anos para sempre;
iv) 19.000 € por ano durante 10 anos;
v) 6.000 € no próximo ano, crescendo posteriormente 5% ao ano, perpetuamente;

Se a taxa de actualização for de 10% diga por que prémio optaria.

Exercício 4

Quando comprou a sua casa, contraiu um empréstimo a 30 anos com um taxa de juro de 6%. A prestação anual são 12 000€. Acabou de fazer um pagamento e decidiu pagar todo o valor em dívida. De quanto será esse pagamento se:

a) Já viveu na casa 12 anos (havendo por isso 18 anos a pagar)
b) Já viveu na casa 12 anos e decidiu pagar o valor em dívida imediatamente antes do 12º pagamento

Exercício 5

Imagine que trabalha para uma farmacêutica que desenvolveu um novo medicamento, que tem uma patente por 17 anos. Espera-se que os lucros sejam 2M€ no 1º ano e que cresçam a uma taxa de 5% durante os próximos 17 anos. Qual será o valor actual do novo medicamento se a taxa de juro anual é 10%?