Lecture Notes

SHIP MANOEUVRING AND SEAKEEPING

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Undergraduate Course

SHIP MANOEUVRING AND SEAKEEPING

(26 lecture hours, 1.5 credits)

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Table of Contents

CHAPTER 1: INTRODUCTION ······ 1
§1.1 Contents of the Course and Its Significance ······ 1
§1.2 Relationships of Ship Manoeuvring Motion and Motion in Waves ······ 1
CHAPTER 2: BRIEF INTRODUCTION TO SHIP MANOEUVRABILITY ······ 1
§2.1 Definition of Ship Manoeuvrability and Its Contents ······ 1
§2.2 Significance of Ship Manoeuvrability ······ 2
CHAPTER 3: EVALUATION OF SHIP MANOEUVRABILITY ······ 4
§3.1 Brief Introduction ······ 4
§3.2 Standard Manoeuvres and the Parameters Evaluating Ship Manoeuvrability ······ 4
§3.3 IMO Standards for Ship Manoeuvrability ······ 7
§3.4 Full-scale Trials ······ 8
CHAPTER 4: PREDICTION OF SHIP MANOEUVRABILITY ······ 8
§4.1 Brief Introduction ······ 8
§4.2 Free-running Model Tests ······ 8
§4.3 Equations of Ship Manoeuvring Motion (Mathematical Models) ······ 9
§4.4 Methods for Determining the Hydrodynamic Forces Acting on a Manoeuvring Ship ······ 13
§4.5 Prediction Based on Mathematical Models and Computer Simulation ······ 16
CHAPTER 5: LINEAR EQUATIONS OF SHIP MANOEUVRING MOTION ······ 16
§5.1 Brief Introduction ······ 16
§5.2 Linear Hydrodynamic Derivatives ······ 17
§5.3 Analysis of Stability (inherent dynamic stability) ······ 20
§5.4 Nomoto Model and K,T Indexes ······ 23
§5.5 Analysis of Turning Ability Based on the Linear Equations ······ 29
CHAPTER 6: CONTROL DEVICES ······ 31
§6.1 Kinds of Control Devices ······ 31
§6.2 Rudder and Its Design ······ 33
CHAPTER 7: MEASURES TO IMPROVE SHIP MANOEUVRABILITY ······ 36
CHAPTER 8: BRIEF INTRODUCTION TO SEAKEEPING ······ 37
§8.1 Definition of Seakeeping and Its Contents ······ 37
§8.2 Ship Motions in Six Degrees of Freedom ······ 38
CHAPTER 9: OCEAN WAVES AND STATISTIC ANALYSIS ······ 38
§9.1 Brief Introduction ······ 38

§9.2 Regular Waves ··· 38
§9.3 Basic Theory of Irregular Waves ··· 40
§9.4 Theoretical Basic of Spectrum Analysis ··· 40
§9.5 Response Relationships of Linear System ··· 42
CHAPTER 10: ROLL MOTION ··· 43
§10.1 Linear Theory of Roll Motion ··· 44
§10.2 Roll Test ··· 50
§10.3 Determining the Hydrodynamic Coefficients of Roll Motion ··· 50
§10.4 Estimation of Roll Amplitude ··· 51
§10.5 Roll Control Devices ··· 52
CHAPTER 11: PITCH AND HEAVE MOTIONS ··· 53
§11.1 Theoretical Computation for Seakeeping Problems ··· 53
§11.2 Pitch and Heave in Irregular Waves ··· 56
§11.3 Derived Responses due to Ship Motion ··· 56
§11.4 Ship Motion in Oblique Waves ··· 57
CHAPTER 12: DESIGN OF SEAKEEPING PERFORMANCES AND FULL-SCALE TRIALS ··· 58
§12.1 Seakeeping Performance Index ··· 58
§12.2 Effects of Ship Principal Dimensions and Form on Seakeeping Performances ··· 59
§12.3 Full-scale Trials of Seakeeping ··· 59
REFERENCES ··· 60

CHAPTER 1: INTRODUCTION（0.5 hour）

Demand: Know the basic concepts of kinematic performances of ships in calm water and in waves; know the similarities and differences of ship manoeuvring[1] motion in calm water and ship oscillating motion in waves, as well as the immanent relationships between the two kinds of ship motion.

§1.1 Contents of the Course and Its Significance

This course is composed of two parts: ship manoeuvring and seakeeping performances. Manoeuvring and seakeeping performances are important hydrodynamic performances of ships. The course introduces the basic concepts and basic theories of ship manoeuvrability and seakeeping performance, as well as the corresponding computation methods and the methods of full-scale trials and model tests to evaluate them. The main task of the course is by means of all the teaching phases to enable the students to master the basic concepts and basic theories, and the corresponding computational and experimental methods. The emphasis is laid on training the ability, especially the ability of innovation and practice to apply the relevant theories and methods to analyze and solve the ship manoeuvring and seakeeping performances related problems in ship design, to ensure that the ship is designed with proper manoeuvring and seakeeping performances.

§1.2 Relationships of Ship Manoeuvring Motion and Motion in Waves

This course deals with ship manoeuvrability and seakeeping performance, the unsteady kinematic and dynamic performances of ship. Both performances are related to ship unsteady motions. Traditionally, ship manoeuvrability deals with the ship manoeuvring motion in the horizontal plane in calm water, whereas seakeeping deals with the ship oscillating motions in six degrees of freedom in ocean waves. Actually, there is an immanent relationship between the two kinds of motions. Especially for a sea-going ship, it will usually travel in waves and need to keep its course or change its course in waves. On the other hand, a ship undergoing manoeuvring motion in waves will undergo oscillating motions in six degrees of freedom simultaneously. Therefore, it is important to investigate the ship manoeuvring and seakeeping performances conjunctly.

CHAPTER 2: BRIEF INTRODUCTION TO SHIP MANOEUVRABILITY（0.5 hour）

Demand: Know the basic concepts of ship manoeuvrability; know the relation between manoeuvrability and navigation safety and economy.

§2.1 Definition of Ship Manoeuvrability and Its Contents

Ship manoeuvrability is the ability of a ship to keep or change its state of motion under the control actions, i.e., to keep the straight-ahead course with constant speed, or to change the speed, the course and/or the position of the ship, according to the intention of the helmsman.

Ship manoeuvrability includes the following contents:

(1) Inherent dynamic stability, also called straight line stability: A ship is dynamically stable on a straight course if it can, after a small disturbance, soon settle on a new straight course without any control actions, see Figure 2.1. The resultant deviation from the original course will depend on the degree of inherent stability of the ship and on the magnitude and duration of the disturbance. As shown in Figure 2.1, for a dynamically unstable ship, it will ultimately enter into an arbitrary unsteady turning motion.

[1] It is “manoeuvring” in British English and “maneuvering” in American English.

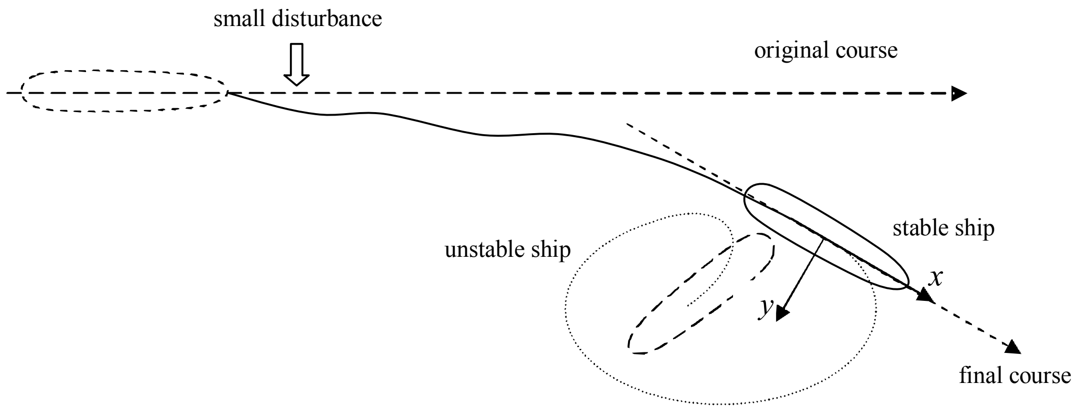


Figure 2.1 Inherent dynamic stability

(2) Course-keeping ability, also called directional stability: The course-keeping ability is the ability of the steered ship to maintain its original course direction, see Figure 2.2. A ship which has inherent dynamic stability can only maintain its original course direction under the control action. Also a ship which is dynamically unstable on straight course can maintain its original course direction by frequently applying the control action.

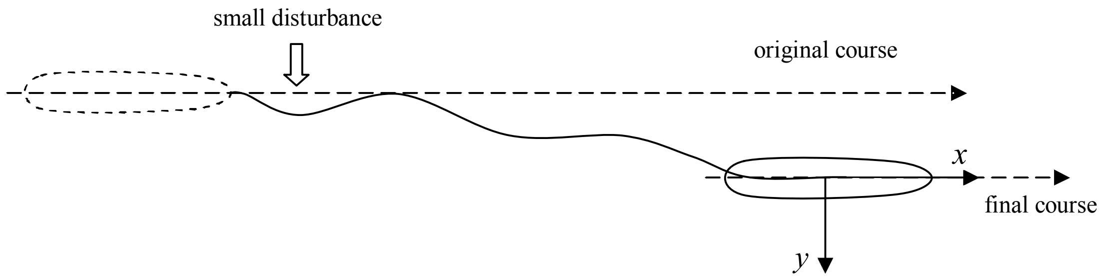


Figure 2.2 Course-keeping ability

(3) Initial turning/course-changing ability: The initial turning ability and course-changing ability are the ability of ship to change its heading as response to a control action. A ship with good initial turning ability and/or course-changing ability will quickly get into turning or change its original course after the control action.

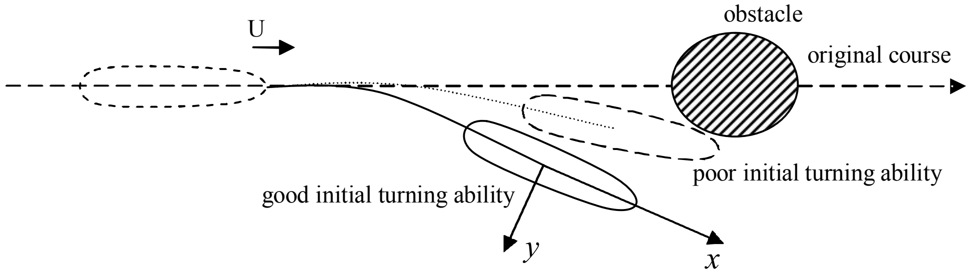
(4) Yaw checking ability: The yaw checking ability is the ability of the steered ship to respond to the counter-rudder action applied in a certain state of turning.

(5) Turning ability: The turning ability is the ability of ship to turn under the hard-over rudder action.

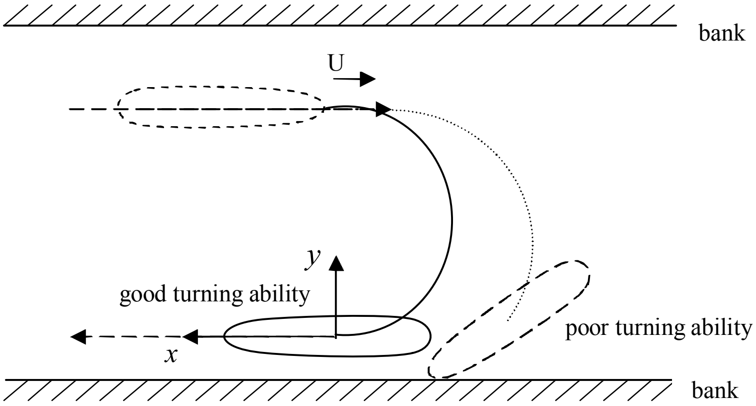
(6) Stopping ability: The stopping ability is the ability of ship to stop with engine stopped (inertia stop) or engine-full astern (crash stop) after a steady approach at full speed.

§2.2 Significance of Ship Manoeuvrability

Ship manoeuvrability is directly related to navigation safety and economy. For a ship manoeuvring under severe environmental conditions and/or in restricted water, marine disasters may occur if the ship has not adequate manoeuvrability. For example, for a ship with poor initial turning ability or poor turning ability, collision with obstacles in the waterway or with the bank of the narrow waterway may not be avoided, as shown in Figure 2.3.



(a) Relation between initial turning ability and navigation safety



(b) Relation between turning ability and navigation safety

Figure 2.3 Relation between ship manoeuvrability and navigation safety

On the other hand, since a dynamically unstable ship or a ship with poor dynamic stability can only keep its course by frequently repeated use of the control device, for such a ship not only the voyage is longer than the planned one, as shown in Figure 2.4, but also more energy is needed to consume by the control device machine.

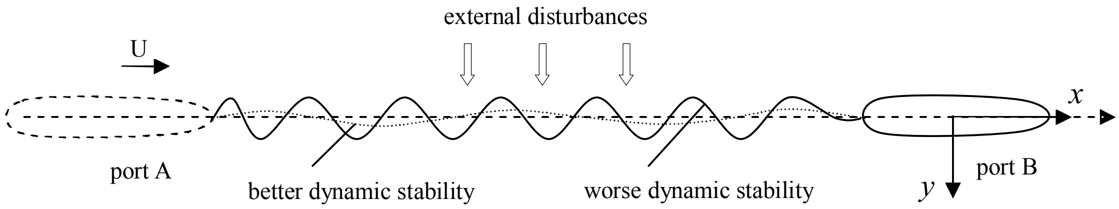


Figure 2.4 Relation between ship manoeuvrability and navigation economy

Actually, ship manoeuvrability itself is not the only factor which affects a safe navigation. Other two factors, the environmental conditions such as wind, waves and current, and the human factor, contribute a lot to navigation safety by influencing the ship manoeuvring in different manner. Though the human factor is the most important factor which affects navigation safety, and the effect of environmental conditions on ship manoeuvrability is an important issue in study of ship manoeuvrability, only the inherent ship manoeuvrability will be dealt with in this course.

CHAPTER 3: EVALUATION OF SHIP MANOEUVRABILITY（1.5 hours）

Demand: Know the various kinds of standard manoeuvres and the parameters used to evaluate ship manoeuvrability; know the definition of the parameters, the methods recommended to obtain the parameters and the quantitative demands of the parameters in the IMO (International Maritime Organization) "Standards for Ship Manoeuvrability".

§3.1 Brief Introduction

Ship manoeuvrability is an important hydrodynamic performance relating to navigation safety. A ship should have adequate manoeuvrability to ensure a safe navigation. To design a ship with good manoeuvrability, it is important for us to judge if a ship has good or poor manoeuvrability. For this purpose some standard manoeuvres are proposed. From these manoeuvres the significant qualities for evaluating ship manoeuvrability can be derived.

§3.2 Standard Manoeuvres and the Parameters Evaluating Ship Manoeuvrability

§3.2.1 Turning test

The turning test is performed to evaluate the turning ability. A turning circle manoeuvre is to be performed to both starboard and port with 35° rudder angle or the maximum design rudder angle permissible at the test speed. The rudder angle is executed following a steady approach with zero yaw rate. The kinematic parameters and the path of the midship point are recorded during the test, see Figure 3.1.

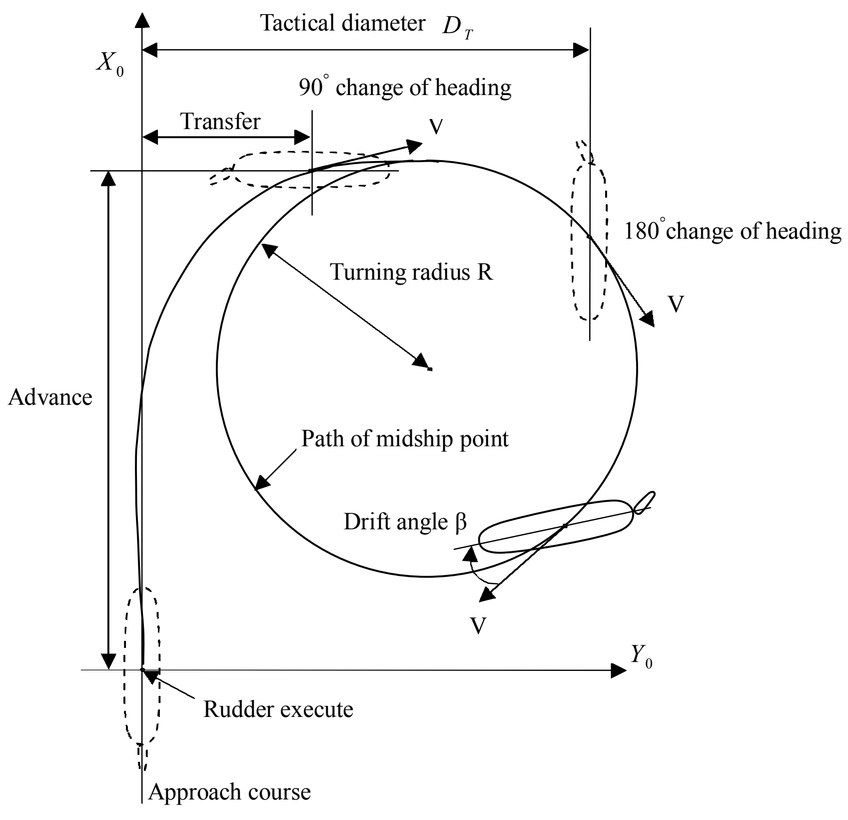


Figure 3.1 Path of the midship point during turning

The parameters obtained from this manoeuvre for evaluating the turning ability are the tactical diameter, the advance and the transfer.

§3.2.2 Zig-zag test

The zig-zag test is performed to evaluate the initial turning, the yaw-checking and the course-keeping abilities. A zig-zag test is to be initiated to both starboard and port and begins by applying a specified amount of rudder angle. The rudder angle is then alternately shifted to either side after a specified deviation from the original heading of the ship is reached[2].

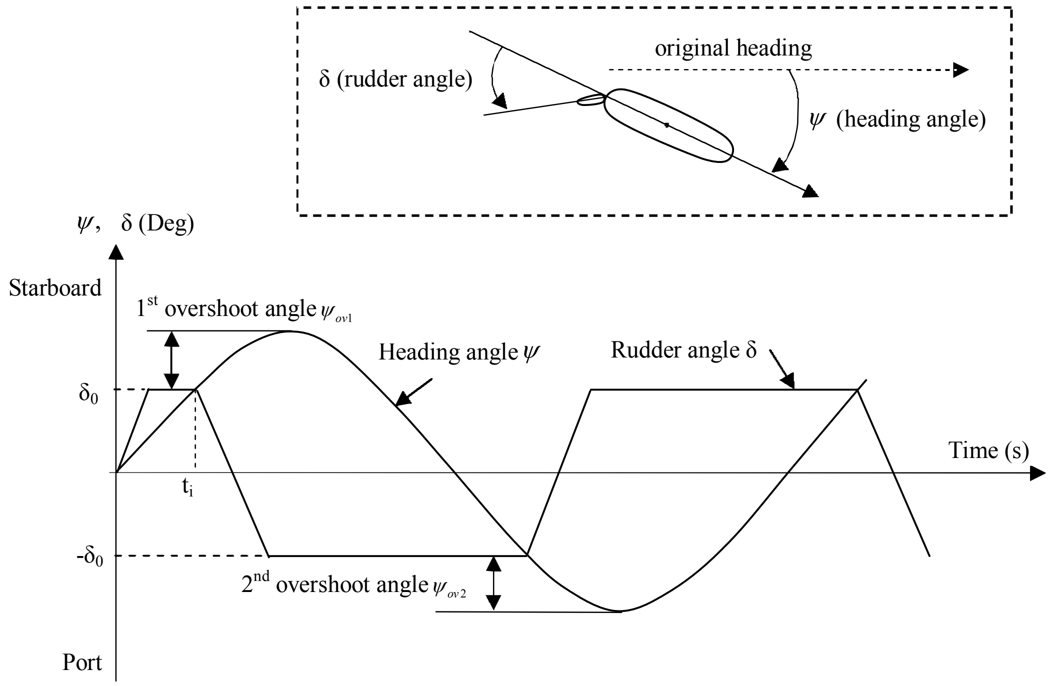


Figure 3.2 Time histories of the rudder angle and heading angle during zig-zag test

Usually two kinds of zig-zag tests, the 10°/10° and 20°/20° zig-zag tests are performed. The 10°/10° zig-zag test uses 10° rudder angles to either side following a heading deviation of 10° from the original course. The 20°/20° zig-zag test uses 20° rudder angles and 20° heading deviation from the original course. The parameters obtained from this manoeuvre for evaluating the manoeuvring characteristics are the overshoot angles ψ_{ov}, the initial turning time to second execute t_i and the time to check yaw, see Figure 3.2.

§3.2.3 Stopping test

The stopping test is performed to evaluate the stopping ability. A full astern stopping test is conducted to determine the track reach of ship from the time when an astern order is given until the ship is stopped dead in the water. The track reach is the length measured along the path of midship point, see Figure 3.3.

2 The zig-zag manoeuvre was first proposed by the German scientist Günther Kempf in 1932.

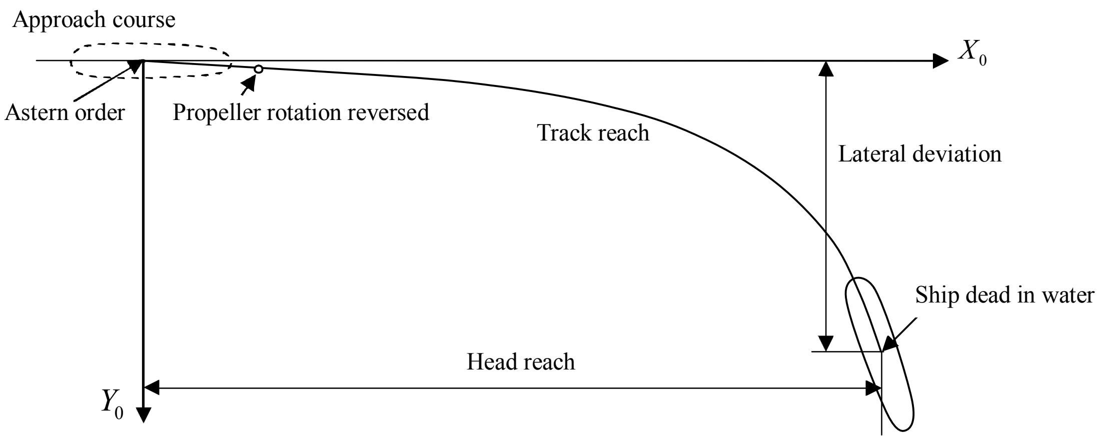


Figure 3.3 Path of the midship point during stopping test

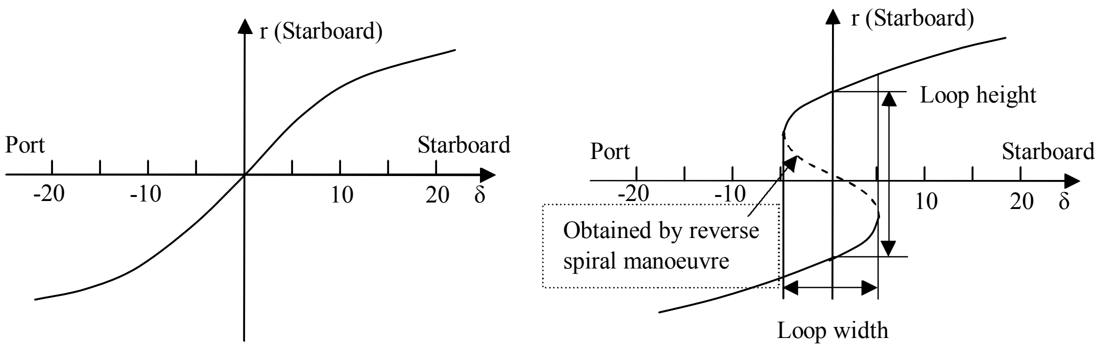
§3.2.4 Spiral test

There are two kinds of spiral test, the direct spiral test, also called Dieudonné's spiral test[3], and the reverse spiral test, also called Bech's reverse spiral test[4], which are performed to evaluate the ship dynamic stability and course-keeping ability.

The direct spiral test is an orderly series of turning circle tests. The rudder angle changes in sequence of +15°→ +10°→ +5°→ 0°→ −5°→ −10°→ −15°→ −10°→ −5°→0°→ +5°→ +10°→ +15°. The test result is expressed as steady turning rate versus rudder angle curve, as shown in Figure 3.4. This test is time consuming and sensitive to weather conditions. Moreover, for a dynamically unstable ship, within a small range of rudder angle the ship may still turn steadily in the original direction although the rudder is already deflected to the other side. In this case the curve of steady turning rate versus rudder angle will change abruptly, as shown in Figure 3.4(b).

In the reverse spiral test, the ship is steered to obtain a constant turning rate and the mean rudder angle required to maintain this rate is measured. The reverse spiral test provides a more rapid procedure to obtain the steady turning rate versus rudder angle curve than the direct spiral test, and can obtain the complete curve for a dynamically unstable ship, as shown in Figure 3.4(b). The loop height and loop width in Figure 3.4(b) reflect how unstable the ship is, therefore Figure 3.4(b) obtained from spiral tests can be used to evaluate the instability of ship.

[3] The direct spiral test was first proposed by the French scientist Jean Dieudonné in 1949-1950 and published in 1953.
[4] The reverse spiral test was first proposed by Mogens Bech in 1966 and published in 1968.

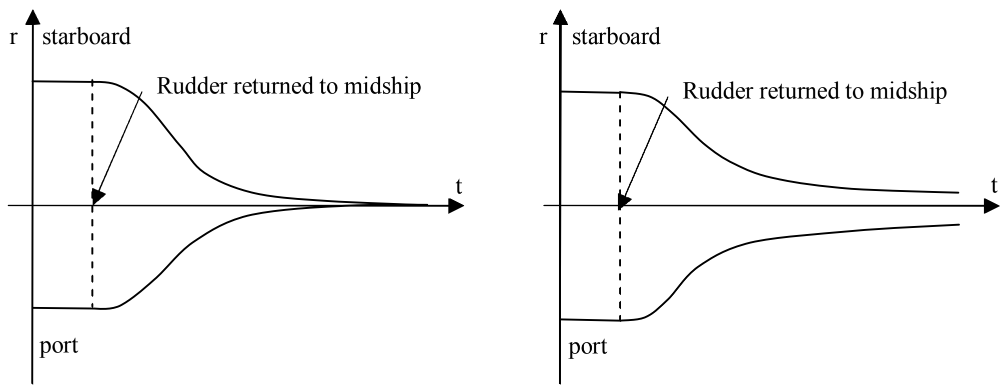


(a) r ~ δ curve for a dynamically stable ship (b) r ~ δ curve for a dynamically unstable ship

Figure 3.4 r ~ δ curve obtained from spiral tests

§3.2.5 Pull-out test

Pull-out manoeuvre was proposed for evaluating the dynamic stability of a ship[5]. It is conducted after completion of a turning circle manoeuvre to starboard or port with rudder angle of approximately 20° by returning the rudder to the midship position and keeping it there until a steady turning rate is reached. For a stable ship, the steady turning rate will decay to zero, whereas for an unstable ship the rate will be a certain value, the so-called residual turning rate, which is a measurement of the instability. Usually, the residual turning rates to starboard and port will be different, as shown in Figure 3.5(b).



(a) Stable ship (b) Unstable ship

Figure 3.5 Time histories of the turning rate during pull-out manoeuvre

§3.3 IMO Standards for Ship Manoeuvrability

Ships with very poor manoeuvrability may result in marine disasters and environmental pollution. To avoid that ships are built with very poor manoeuvrability, International Maritime Organization (IMO), the international authoritative organization responsible for maritime safety and environment protection, has made long-term efforts to develop criteria and evaluation methods for ship manoeuvrability since late 60s last century. In December 2002, IMO adopted the resolution MSC.137 (76), "Standards for Ship Manoeuvrability". The standards are to be used to evaluate the ship manoeuvrability and to assist those responsible for ship design, construction, repair and operation to ensure that ships comply with the standards.

5 The pull-out test was first proposed by Roy Burcher in 1969.

According to the "Standards for Ship Manoeuvrability", a ship is considered satisfactory if the following criteria are complied with:

1. Turning ability
   The advance should not exceed 4.5 ship lengths and the tactical diameter should not exceed 5 ship lengths in the turning circle manoeuvre.
2. Initial turning ability
   With the application of 10° rudder angle to starboard/port, the ship should not have traveled more than 2.5 ship lengths by the time when the heading has changed by 10° from the original heading. (It is equivalent to require that the initial turning time to second execute in 10°/10° zig-zag test satisfies t_i<2.5L/V, where L is ship length and V is ship speed).
3. Yaw-checking and course-keeping abilities

3.1 The first overshoot angle in 10°/10° zig-zag test should not exceed 10° if L/V is less than 10 s; 20° if L/V is 30 s or more; and 5+1/2(L/V) degrees if L/V is 10 s or more, but less than 30 s.

3.2 The second overshoot angle in 10°/10° zig-zag test should not exceed 25° if L/V is less than 10 s; 40° if L/V is 30 s or more; and 17.5+0.75(L/V) degrees if L/V is 10 s or more, but less than 30 s.

3.3 The first overshoot angle in 20°/20° zig-zag test should not exceed 25°.

4. Stopping ability
   The track reach in the full astern stopping test should not exceed 15 ship lengths. However, this value may be modified by the administration where ships of large displacement make this criterion impracticable, but should in no case exceed 20 ship lengths.

§3.4 Full-scale Trials

To demonstrate the compliance of a new-built ship with the IMO standards, full-scale trials of standard manoeuvres are to be conducted before delivery. The characteristics are measured from the test records and are to be compared with the IMO criteria.

CHAPTER 4: PREDICTION OF SHIP MANOEUVRABILITY（3.5 hours）

Demand: Know the various kinds of free-running model tests and master the methods to obtain the parameters evaluating manoeuvrability from the tests; know how to derive the equations of ship manoeuvring motion; know the various kinds of methods for determining the hydrodynamic forces acting on a manoeuvring ship; master the method of manoeuvrability prediction by means of the mathematical model and computer simulation technique.

§4.1 Brief Introduction

According to the IMO "Standards for Ship Manoeuvrability", manoeuvring performance of a ship should be designed to comply with the standards during the design stage. Therefore, prediction of the ship manoeuvring performance should be conducted at the early design stage. At present, there are two methods for this purpose. The first one is the method of free-running model tests, and the second one is the method of computer simulation using the mathematical models.

§4.2 Free-running Model Tests

Free-running model tests are conducted with a scale model. The standard manoeuvres are performed and the parameters evaluating manoeuvring characteristics are measured directly from the test records. This kind of method is the most direct method for predicting the manoeuvring performance and is regarded as the most reliable prediction method. However, the method is time-consuming and costs a lot of money, and is inconvenient for applying at the design stage, since series of model tests are usually needed to conduct. Moreover, it should be noted that due to the so-called scale effects, there exists usually a distinct difference between the predicted performance and the actual one.

§4.3 Equations of Ship Manoeuvring Motion (Mathematical Models)

With the rapid development of the computer technology and its successful application in ship engineering, the method of computer simulation using the mathematical models becomes more and more popular. It provides a convenient tool for predicting ship manoeuvrability at the design stage. One of the preconditions for applying this kind of method is modeling of the equations describing the manoeuvring motion.

§4.3.1 Coordinate systems

To investigate ship manoeuvrability by means of mathematical tool, two right-handed coordinate systems are adopted: the earth-fixed coordinate system $o_0\text{-}x_0y_0z_0$ and the body-fixed coordinate system $o\text{-}xyz$ which moves together with the ship, see Figure 4.1. The $o_0\text{-}x_0y_0$ plane and the $o\text{-}xy$ plane lie on the undisturbed free surface, with the x_0 axis pointing to the direction of the original course of the ship, whereas the z_0 axis and the z axis point downwards vertically. The angle between the directions of x_0 axis and x axis is defined as the heading angle, ψ. At the moment as manoeuvring motion is getting start, the two coordinate systems coincide with each other. At any later moment, the position of the ship is determined by the coordinates x_{0G} and y_{0G} of the ship center of gravity in the earth-fixed coordinate system, and the orientation of the ship is determined by the heading angle ψ.

The ship manoeuvring motion in the horizontal plane is described by the speed $\vec{V}$ of translational motion and the yaw rate $r = \dot{\psi}$ of rotational motion about the z axis. The components of the speed $\vec{V}$ in the directions of x axis and y axis are u and v, respectively. The angle between the directions of speed $\vec{V}$ and x axis is defined as the drift angle, β. Obviously we have

$$u = V\cos\beta, \quad v = -V\sin\beta; \qquad \text{with } V = \left|\vec{V}\right|.$$

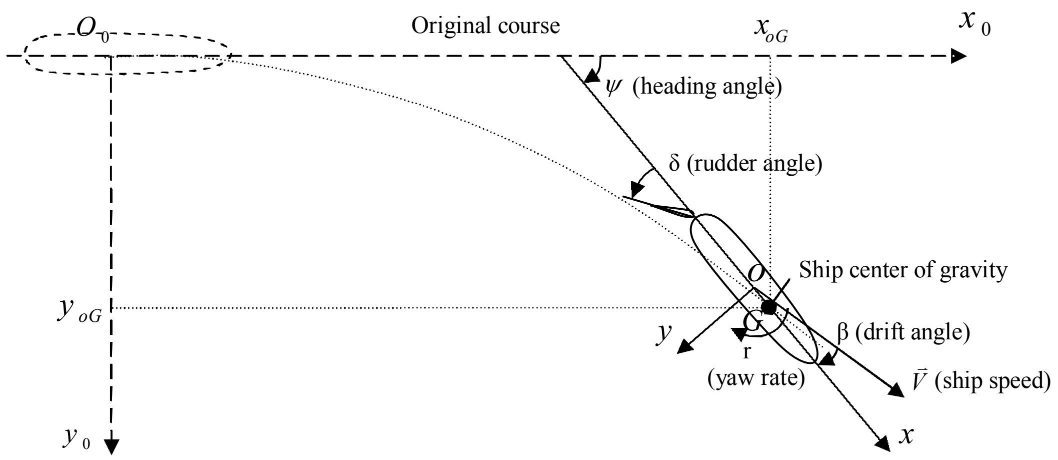


Figure 4.1 Coordinate systems

§4.3.2 Mathematical models

Here "mathematical models" mean the equations of ship manoeuvring motion. The equations of motion can be derived according to the Newton second law, the Newtonian law of motion.

The earth-fixed coordinate system is an inertial frame in which the Newtonian law of motion is valid. In this coordinate system, we have the equations of motion as follows.

$$\begin{aligned} X_0 &= m\ddot{x}_{0G}, \\ Y_0 &= m\ddot{y}_{0G}, \\ N_0 &= I_{zG}\ddot{\psi}. \end{aligned} \tag{4.1}$$

where X_0 and Y_0 are the components of external force acting on the ship in the directions of x_0 axis and y_0 axis, respectively; N_0 is the component of external moment about the z axis; m is the mass of the ship; I_{zG} is the moment of inertia of the ship about z axis; $\ddot{x}_{0G}$ and $\ddot{y}_{0G}$ are the components of acceleration in the directions of x_0 axis and y_0 axis, respectively; $\ddot{\psi}$ is the yaw acceleration.

For purpose of ship manoeuvrability prediction, it is more convenient to use the equations of motion in the body-fixed coordinate system. To establish these equations, the relations between the kinematic parameters defined in the earth-fixed and the body-fixed coordinate systems are to be utilized. With the original of the body-fixed coordinate system lying on the center of gravity, we have

$$\begin{aligned} x_0 &= x_{0G} + x\cos\psi - y\sin\psi, \\ y_0 &= y_{0G} + x\sin\psi + y\cos\psi, \\ z_0 &= z. \end{aligned}$$

or in reverse,

$$\begin{aligned} x &= \ \ (x_0 - x_{0G})\cos\psi + (y_0 - y_{0G})\sin\psi, \\ y &= -(x_0 - x_{0G})\sin\psi + (y_0 - y_{0G})\cos\psi, \\ z &= z_0. \end{aligned}$$

Denoting $x_0' = x_0 - x_{0G}$, $y_0' = y_0 - y_{0G}$ and $z_0' = z_0$, we have

$$\begin{aligned} x_0' &= x\cos\psi - y\sin\psi, \\ y_0' &= x\sin\psi + y\cos\psi, \\ z_0' &= z. \end{aligned}$$

and in reverse,

$$\begin{aligned} x &= x_0'\cos\psi + y_0'\sin\psi, \\ y &= -x_0'\sin\psi + y_0'\cos\psi, \\ z &= z_0'. \end{aligned}$$

Denoting the components of external force in the directions of x axis and y axis by X and Y, and the components of ship speed in the directions of x axis and y axis by u_G and v_G, respectively, we have

$$\begin{aligned} X &= X_0\cos\psi + Y_0\sin\psi, \\ Y &= -X_0\sin\psi + Y_0\cos\psi. \end{aligned} \tag{4.2}$$

$$\begin{aligned} \dot{x}_{0G} &= u_G\cos\psi - v_G\sin\psi, \\ \dot{y}_{0G} &= u_G\sin\psi + v_G\cos\psi. \end{aligned} \tag{4.3}$$

Differentiating Equation (4.3) with respect to time yields

$$\begin{aligned} \ddot{x}_{0G} &= \dot{u}_G \cos\psi - u_G\,\dot{\psi}\sin\psi - \dot{v}_G \sin\psi - v_G\,\dot{\psi}\cos\psi, \\ \ddot{y}_{0G} &= \dot{u}_G \sin\psi + u_G\,\dot{\psi}\cos\psi + \dot{v}_G \cos\psi - v_G\,\dot{\psi}\sin\psi. \end{aligned} \tag{4.4}$$

Substituting Equations (4.1) and (4.4) into Equation (4.2), we obtain the equations of motion in the body-fixed coordinate system, with the original of the system lying on the ship center of gravity, as

$$\begin{aligned} X &= m(\dot{u}_G - v_G\dot{\psi}), \\ Y &= m(\dot{v}_G + u_G\dot{\psi}). \end{aligned} \tag{4.5}$$

On the other hand, since the external moment about the z axis through the center of gravity is same in the body-fixed coordinate system as in the earth-fixed coordinate system, we have the equation of yaw motion in the body-fixed coordinate system as Equation (4.1).

In practice it is more convenient when the original of the body-fixed coordinate system does not lie on the center of gravity, but on the midship point. Assuming that the ship is symmetrical about its longitudinal centerplane, the center of gravity has the coordinates ($x_G, 0, z_G$) in the body-fixed coordinate system with the original lying on the midship point. In such a coordinate system the components of ship speed at the center of gravity, u_G and v_G, and at the original, u and v, have the relations as follows

$$u_G = u, \quad v_G = v + x_G\dot{\psi}.$$

Therefore, we obtain the equations of motion in the body-fixed coordinate system with the original lying on the midship point in the form

$$\begin{aligned} X &= m(\dot{u} - v\dot{\psi} - x_G\dot{\psi}^2), \\ Y &= m(\dot{v} + u\dot{\psi} + x_G\ddot{\psi}). \end{aligned} \tag{4.6}$$

On the other hand, since

$$N_0 = N - Y \cdot x_G, \quad I_z = I_{zG} + m\,x_G^2,$$

where N is the external moment and I_z is the moment of inertia about the z axis, respectively. From the third equation of Equation (4.1) we have the equation of yaw motion in the body-fixed coordinate system with the original lying on the midship point as follows

$$N = I_z\ddot{\psi} + m\,x_G(\dot{v} + u\dot{\psi}).$$

As a result, we have the equations of motion in the body-fixed coordinate system, with the original of the coordinate system lying on the midship point, in the form

$$\begin{aligned} X &= m(\dot{u} - vr - x_G r^2), \\ Y &= m(\dot{v} + ur + x_G\dot{r}), \\ N &= I_z\dot{r} + m\,x_G(\dot{v} + ur). \end{aligned} \tag{4.7}$$

where $r = \dot{\psi}$ is the yaw rate about the z axis.

§4.3.3 Expressions of the hydrodynamic forces

X, Y and N in Equation (4.7) are the components of external force and moment acting on the ship. These force and moment include the hydrodynamic force and moment in calm water as well as various environmental exciting force and moment due to wind, waves and current, etc. In this course we will only deal with the hydrodynamic force and moment in calm water.

There are two kinds of manner for expression of the hydrodynamic force and moment, one is that introduced first by Prof. Martin A. Abkowitz from MIT in USA and the other one is due to the Mathematical Modeling Group (MMG) from Japan.

In 1964, Prof. Abkowitz proposed a method for expression of the hydrodynamic force and moment by using the Taylor expansion. The hydrodynamic force and moment are expressed as functions of the kinematical parameters and the rudder angle in the form

$$X = X(u, v, r, \dot{u}, \dot{v}, \dot{r}, \delta),$$
$$Y = Y(u, v, r, \dot{u}, \dot{v}, \dot{r}, \delta),$$
$$N = N(u, v, r, \dot{u}, \dot{v}, \dot{r}, \delta),$$

and then expanded in Taylor series about the initial steady state of forward motion with constant speed, i.e., $u_0 = U, v_0 = 0, r_0 = 0, \dot{u}_0 = 0, \dot{v}_0 = 0, \dot{r}_0 = 0, \delta_0 = 0.$ It results in

$$\begin{aligned} X = X_0 &+ \frac{\partial X}{\partial u}(u-U) + \frac{\partial X}{\partial v}v + \frac{\partial X}{\partial r}r + \frac{\partial X}{\partial \dot{u}}\dot{u} + \frac{\partial X}{\partial \dot{v}}\dot{v} + \frac{\partial X}{\partial \dot{r}}\dot{r} + \frac{\partial X}{\partial \delta}\delta \\ &+ \frac{1}{2!}\left[\frac{\partial}{\partial u}(u-U) + \frac{\partial}{\partial v}v + \frac{\partial}{\partial r}r + \frac{\partial}{\partial \dot{u}}\dot{u} + \frac{\partial}{\partial \dot{v}}\dot{v} + \frac{\partial}{\partial \dot{r}}\dot{r} + \frac{\partial}{\partial \delta}\delta\right]^2 X \\ &+ \cdots + \frac{1}{n!}\left[\frac{\partial}{\partial u}(u-U) + \frac{\partial}{\partial v}v + \frac{\partial}{\partial r}r + \frac{\partial}{\partial \dot{u}}\dot{u} + \frac{\partial}{\partial \dot{v}}\dot{v} + \frac{\partial}{\partial \dot{r}}\dot{r} + \frac{\partial}{\partial \delta}\delta\right]^n X + \cdots, \end{aligned} \tag{4.8a}$$

$$\begin{aligned} Y = Y_0 &+ \frac{\partial Y}{\partial u}(u-U) + \frac{\partial Y}{\partial v}v + \frac{\partial Y}{\partial r}r + \frac{\partial Y}{\partial \dot{u}}\dot{u} + \frac{\partial Y}{\partial \dot{v}}\dot{v} + \frac{\partial Y}{\partial \dot{r}}\dot{r} + \frac{\partial Y}{\partial \delta}\delta \\ &+ \frac{1}{2!}\left[\frac{\partial}{\partial u}(u-U) + \frac{\partial}{\partial v}v + \frac{\partial}{\partial r}r + \frac{\partial}{\partial \dot{u}}\dot{u} + \frac{\partial}{\partial \dot{v}}\dot{v} + \frac{\partial}{\partial \dot{r}}\dot{r} + \frac{\partial}{\partial \delta}\delta\right]^2 Y \\ &+ \cdots + \frac{1}{n!}\left[\frac{\partial}{\partial u}(u-U) + \frac{\partial}{\partial v}v + \frac{\partial}{\partial r}r + \frac{\partial}{\partial \dot{u}}\dot{u} + \frac{\partial}{\partial \dot{v}}\dot{v} + \frac{\partial}{\partial \dot{r}}\dot{r} + \frac{\partial}{\partial \delta}\delta\right]^n Y + \cdots, \end{aligned} \tag{4.8b}$$

$$\begin{aligned} N = N_0 &+ \frac{\partial N}{\partial u}(u-U) + \frac{\partial N}{\partial v}v + \frac{\partial N}{\partial r}r + \frac{\partial N}{\partial \dot{u}}\dot{u} + \frac{\partial N}{\partial \dot{v}}\dot{v} + \frac{\partial N}{\partial \dot{r}}\dot{r} + \frac{\partial N}{\partial \delta}\delta \\ &+ \frac{1}{2!}\left[\frac{\partial}{\partial u}(u-U) + \frac{\partial}{\partial v}v + \frac{\partial}{\partial r}r + \frac{\partial}{\partial \dot{u}}\dot{u} + \frac{\partial}{\partial \dot{v}}\dot{v} + \frac{\partial}{\partial \dot{r}}\dot{r} + \frac{\partial}{\partial \delta}\delta\right]^2 N \\ &+ \cdots + \frac{1}{n!}\left[\frac{\partial}{\partial u}(u-U) + \frac{\partial}{\partial v}v + \frac{\partial}{\partial r}r + \frac{\partial}{\partial \dot{u}}\dot{u} + \frac{\partial}{\partial \dot{v}}\dot{v} + \frac{\partial}{\partial \dot{r}}\dot{r} + \frac{\partial}{\partial \delta}\delta\right]^n N + \cdots, \end{aligned} \tag{4.8c}$$

where X_0, Y_0 and N_0 as well as all the derivatives take the value at the initial steady state of forward motion with $u_0 = U, v_0 = 0, r_0 = 0, \dot{u}_0 = 0, \dot{v}_0 = 0, \dot{r}_0 = 0, \delta_0 = 0.$

The derivatives in Equation (4.8) are called hydrodynamic derivatives. For simplicity these derivatives are usually expressed as

$$\frac{\partial X}{\partial u} = X_u, \quad \frac{\partial X}{\partial v} = X_v, \quad \frac{\partial X}{\partial r} = X_r, \quad \frac{\partial X}{\partial \dot{u}} = X_{\dot{u}}, \quad \frac{\partial X}{\partial \dot{v}} = X_{\dot{v}}, \quad \frac{\partial X}{\partial \dot{r}} = X_{\dot{r}}, \quad \frac{\partial X}{\partial \delta} = X_\delta; \cdots.$$

Substituting Equation (4.8) into Equation (4.7), the equations of ship manoeuvring motion are derived. This kind of equations is called Abkowitz model, or whole ship model.

Another method for expression of the hydrodynamic force and moment was proposed by the Japanese Mathematical Modeling Group, JMMG, in the late 1970s last century. In this expression the hydrodynamic force and moment are decomposed into three parts, i.e., the parts acting on the ship hull, the propeller and the rudder, respectively, in the form

$$\begin{aligned} X &= X_H + X_P + X_R, \\ Y &= Y_H + Y_P + Y_R, \\ N &= N_H + N_P + N_R, \end{aligned} \tag{4.9}$$

where the subscripts "H", "P" and "R" denote the hull, the propeller and the rudder, respectively.

Referring to the Abkowitz model, expressions for the components of hydrodynamic force and moment in Equation (4.9) in form of hydrodynamic derivatives as well as the coefficients accounting for the interaction effects of the hull, propeller and rudder were proposed. Substituting Equation (4.9) into Equation (4.7), the equations of ship manoeuvring motion are derived. This kind of equations is called MMG model, or modular model.

§4.4 Methods for Determining the Hydrodynamic Forces Acting on a Manoeuvring Ship

Obviously, as a precondition to use the equations of motion to simulate the manoeuvring motion, the hydrodynamic derivatives in the equations should be determined. There are various methods for this purpose. In the following, the basic methods which can be used to determine the hydrodynamic force and moment acting on a ship in manoeuvring motion will be briefly introduced.

§4.4.1 Captive model tests

Captive model tests are tests conducted with a scale model in ship model basins, where the models are forced to move in a prescribed manner. These tests include oblique-towing test in a conventional long and narrow towing tank, as shown in Figure 4.2, rotating-arm test in a rotating-arm facility, as shown in Figure 4.3, and the planar motion test using Planar Motion Mechanism (PMM) in a long and narrow towing tank as shown in Figure 4.4(a), (b) and (c), as well as Circular Motion Test (CMT) in a big towing tank or in a seakeeping and manoeuvring basin, see Figure 4.5. By analyzing the forces and moments measured on the model, the hydrodynamic derivatives can be determined.

It should be pointed out that captive model tests suffer from scale effects as in the free-running model tests, therefore extrapolation corrections are usually needed.

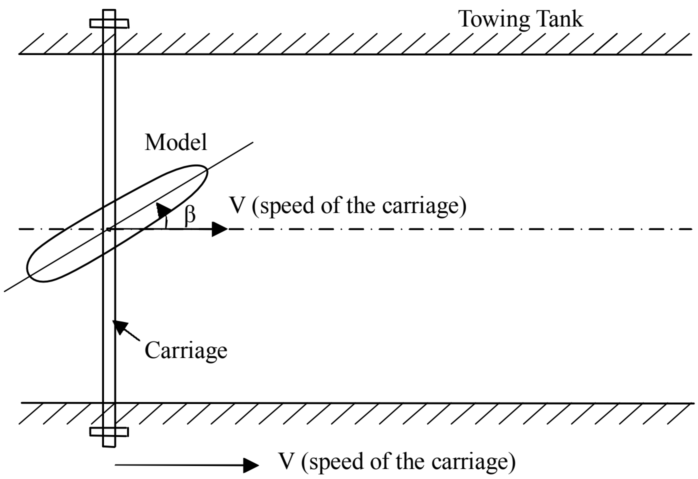


Figure 4.2 Oblique-towing test in towing tank

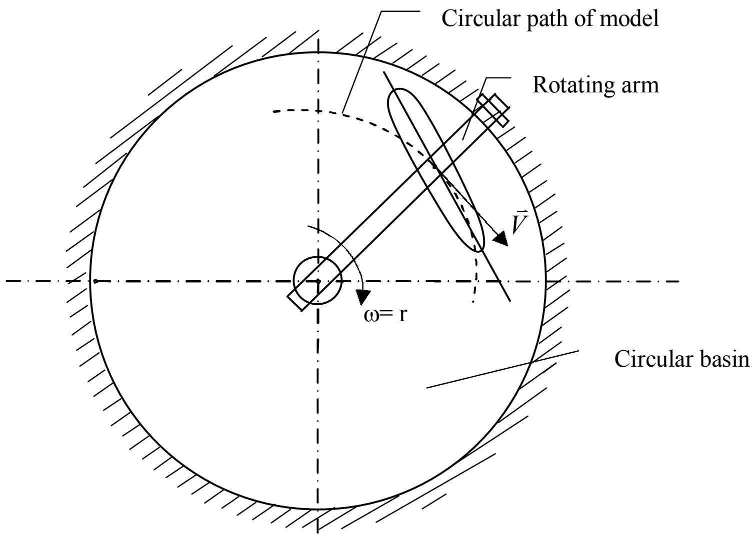
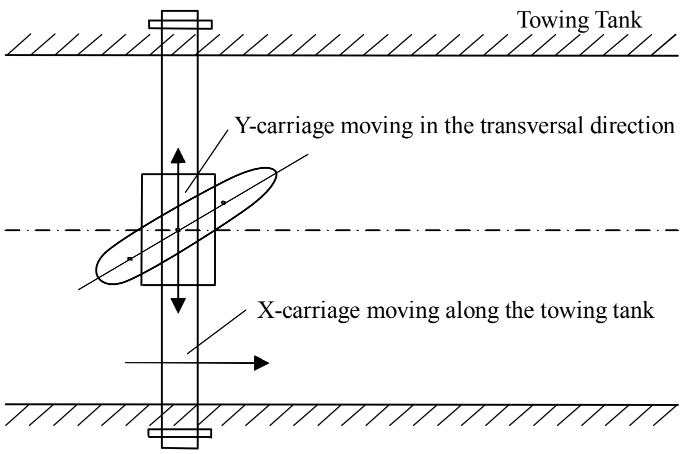
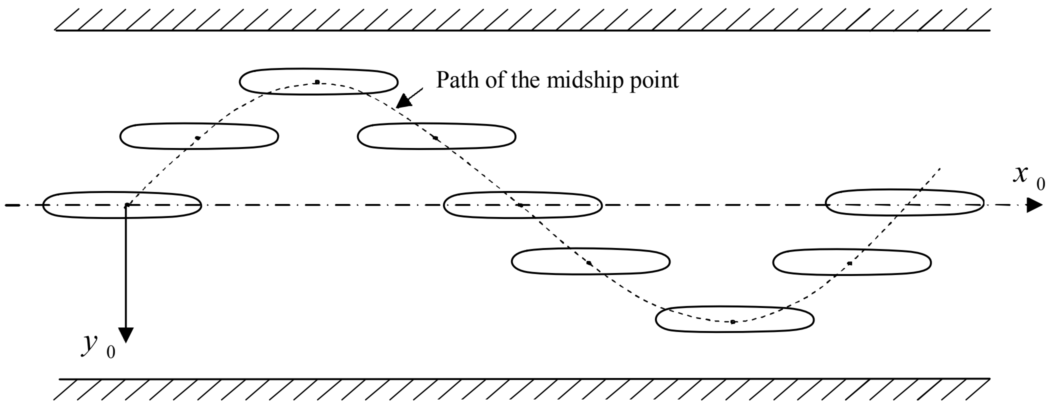


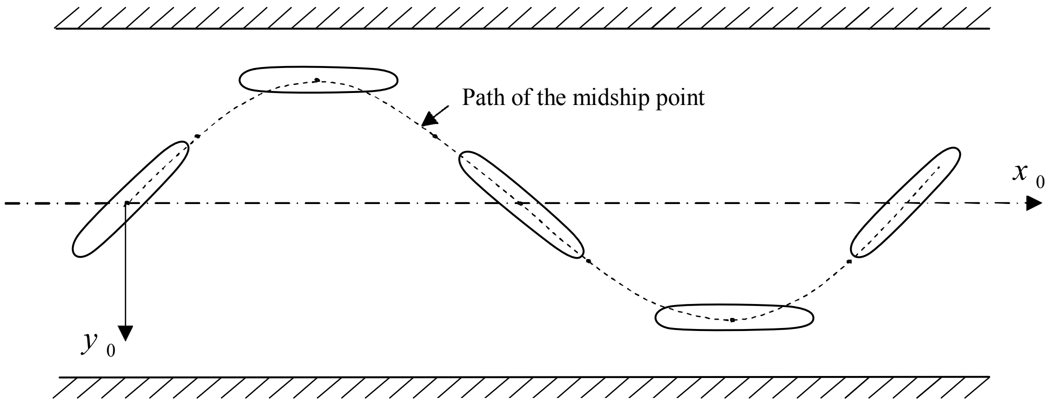
Figure 4.3 Rotating-arm test in rotating-arm facility



(a) PMM in a towing tank



(b) Pure sway test



(c) Pure yaw test

Figure 4.4 PMM tests in a towing tank

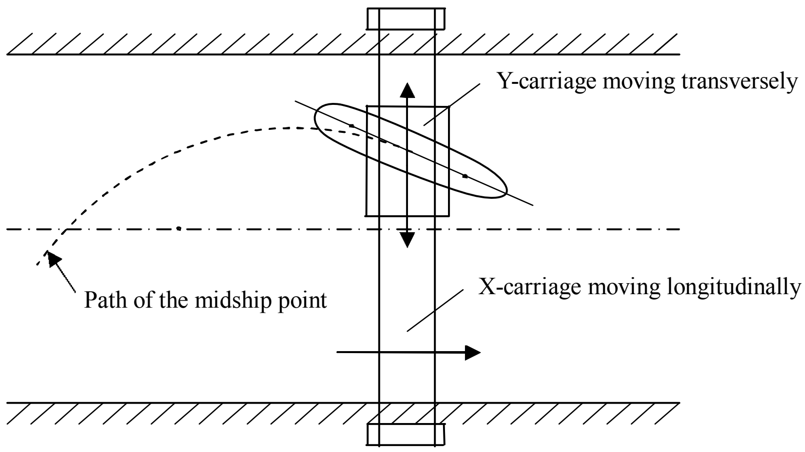


Figure 4.5 Circular Motion Test

§4.4.2 System identification technique

The hydrodynamic derivatives in the equations of motion can also be determined from the full-scale trials or free-running model tests by system identification methods. The data of control parameters such as rudder angle are measured and used as input, whereas the data of kinematic parameters such as speed, acceleration are measured and used as output, then the system parameters, i.e., the hydrodynamic derivatives in the dynamic model, the equations of motion, are determined by means of the parameter identification methods.

§4.4.3 Semi-theoretical and semi-empirical methods

The hydrodynamic derivatives can also be determined by semi-theoretical and semi-empirical methods. These methods include the methods of data base and the estimation methods using semi-empirical formulae. Systematical captive model tests need to be conducted, and the data of measured hydrodynamic force and moment are to be gathered, from which a data base or semi-empirical formulae can be derived. Then the data base and the semi-empirical formulae can be used to estimate the hydrodynamic derivatives in the equations of motion conveniently.

§4.4.4 Numerical methods

The hydrodynamic force and moment acting on a ship in manoeuvring motion can be calculated by theoretical and/or numerical methods. Traditionally, simple methods based on potential theory such as the low-aspect-ratio wing theory and the slender-body theory are used to calculate the hydrodynamic force and moment. With the rapid development of computer technology and numerical methods in the field of fluid dynamics, Computational Fluid Dynamics (CFD) technique has become more and more popular in the field of ship hydrodynamics. Complete three-dimensional numerical methods are now available for calculating the hydrodynamic force and moment acting on a ship in manoeuvring motion. These 3D numerical methods can be classified into two categories, i.e., the methods based on potential theory, such as Boundary Element Method (BEM), also called panel method, and the viscous flow methods, such as the method based on solution of Reynolds-Averaged Navier-Stokes (RANS) Equations by means of, for example, Finite Volume Method (FVM). Great progress has been achieved in this aspect during the last decade.

§4.5 Prediction Based on Mathematical Models and Computer Simulation

Compared with the method of free-running model tests, the method of numerical simulation using mathematical models is a useful and convenient method for predicting ship manoeuvrability at the design stage. Once the hydrodynamic derivatives are determined, the equations of ship manoeuvring motion, which are a set of differential equations, can be solved by numerical methods such as the Runge-Kutta methods to determine the components of manoeuvring motion $u(t)$, $v(t)$ and $r(t)$ at any time t. Then the position and the orientation of the ship can be determined by the coordinates of the ship center of gravity in the earth-fixed coordinate system and the heading angle of the ship, which can be obtained by numerical integrations as follows

$$x_{0G}(t)=\int_0^t \dot{x}_{0G}(t)\,dt,\quad y_{0G}(t)=\int_0^t \dot{y}_{0G}(t)\,dt;\quad \psi(t)=\int_0^t \dot{\psi}(t)\,dt; \tag{4.10}$$

where

$$\begin{aligned}
&\dot{x}_{0G}(t)=u(t)\cos\psi(t)-[v(t)+x_G r(t)]\sin\psi(t),\\
&\dot{y}_{0G}(t)=u(t)\sin\psi(t)+[v(t)+x_G r(t)]\cos\psi(t),\\
&\dot{\psi}(t)=r(t).
\end{aligned} \tag{4.11}$$

In this way, any standard manoeuvres can be simulated by a computer and the manoeuvring characteristics of the ship can be predicted.

CHAPTER 5: LINEAR EQUATIONS OF SHIP MANOEUVRING MOTION（4 hours）

Demand: Know the physical meanings and characteristics of the linear hydrodynamic derivatives in the equations of ship manoeuvring motion; master the method of analyzing the course-keeping ability and turning ability based on the linear equations of ship manoeuvring motion; know how to derive the Nomoto model and its application in course control problems; understand and master the relationship between the K, T indexes in the Nomoto model and ship manoeuvrability.

§5.1 Brief Introduction

As stated in the last chapter, the equations of motion, i.e., the mathematical models, can be used to simulate any standard manoeuvres, and hence to predict ship manoeuvrability. In this chapter we will use the simplified equations, the linear equations of ship manoeuvring motion to analyze ship manoeuvrability.

As shown in Equation (4.8), the hydrodynamic force and moment acting on a ship in manoeuvring motion are expressed in series, with the hydrodynamic derivatives as coefficients. Under the assumption that the changes in velocities and accelerations $\Delta u = u - U,\ \Delta v = v,\ \Delta r = r,\ \Delta\dot{u} = \dot{u},\ \Delta\dot{v} = \dot{v},\ \Delta\dot{r} = \dot{r}$ due to the manoeuvring motion and the rudder angle δ are small, the high-order terms in the expressions of hydrodynamic force and moment can be neglected. This results in the linear equations of ship manoeuvring motion as follows

$$\begin{aligned} m(\dot{u} - vr - x_G r^2) &= X_0 + X_u(u-U) + X_v v + X_r r + X_{\dot{u}}\dot{u} + X_{\dot{v}}\dot{v} + X_{\dot{r}}\dot{r} + X_\delta \delta, \\ m(\dot{v} + ur + x_G \dot{r}) &= Y_0 + Y_u(u-U) + Y_v v + Y_r r + Y_{\dot{u}}\dot{u} + Y_{\dot{v}}\dot{v} + Y_{\dot{r}}\dot{r} + Y_\delta \delta, \\ I_z \dot{r} + m x_G(\dot{v} + ur) &= N_0 + N_u(u-U) + N_v v + N_r r + N_{\dot{u}}\dot{u} + N_{\dot{v}}\dot{v} + N_{\dot{r}}\dot{r} + N_\delta \delta. \end{aligned} \tag{5.1}$$

For consistency, the terms $vr,\ x_G r^2$ in Equation (5.1) should be neglected; and for the same reason, $ur = (u - U + U)r = (u - U)r + Ur \approx Ur$. On the other hand, the hydrodynamic force and moment at the initial steady state of forward motion $X_0,\ Y_0, N_0$ should vanish. It results from Equation (5.1)

$$\begin{aligned} m\dot{u} &= X_u(u-U) + X_v v + X_r r + X_{\dot{u}}\dot{u} + X_{\dot{v}}\dot{v} + X_{\dot{r}}\dot{r} + X_\delta \delta, \\ m(\dot{v} + Ur + x_G \dot{r}) &= Y_u(u-U) + Y_v v + Y_r r + Y_{\dot{u}}\dot{u} + Y_{\dot{v}}\dot{v} + Y_{\dot{r}}\dot{r} + Y_\delta \delta, \\ I_z \dot{r} + m x_G(\dot{v} + Ur) &= N_u(u-U) + N_v v + N_r r + N_{\dot{u}}\dot{u} + N_{\dot{v}}\dot{v} + N_{\dot{r}}\dot{r} + N_\delta \delta. \end{aligned} \tag{5.2}$$

§5.2 Linear Hydrodynamic Derivatives

The linear hydrodynamic derivatives have explicit physical meanings. They mean the force or moment induced by a unit component of velocity or acceleration in manoeuvring motion, while the other components keep vanished. Mathematically they are the gradients of the curve of force or moment versus the component of velocity or acceleration at the origin. Obviously, for ships with port-starboard symmetry, the linear hydrodynamic derivatives $X_v,\ X_r,\ X_{\dot{v}}, X_{\dot{r}},\ X_\delta$ and $Y_{\dot{u}}, Y_u,\ N_{\dot{u}}, N_u$ should vanish due to the symmetry of the flow about the ship centerplane, as shown in Figure 5.1 for X_v as an example. Therefore, we obtain the linear equations of ship manoeuvring motion as follows

$$\begin{aligned} &(m - X_{\dot{u}})\dot{u} - X_u(u-U) = 0, \\ &(m - Y_{\dot{v}})\dot{v} - Y_v v + (m x_G - Y_{\dot{r}})\dot{r} + (mU - Y_r)r = Y_\delta \delta, \\ &(m x_G - N_{\dot{v}})\dot{v} - N_v v + (I_z - N_{\dot{r}})\dot{r} + (m x_G U - N_r)r = N_\delta \delta. \end{aligned} \tag{5.3}$$

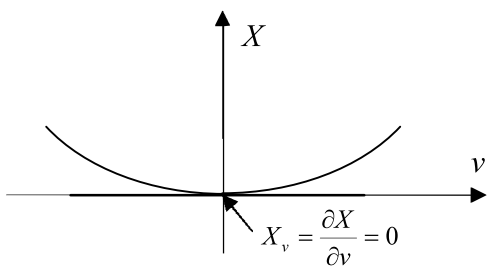


Figure 5.1 Typical X versus v curve

Sometimes it is more convenient to use the non-dimensional form of the equations of motion. Dividing the first two equations of Equation (5.3) by $\frac{1}{2}\rho L^2 U^2$ and dividing the last equation by $\frac{1}{2}\rho L^3 U^2$ respectively, where ρ is the mass density of water, L is the ship length and U is the ship speed at the initial steady state of forward motion, we obtain the non-dimensional form of the linear equations of ship manoeuvring motion

$$\begin{aligned}&(m'-X'_{\dot u})\dot u'-X'_u(u'-U')=0\\&(m'-Y'_{\dot v})\dot v'-Y'_v v'+(m'x'_G-Y'_{\dot r})\dot r'+(m'U'-Y'_r)r'=Y'_\delta\delta\\&(m'x'_G-N'_{\dot v})\dot v'-N'_v v'+(I'_z-N'_{\dot r})\dot r'+(m'x'_G U'-N'_r)r'=N'_\delta\delta\end{aligned} \tag{5.4}$$

where the non-dimensional parameters and coefficients are defined as

$$m'=\frac{m}{\frac{1}{2}\rho L^3},\quad X'_{\dot u}=\frac{X_{\dot u}}{\frac{1}{2}\rho L^3},\quad \dot u'=\frac{\dot u L}{U^2},\quad X'_u=\frac{X_u}{\frac{1}{2}\rho L^2 U},\quad u'=\frac{u}{U},\quad U'=1;$$

$$Y'_{\dot v}=\frac{Y_{\dot v}}{\frac{1}{2}\rho L^3},\quad \dot v'=\frac{\dot v L}{U^2},\quad Y'_v=\frac{Y_v}{\frac{1}{2}\rho L^2 U},\quad v'=\frac{v}{U},$$

$$x'_G=\frac{x_G}{L},\quad Y'_{\dot r}=\frac{Y_{\dot r}}{\frac{1}{2}\rho L^4},\quad \dot r'=\frac{\dot r L^2}{U^2},\quad Y'_r=\frac{Y_r}{\frac{1}{2}\rho L^3 U},\quad r'=\frac{rL}{U},\quad Y'_\delta=\frac{Y_\delta}{\frac{1}{2}\rho L^2 U^2};$$

$$N'_{\dot v}=\frac{N_{\dot v}}{\frac{1}{2}\rho L^4},\quad N'_v=\frac{N_v}{\frac{1}{2}\rho L^3 U},\quad I'_z=\frac{I_z}{\frac{1}{2}\rho L^5},\quad N'_{\dot r}=\frac{N_{\dot r}}{\frac{1}{2}\rho L^5},\quad N'_r=\frac{N_r}{\frac{1}{2}\rho L^4 U},\quad N'_\delta=\frac{N_\delta}{\frac{1}{2}\rho L^3 U^2}.$$

In Equation (5.3), the hydrodynamic forces proportional to the accelerations are inertia forces, whereas those proportional to the velocities are damping forces. Correspondingly, the coefficients $-X_{\dot u}, -Y_{\dot v}$, $-Y_{\dot r}$ and $-N_{\dot v}$ are called added masses and can be expressed as m_{11}, m_{22}, m_{26} and m_{62}; and $-N_{\dot r}$ is called added moment of inertia and can be expressed as m_{66}; whereas the coefficients $-X_u, -Y_v, -Y_r, -N_v$ and $-N_r$ are called damping coefficients. On the other hand, the coefficients Y_δ and N_δ are called control derivatives.

According to the analysis of the flow about the ship, we can determine the magnitude and the sign of the linear hydrodynamic derivatives as follows:

$X_{\dot u}$ is the longitudinal force induced by a unit longitudinal acceleration. This force tends to arrest the acceleration. That means, for a positive acceleration the force will be negative, and vice versa, as shown in Figure 5.2. As a result, the gradient of the $X \sim \dot u$ curve at the origin is negative, which means $X_{\dot u}$ is negative.

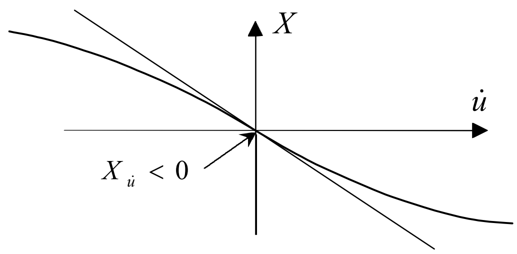


Figure 5.2 Typical X versus $\dot u$ curve

$Y_{\dot v}$ is the lateral force induced by a unit transverse acceleration. Similar as $X_{\dot u}$, we can find $Y_{\dot v}$ is negative, see Figure 5.3. On the other hand, since the forces acting on the fore half part and on the aft half part of the ship induced by the transverse acceleration are in the same direction, as shown in Figure 5.4, the absolute value of total force will be large, therefore $Y_{\dot{v}}$ has large magnitude.

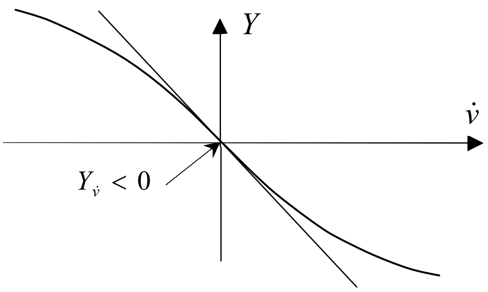


Figure 5.3 Typical Y versus $\dot{v}$ curve

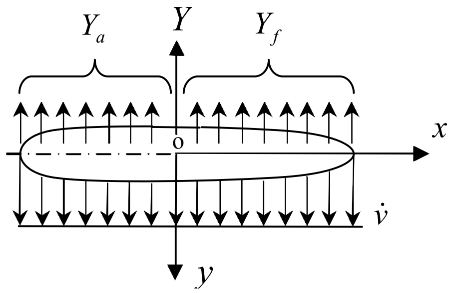


Figure 5.4 Force on a ship with acceleration $\dot{v}$

From intuition we can image that $Y_{\dot{v}}$ is larger than $X_{\dot{u}}$. Actually, for a conventional ship the magnitudes of these two hydrodynamic derivatives have approximately the magnitude

$$-X_{\dot{u}} \approx (0.05 \sim 0.15)m; \quad -Y_{\dot{v}} \approx (0.9 \sim 1.2)m.$$

X_u and Y_v are the longitudinal force and lateral force induced by a unit longitudinal velocity and a unit lateral velocity, respectively. Similar analysis can be made for X_u and Y_v as for $X_{\dot{u}}$ and $Y_{\dot{v}}$ to find that X_u and Y_v are negative, and Y_v has large magnitude, whereas X_u has moderate magnitude.

$N_{\dot{r}}$ and N_r are the yaw moments induced by a unit yaw acceleration and by a unit yaw rate, respectively. Since a positive yaw acceleration or a positive yaw rate will induce a negative yaw moment, we can find $N_{\dot{r}}$ and N_r are negative. Moreover, since the moments acting on the fore half part and on the aft half part of the ship induced by the yaw acceleration or by the yaw rate are in the same direction, the absolute value of total moment will be large, therefore $N_{\dot{r}}$ and N_r have large magnitude. Figure 5.5 and Figure 5.6 show the case of $N_{\dot{r}}$ as an example.

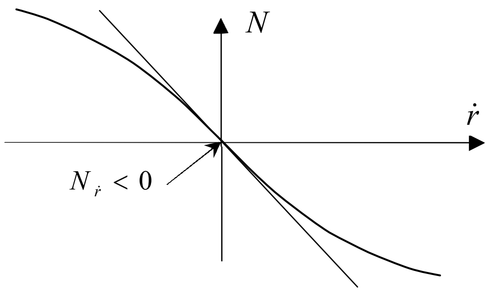


Figure 5.5 Typical N versus $\dot{r}$ curve

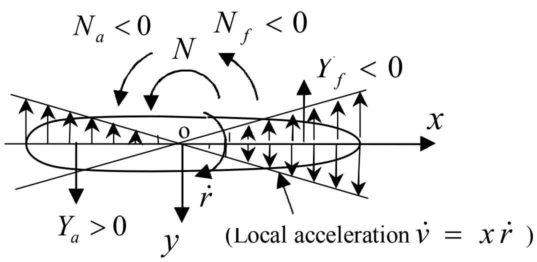


Figure 5.6 Moment on a ship with yaw acceleration $\dot{r}$

$Y_{\dot{r}}$ and Y_r are the lateral forces induced by a unit yaw acceleration and by a unit yaw rate, respectively. Since the forces acting on the fore half part and on the aft half part of the ship induced by the yaw acceleration or by the yaw rate are in the opposite direction (see Figure 5.6 as example), the total force will be very small (For a ship with fore-and-aft symmetry, it should vanish), and the sign of the total force will be the same as the larger one of the fore and aft forces, therefore $Y_{\dot{r}}$ and Y_r have very small magnitude and uncertain sign. A similar analysis can be made for $N_{\dot{v}}$ and N_v as for $Y_{\dot{r}}$ and Y_r with conclusion that $N_{\dot{v}}$ and N_v have very small magnitude and uncertain sign (see Figure 5.7 as example for $N_{\dot{v}}$).

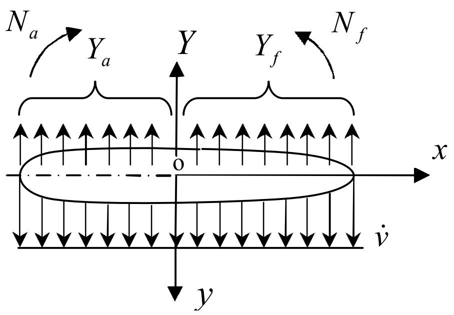


Figure 5.7 Moment on a ship with acceleration $\dot{v}$

The results of the analysis above are summarized in Table 5.1.

Table 5.1 Magnitude and sign of the linear hydrodynamic derivatives

item	$X_{\dot{u}}$	X_u	$Y_{\dot{v}}$	Y_v	$N_{\dot{r}}$	N_r	$Y_{\dot{r}}$	Y_r	$N_{\dot{v}}$	N_v
magnitude	moderate	moderate	very large	very large	very large	very large	very small	very small	very small	very small
sign	negative	negative	negative	negative	negative	negative	uncertain	uncertain	uncertain	uncertain

On the other hand, control derivatives Y_δ and N_δ denote the control force and moment induced by a unit rudder angle. With rudder angle to starboard defined as positive, a negative force and a positive moment will be induced by a positive rudder angle. Hence Y_δ is negative, whereas N_δ is positive. Obviously, rudder is to be designed to give as large control force and moment as possible, therefore Y_δ and N_δ should be as large as possible.

§5.3 Analysis of Stability (inherent dynamic stability)

It can be seen that in Equation (5.3) the first equation is uncoupled from the last two equations. Therefore, it can be solved separately. On the other hand, the last two equations are coupled each other and hence have to be solved together.

The first equation of Equation (5.3) can be rewritten as

$$\frac{d(u-U)}{dt}-\frac{X_u}{m-X_{\dot{u}}}(u-U)=0. \tag{5.5}$$

Equation (5.5) is an ordinary differential equation of first order with constant coefficients, whose solution is given by

$$u-U=c_3e^{\sigma_3 t}, \tag{5.6}$$

where $\sigma_3=X_u/(m-X_{\dot{u}})$, and c_3 is an integral constant to be determined by the initial condition.

Since $X_u<0$ and $m-X_{\dot{u}}>0$, we have $\sigma_3<0$. Therefore, as $t\to\infty$, the term on the right hand side of Equation (5.6) will vanish, resulting in $u\to U$. That means, the longitudinal motion of the ship will finally tend to the original steady state with a constant forward speed.

To analyze the inherent dynamic stability of the ship (i.e. the dynamic stability without control actions), we set $\delta=0$ and assume that the lateral motion and yaw motion are induced by a small disturbance. We obtain from the last two equations of Equation (5.3)

$$(m-Y_{\dot v})\dot v - Y_v v + (mx_G - Y_{\dot r})\dot r + (mU - Y_r)r = 0,$$
$$(mx_G - N_{\dot v})\dot v - N_v v + (I_z - N_{\dot r})\dot r + (mx_G U - N_r)r = 0, \tag{5.7}$$

which can be rewritten as

$$(m-Y_{\dot v})\dot v - Y_v v = -(mx_G - Y_{\dot r})\dot r - (mU - Y_r)r,$$
$$(mx_G - N_{\dot v})\dot v - N_v v = -(I_z - N_{\dot r})\dot r - (mx_G U - N_r)r. \tag{5.8}$$

Solving Equation (5.8) for $\dot v$ and v yields

$$v = \frac{-(m-Y_{\dot v})[(mx_G U - N_r)r + (I_z - N_{\dot r})\dot r] + (mx_G - N_{\dot v})[(mU - Y_r)r + (mx_G - Y_{\dot r})\dot r]}{Y_v(mx_G - N_{\dot v}) - N_v(m - Y_{\dot v})},$$
$$\dot v = \frac{-Y_v[(mx_G U - N_r)r + (I_z - N_{\dot r})\dot r] + N_v[(mU - Y_r)r + (mx_G - Y_{\dot r})\dot r]}{Y_v(mx_G - N_{\dot v}) - N_v(m - Y_{\dot v})}. \tag{5.9}$$

Differentiating the first equation of Equation (5.9) with respect to time and letting the result equal to the second equation, we eliminate $\dot v, v$ from Equation (5.9) and obtain

$$A\ddot r + B\dot r + Cr = 0, \quad \text{or} \quad \ddot r + \frac{B}{A}\dot r + \frac{C}{A} r = 0, \tag{5.10}$$

where

$$A = (m - Y_{\dot v})(I_z - N_{\dot r}) - (mx_G - N_{\dot v})(mx_G - Y_{\dot r}),$$
$$B = -Y_v(I_z - N_{\dot r}) + N_v(mx_G - Y_{\dot r}) + (m - Y_{\dot v})(mx_G U - N_r) - (mx_G - N_{\dot v})(mU - Y_r),$$
$$C = -Y_v(mx_G U - N_r) + N_v(mU - Y_r).$$

Equation (5.10) is an ordinary differential equation of second order with constant coefficients, whose solution is given by

$$r = r_1 e^{\sigma_1 t} + r_2 e^{\sigma_2 t}, \tag{5.11}$$

where r_1 and r_2 are integral constants to be determined by the initial condition, σ_1 and σ_2 are given by

$$\begin{pmatrix}\sigma_1 \\ \sigma_2\end{pmatrix} = \frac{1}{2}\left[-\frac{B}{A} \pm \sqrt{\left(\frac{B}{A}\right)^2 - 4\frac{C}{A}}\right]. \tag{5.12}$$

Similarly, if we eliminate $\dot r, r$ from Equation (5.7), we obtain

$$A\ddot v + B\dot v + Cv = 0, \quad \text{or} \quad \ddot v + \frac{B}{A}\dot v + \frac{C}{A} v = 0, \tag{5.13}$$

with the solution

$$v = v_1 e^{\sigma_1 t} + v_2 e^{\sigma_2 t}, \tag{5.14}$$

where v_1 and v_2 are integral constants to be determined by the initial condition, σ_1 and σ_2 are given by Equation (5.12).

As we can see, if both σ_1 and σ_2 are negative real numbers or imaginary numbers with negative real parts, we will have $e^{\sigma_1 t} \to 0$ and $e^{\sigma_2 t} \to 0$ as $t \to \infty$. That means, after a small disturbance, $r \to 0$ and $v \to 0$ as $t \to \infty$, the ship will finally tend to a new straight course with constant forward speed. At the meantime, the ship will have a heading deviation from the original heading which is given by

$$\psi = \int_0^{t_0} r(t)dt, \tag{5.15}$$

where t_0 is the moment when r vanishes. Obviously, a ship with better inherent dynamic stability will tend to a new straight course faster and will have smaller heading deviation.

From the above analysis, it can be seen that a ship affected by a small disturbance can not keep its original course without control action. It can only turn into a new straight course with a heading deviation from the original course if it has inherent dynamic stability. Contrarily, if the ship is unstable, the yaw rate given by Equation (5.11) will not vanish as $t \to \infty$, therefore the ship will turn into an unsteady turning motion. See Figure 2.1 for reference.

Both σ_1 and σ_2 are negative real numbers or imaginary numbers with negative real parts is the criterion for inherent dynamic stability. Further analysis of the stability is made in the following.

For arbitrary $\frac{B}{A}$, if $\frac{C}{A} < 0$, then

$$\sigma_1 = \frac{1}{2}\left[-\frac{B}{A} + \sqrt{\left(\frac{B}{A}\right)^2 - 4\frac{C}{A}}\right] > 0 \text{ and } \sigma_2 = \frac{1}{2}\left[-\frac{B}{A} - \sqrt{\left(\frac{B}{A}\right)^2 - 4\frac{C}{A}}\right] < 0.$$

Therefore, to ensure both σ_1 and σ_2 are negative real numbers or imaginary numbers with negative real parts, we must have $\frac{C}{A} > 0$.

Then, assuming $\frac{C}{A} > 0$,

(1) if $\left(\frac{B}{A}\right)^2 - 4\frac{C}{A} > 0$, σ_1 and σ_2 are two different real numbers. Since $\sqrt{\left(\frac{B}{A}\right)^2 - 4\frac{C}{A}} < \left|\frac{B}{A}\right|$, both σ_1 and σ_2 are positive real numbers if $\frac{B}{A} < 0$; both σ_1 and σ_2 are negative real numbers if $\frac{B}{A} > 0$. In this case, to ensure that both σ_1 and σ_2 are negative real numbers we must have $\frac{B}{A} > 0$.

(2) if $\left(\frac{B}{A}\right)^2 - 4\frac{C}{A} = 0$, σ_1 and σ_2 are real numbers and equal: $\sigma_1 = \sigma_1 = -\frac{B}{2A}$. In this case, to ensure that both σ_1 and σ_2 are negative real numbers we must have $\frac{B}{A} > 0$.

(3) if $\left(\frac{B}{A}\right)^2 - 4\frac{C}{A} < 0$, σ_1 and σ_2 are two different imaginary numbers with the real part $-\frac{B}{2A}$. In this case, to ensure that both σ_1 and σ_2 are imaginary numbers with negative real part we must have $\frac{B}{A} > 0$.

In conclusion, to ensure both σ_1 and σ_2 are negative real numbers or imaginary numbers with negative real parts, we must have $\frac{B}{A} > 0$ and $\frac{C}{A} > 0$.

Actually, by requiring $\sigma_1 \cdot \sigma_2 > 0$ and $\sigma_1 + \sigma_2 < 0$ (which means that both σ_1 and σ_2 are negative real numbers or imaginary numbers with negative real parts), it is more straightforward to see the criterion for inherent dynamic stability:

$$\sigma_1 \cdot \sigma_2 = \frac{1}{2}\left[-\frac{B}{A} + \sqrt{\left(\frac{B}{A}\right)^2 - 4\frac{C}{A}}\right] \cdot \frac{1}{2}\left[-\frac{B}{A} - \sqrt{\left(\frac{B}{A}\right)^2 - 4\frac{C}{A}}\right] = \frac{C}{A} > 0 .$$

$$\sigma_1 + \sigma_2 = \frac{1}{2}\left[-\frac{B}{A} + \sqrt{\left(\frac{B}{A}\right)^2 - 4\frac{C}{A}}\right] + \frac{1}{2}\left[-\frac{B}{A} - \sqrt{\left(\frac{B}{A}\right)^2 - 4\frac{C}{A}}\right] = -\frac{B}{A} < 0 \quad \Rightarrow \frac{B}{A} > 0 .$$

Since $m - Y_{\dot v}$ and $I_z - N_{\dot r}$ are positive and very large, whereas $mx_G - N_{\dot v}$ and $mx_G - Y_{\dot r}$ are small with uncertain sign, it can be concluded that

$$A = (m - Y_{\dot v})(I_z - N_{\dot r}) - (mx_G - N_{\dot v})(mx_G - Y_{\dot r}) > 0.$$

Moreover, since $-Y_v$, $I_z - N_{\dot r}$, $m - Y_{\dot v}$ and $-N_r$ are positive and very large, N_v, $mx_G - Y_{\dot r}$, $mx_G U$, $mx_G - N_{\dot v}$ and Y_r are small with uncertain sign, it can be concluded that

$$B = -Y_v(I_z - N_{\dot r}) + N_v(mx_G - Y_{\dot r}) + (m - Y_{\dot v})(mx_G U - N_r) - (mx_G - N_{\dot v})(mU - Y_r) > 0.$$

Therefore, the criterion for inherent dynamic stability is reduced to

$$C = -Y_v(mx_G U - N_r) + N_v(mU - Y_r) > 0. \tag{5.16}$$

The corresponding non-dimensional inequality is

$$C' = -Y_v'(m'x_G'U' - N_r') + N_v'(m'U' - Y_r') > 0. \tag{5.17}$$

Inequality (5.16) shows that the criterion for inherent dynamic stability includes only four linear hydrodynamic derivatives. These derivatives are also called the stability derivatives. A ship with inherent dynamic stability has a positive C. The larger the magnitude of the positive C, the better is the dynamic stability.

Since $-Y_v > 0$ and $mU - Y_r > 0$, dividing Inequality (5.16) by $-Y_v(mU - Y_r)$ yields

$$\frac{N_r - mx_G U}{Y_r - mU} - \frac{N_v}{Y_v} = l_r - l_v > 0, \tag{5.18}$$

where

$$l_r = \frac{N_r - mx_G U}{Y_r - mU}, \quad l_v = \frac{N_v}{Y_v}$$

are the moment arms due to the forces induced by the yaw motion and the sway motion, respectively. Inequality (5.18) shows that the force due to the yaw motion must act on the ship before the force due to the sway motion, in order to make the ship dynamically stable.

§5.4 Nomoto Model and K,T Indexes

The mathematical models discussed in §4.3 are the so-called hydrodynamic force models. Now we will derive another kind of models, the response models, from the hydrodynamic force models. Such models are mainly used to study the course control problems or to design automatic control devices such as autopilots. We will also see that although this kind of models has simpler form, the coefficients of the models have explicit relation with the manoeuvring characteristics and can be used to evaluate ship manoeuvrability.

The last two equations of Equation (5.3) are rewritten in the form

$$\begin{aligned} &(m-Y_{\dot v})\dot v - Y_v v = Y_\delta \delta - (mx_G - Y_{\dot r})\dot r - (mU - Y_r)r, \\ &(mx_G - N_{\dot v})\dot v - N_v v = N_\delta \delta - (I_z - N_{\dot r})\dot r - (mx_G U - N_r)r. \end{aligned} \tag{5.19}$$

Solving Equation (5.19) for $\dot v$ and v yields

$$\begin{aligned} v &= \frac{(m-Y_{\dot v})[N_\delta \delta - (mx_G U - N_r)r - (I_z - N_{\dot r})\dot r] - (mx_G - N_{\dot v})[Y_\delta \delta - (mU - Y_r)r - (mx_G - Y_{\dot r})\dot r]}{Y_v(mx_G - N_{\dot v}) - N_v(m - Y_{\dot v})}, \\ \dot v &= \frac{Y_v[N_\delta \delta - (mx_G U - N_r)r - (I_z - N_{\dot r})\dot r] - N_v[Y_\delta \delta - (mU - Y_r)r - (mx_G - Y_{\dot r})\dot r]}{Y_v(mx_G - N_{\dot v}) - N_v(m - Y_{\dot v})}. \end{aligned} \tag{5.20}$$

Differentiating the first equation of Equation (5.20) with respect to time and letting the result equal to the second equation, we obtain

$$T_1T_2\ddot r + (T_1 + T_2)\dot r + r = K\delta + KT_3\dot\delta, \tag{5.21}$$

where

$$\begin{aligned} T_1T_2 &= \frac{(m-Y_{\dot v})(I_z - N_{\dot r}) - (mx_G - N_{\dot v})(mx_G - Y_{\dot r})}{-Y_v(mx_G U - N_r) + N_v(mU - Y_r)}, \\ T_1 + T_2 &= \frac{-Y_v(I_z - N_{\dot r}) + N_v(mx_G - Y_{\dot r}) + (m - Y_{\dot v})(mx_G U - N_r) - (mx_G - N_{\dot v})(mU - Y_r)}{-Y_v(mx_G U - N_r) + N_v(mU - Y_r)}, \\ K &= \frac{-Y_v N_\delta + N_v Y_\delta}{-Y_v(mx_G U - N_r) + N_v(mU - Y_r)}, \\ KT_3 &= \frac{-Y_\delta(mx_G - N_{\dot v}) + N_\delta(m - Y_{\dot v})}{-Y_v(mx_G U - N_r) + N_v(mU - Y_r)}, \\ T_3 &= \frac{KT_3}{K} = \frac{-Y_\delta(mx_G - N_{\dot v}) + N_\delta(m - Y_{\dot v})}{-Y_v N_\delta + N_v Y_\delta}. \end{aligned}$$

Comparing the expressions of the coefficients A, B, C in Equation (5.10) and the coefficients T_1T_2, $T_1 + T_2$ in Equation (5.21), noting that σ_1 and σ_2 are given by Equation (5.12), we can find the relationships between σ_1, σ_2 and T_1, T_2 as follows

$$\frac{C}{A} = \sigma_1 \cdot \sigma_2 = \frac{1}{T_1T_2} = \left(-\frac{1}{T_1}\right)\cdot\left(-\frac{1}{T_2}\right) \quad \text{and} \quad -\frac{B}{A} = \sigma_1 + \sigma_2 = -\frac{T_1 + T_2}{T_1T_2} = \left(-\frac{1}{T_1}\right) + \left(-\frac{1}{T_2}\right).$$

Equation (5.21) is the response model of second order. It expresses the relation between the yaw motion and the rudder angle and can be used to describe the yaw motion response to the control action.

Assuming that the ship is approximately fore-and-aft symmetrical, Equation (5.21) can be further reduced to the response model of first order.

For a ship which is approximately fore-and-aft symmetrical, we have

$$x_G \approx 0, \quad Y_r \approx 0, \quad Y_{\dot r} \approx 0, \quad N_v \approx 0, \quad N_{\dot v} \approx 0.$$

It follows that,

$$K \approx -\frac{N_\delta}{N_r},\quad T_1+T_2 \approx -\frac{I_z-N_{\dot r}}{N_r}-\frac{m-Y_{\dot v}}{Y_v},\ T_3 \approx -\frac{m-Y_{\dot v}}{Y_v},\quad T_1+T_2-T_3 \approx -\frac{I_z-N_{\dot r}}{N_r}.$$

On the other hand, from the third equation of Equation (5.3) we have approximately

$$(I_z-N_{\dot r})\dot r - N_r r = N_\delta \delta,$$

or rewritten in the form

$$-\frac{I_z-N_{\dot r}}{N_r}\dot r + r = -\frac{N_\delta}{N_r}\delta .$$

Denoting $T = T_1+T_2-T_3$, we have

$$T\dot r + r = K\delta . \tag{5.22}$$

Equation (5.22) is the response model of first order. It was firstly derived by Japanese professor K. Nomoto in 1957 by using the method of Laplace Transformation. Therefore, the response model of first order is also called Nomoto model. K, T in this model are called manoeuvrability indexes, since, as we can see in the following, they have explicit relations with manoeuvring characteristics.

The response models of second order and first order can be used to predict the yaw motion under the control action. In the following we will use the response model of first order to investigate the yaw motion response of the ship to the rudder action.

It is assumed that the ship is initially in its straight course with a steady forward speed; the rudder is ordered to turn to a certain angle δ_0 during the time t_0 with a uniform rudder turning rate and then held fixed, see Figure 5.8. The change of rudder angle with time is given by

$$\begin{cases} \delta = \dfrac{\delta_0}{t_0}t, & 0 \le t \le t_0 \\ \delta = \delta_0, & t > t_0 \end{cases} \tag{5.23}$$

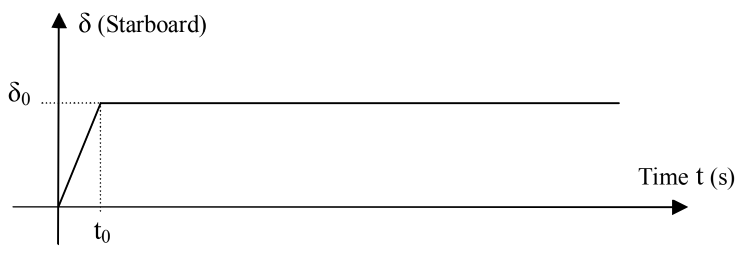


Figure 5.8 Time history of rudder angle

(1) At the first phase, $0 \le t \le t_0$, it results from Equations (5.22) and (5.23)

$$\dot r + \frac{1}{T}r = \frac{K\delta_0}{Tt_0}t \tag{5.24}$$

Equation (5.24) is an ordinary differential equation of first order, whose solution is given by

$$r = e^{-\frac{t}{T}}\left[\frac{K\delta_0}{t_0}e^{\frac{t}{T}}(t-T)+c\right], \tag{5.25}$$

where c is an integral constant. According to the initial condition $r=0$ at $t=0$, c is found as

$$c = \frac{K\delta_0}{t_0}T. \tag{5.26}$$

Substituting Equation (5.26) in Equation (5.25) results in

$$r = \frac{K\delta_0}{t_0}(t-T+Te^{-\frac{t}{T}}). \tag{5.27}$$

At $t=t_0$, it results from Equation (5.27)

$$r = \frac{K\delta_0}{t_0}(t_0-T+Te^{-\frac{t_0}{T}}). \tag{5.28}$$

Integrating Equation (5.27) with respect to time t results in

$$\psi = \frac{K\delta_0}{t_0}\left[\frac{t^2}{2}-Tt+T^2(1-e^{-\frac{t}{T}})\right]. \tag{5.29}$$

Differentiating Equation (5.27) with respect to time t results in

$$\dot{r} = \frac{K\delta_0}{t_0}(1-e^{-\frac{t}{T}}). \tag{5.30}$$

(2) At the second phase, $t>t_0$, it results from Equations (5.22) and (5.23)

$$\dot{r}+\frac{1}{T}r = \frac{K\delta_0}{T}. \tag{5.31}$$

Equation (5.31) is a differential equation of first order, whose solution is given by

$$r = e^{-\frac{t}{T}}\left(K\delta_0 e^{\frac{t}{T}}+c\right), \tag{5.32}$$

where c is an integral constant. According to the initial condition, Equation (5.28), c is found as

$$c = \frac{K\delta_0}{t_0}T\left(1-e^{\frac{t_0}{T}}\right). \tag{5.33}$$

Substituting Equation (5.33) in Equation (5.32) results in

$$r = \frac{K\delta_0}{t_0}\left[t_0+T(1-e^{\frac{t_0}{T}})e^{-\frac{t}{T}}\right]. \tag{5.34}$$

Integrating Equation (5.34) with respect to time t (from t_0 to t) results in

$$\psi = \psi(t_0)+\int_{t_0}^{t} r dt = \frac{K\delta_0}{t_0}\left[t_0(t-T)-\frac{t_0^2}{2}+T^2(e^{\frac{t_0}{T}}-1)e^{-\frac{t}{T}}\right]. \tag{5.35}$$

In deriving Equation (5.35) we have utilized $\psi(t_0)$ obtained from Equation (5.29).

On the other hand, differentiating Equation (5.34) with respect to time t results in

$$\dot{r} = \frac{K\delta_0}{t_0}(e^{\frac{t_0}{T}} - 1)e^{-\frac{t}{T}}. \tag{5.36}$$

Since $I_z - N_{\dot{r}} > 0$ and $N_r < 0$, we have $T > 0$. Therefore $e^{-\frac{t}{T}} \to 0$ as $t \to \infty$. It results form Equation (5.35), (5.34) and (5.36) that as $t \to \infty$

$$\psi \to K\delta_0\left[(t-T) - \frac{t_0}{2}\right],\ \ r \to K\delta_0 \ \text{ and } \ \dot{r} \to 0.$$

This means, as $t \to \infty$ the ship will trend into steady turn with a steady yaw rate $r_0 = K\delta_0$ and the heading angle changing with the time t linearly as

$$\psi = K\delta_0\left[t - (T + \frac{t_0}{2})\right] = K\delta_0(t-T) - \frac{r_0 t_0}{2}.$$

Denoting the ship speed in steady turning phase as V_0 and the turning radius as R, the diameter of the turning circle, $D_0 = 2R$, can be determined by

$$D_0 = 2\frac{V_0}{r_0} = 2\frac{V_0}{K\delta_0}. \tag{5.37}$$

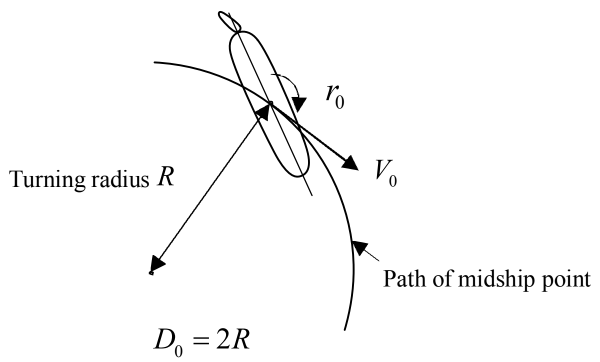


Figure 5.9 Turning radius in steady turning phase

From Equation (5.27) and (5.34) we can see that the yaw rate is proportional to K. At a certain rudder angle, larger K will give larger yaw rate. Moreover, in the steady turning phase the yaw rate $r_0 = K\delta_0$, which means larger K will give larger steady yaw rate r_0 and hence smaller diameter of the turning circle D_0. Therefore, the parameter K can be used to evaluate the turning ability. Larger K implies better turning ability.

On the other hand, as discussed above, for a normal ship, $T > 0$. At a certain moment t, smaller T will result in smaller $e^{-\frac{t}{T}}$ ($T \downarrow \Rightarrow \frac{t}{T} \uparrow \Rightarrow e^{\frac{t}{T}} \uparrow \Rightarrow e^{-\frac{t}{T}} \downarrow$); as indicated by Equation (5.30), this means a larger yaw acceleration $\dot{r}$ at the initial turning phase. Therefore, smaller T implies better initial turning ability or course-changing ability. Moreover, since $e^{-\frac{t}{T}} \to 0$ as $t \to \infty$, for a smaller T, $e^{-\frac{t}{T}}$ will vanish faster. It means, as indicated by Equation (5.34), the ship will tend to steady turning faster. Once again, this indicates that smaller T will give better initial turning ability.

Smaller T, actually, will give also better dynamic stability on a straight course. To show this, we assume that a ship is originally traveling in a straight course and then affected by a small disturbance. Without control action, the yaw motion of the ship is described by the following equation according to Equation (5.22)

$$T\dot{r} + r = 0. \tag{5.38}$$

The solution of Equation (5.38) is given by

$$r = ce^{-\frac{t}{T}}, \tag{5.39}$$

where c is an integral constant determined by the initial condition $r = r_0$ at $t = 0$, resulting in $c = r_0$.

Equation (5.39) describes the yaw motion of the ship after a small disturbance. It can be seen that as $t \to \infty$ the yaw rate will vanish after the disturbance; smaller T will make the yaw rate vanishing faster and hence give a better dynamic stability on a straight course.

It should bear in mind that the analysis above using the response model of first order, Equation (5.22), is based on the assumption of $x_G \approx 0,\ \ Y_r \approx 0,\ \ Y_{\dot{r}} \approx 0,\ \ N_v \approx 0,\ \ N_{\dot{v}} \approx 0$.

As discussed above, the parameter K can be used to evaluate the turning ability, whereas the parameter T can be used to evaluate the dynamic stability on a straight course, the initial turning ability and the course-changing ability. For good ship manoeuvrability we wish the ship having larger K and smaller T. However, as we will see in the following, a large K usually accompanies a large T; or contrarily, a small K accompanies a small T.

We recall the definition of the manoeuvrability indexes

$$K \approx -\frac{N_\delta}{N_r}, \quad T \approx -\frac{I_z - N_{\dot{r}}}{N_r}$$

where N_δ is the control moment derivative due to the rudder angle, N_r is the damping moment derivative due to the yaw motion, I_z is the moment of inertia of the ship and $-N_{\dot{r}}$ is the added moment of inertia.

To obtain a large K for good turning ability, we wish a small N_r. This will, however, result in a large T and hence a poor dynamic stability on straight course and a poor initial turning ability at the same time. Therefore, we can not improve the ship manoeuvrability only by changing the value of damping moment for yaw motion. On the other hand, obtaining large control moment derivative by designing the rudder may result in good turning ability without loss of other manoeuvring performances. Therefore, designing the rudder to obtain large control moment is the effective way to improve ship manoeuvrability.

To obtain a small T for better dynamic stability on straight course and initial turning ability, one might try to design the ship with small moment of inertia and small added moment of inertia. However, the moment of inertia and added moment of inertia are mainly determined by the principal dimensions and ship form coefficients which are chosen according to other ship performances. Hence there is no big space for choosing the principal dimensions and ship form coefficients to obtain small T. For large ship with full form such as VLCC, T is usually very large due to the large inertia. Therefore, such ships are usually dynamically unstable and have poor initial turning ability and course-changing ability.

As stated above, the manoeuvrability indexes K and T can be conveniently used to evaluate the manoeuvring performances of ships. For comparison purpose, it is more convenient to use non-dimensional manoeuvrability indexes. Since K has the dimension of $1/s$ and T has the dimension of s, the non-dimensional forms of these indexes are give by

$$K' = \left(\frac{L}{U}\right)K, \quad T' = \left(\frac{U}{L}\right)T.$$

§5.5 Analysis of Turning Ability Based on the Linear Equations

In last subsection, the response model of first order is used to analyze ship yaw motion and the relationship between the manoeuvrability indexes K, T and ship manoeuvrability. In this subsection we will use the linear hydrodynamic force model to analyze the turning motion of ships.

It is assumed that a ship originally travels with a steady forward speed in a straight course, and the rudder is ordered to a certain rudder angle to starboard or port with a constant rudder turning rate and then held fixed, see Figure 5.8. Ultimately, the ship tends to a steady turning motion.

The whole process can be divided into three phases: The rudder-turning phase, the transition phase and the steady turning phase. In the following we will analyze the motion in the three phases in detail by using the last two equations of Equation (5.3):

$$\begin{aligned}&(m - Y_{\dot v})\dot v - Y_v v + (mx_G - Y_{\dot r})\dot r + (mU - Y_r)r = Y_\delta \delta,\\&(mx_G - N_{\dot v})\dot v - N_v v + (I_z - N_{\dot r})\dot r + (mx_G U - N_r)r = N_\delta \delta.\end{aligned} \tag{5.40}$$

(1) Rudder-turning phase

At this phase, $0 \le t \le t_0$, , the rudder angle δ is given by $\delta = \delta_0 t / t_0$, where δ_0 is the rudder angle applied and t_0 is the time of rudder-turning. The control force and moment of the rudder will produce a sway acceleration and a yaw acceleration. But the time of rudder-turning is so small and the inertia the ship is so large that at this phase no notable sway speed and yaw rate are induced.

Therefore, we have at the rudder-turning phase

$$\delta = \frac{\delta_0}{t_0} t; \quad \dot v \ne 0, \dot r \ne 0; \quad v \approx 0, r \approx 0.$$

It results from Equation (5.40)

$$\begin{aligned}&(m - Y_{\dot v})\dot v + (mx_G - Y_{\dot r})\dot r = Y_\delta \delta.\\&(mx_G - N_{\dot v})\dot v + (I_z - N_{\dot r})\dot r = N_\delta \delta.\end{aligned} \tag{5.41}$$

From Equation (5.41) we obtain $\dot v$ and $\dot r$ as follows

$$\begin{aligned}\dot v &= \frac{(I_z - N_{\dot r})Y_\delta - (mx_G - Y_{\dot r})N_\delta}{(m - Y_{\dot v})(I_z - N_{\dot r}) - (mx_G - N_{\dot v})(mx_G - Y_{\dot r})}\delta,\\ \dot r &= \frac{(m - Y_{\dot v})N_\delta - (mx_G - N_{\dot v})Y_\delta}{(m - Y_{\dot v})(I_z - N_{\dot r}) - (mx_G - N_{\dot v})(mx_G - Y_{\dot r})}\delta.\end{aligned} \tag{5.42}$$

Equation (5.42) shows that the sway acceleration $\dot v$ and the yaw acceleration $\dot r$ change with the rudder angle proportionally at the rudder-turning phase. Noting that the denominator in Equation (5.42) is the coefficient A in Equation (5.10), which is usually positive, since

$$\begin{aligned}&(I_z - N_{\dot r})Y_\delta - (mx_G - Y_{\dot r})N_\delta \approx (I_z - N_{\dot r})Y_\delta < 0,\\&(m - Y_{\dot v})N_\delta - (mx_G - N_{\dot v})Y_\delta \approx (m - Y_{\dot v})N_\delta > 0,\end{aligned}$$

for a positive rudder angle, the sway acceleration $\dot v$ will be negative and the yaw acceleration $\dot r$ will be positive. This means, at the rudder-turning phase, a positive rudder angle will create a negative sway acceleration and a positive yaw acceleration.

Assuming that x_G, $Y_{\dot{r}}$ and $N_{\dot{v}}$ are very small and the relevant terms can be neglected, we have

$$\dot{v} \approx \frac{Y_\delta}{m - Y_{\dot{v}}}\delta; \quad \dot{r} \approx \frac{N_\delta}{I_z - N_{\dot{r}}}\delta = \frac{K}{T}\delta. \tag{5.43}$$

Equation (5.43) shows that larger K and smaller T will give a larger $\dot{r}$, hence a better initial turning ability.

(2) Transition phase

At this phase, $\delta = \delta_0$; $\dot{v} \neq 0, \dot{r} \neq 0$; $v \neq 0, r \neq 0$. It results from Equation (5.40)

$$\begin{aligned} &(m - Y_{\dot{v}})\dot{v} - Y_v v + (mx_G - Y_{\dot{r}})\dot{r} + (mU - Y_r)r = Y_\delta \delta_0, \\ &(mx_G - N_{\dot{v}})\dot{v} - N_v v + (I_z - N_{\dot{r}})\dot{r} + (mx_G U - N_r)r = N_\delta \delta_0. \end{aligned} \tag{5.44}$$

At the transition phase, the control force and moment remain unchanging, whereas the kinematical quantities of the ship change with time. We can solve Equation (5.44) by numerical integration method, for example, by the Runge-Kutta method, to obtain the kinematical quantities $\dot{v}(t), \dot{r}(t), v(t)$ and $r(t)$.

At the initial transition stage, the ship moves laterally to the outside (other side as the rudder deflection) under rudder force action and turns to inside (the same side as the rudder deflection) under rudder moment action. With the development of $\dot{v}(t), \dot{r}(t), v(t)$ and $r(t)$, the hydrodynamic force and moment acting on the ship hull increase gradually. Ultimately, the hydrodynamic force and moment acting on the hull and the control force and moment due to the rudder angle reach equilibrium, the ship then enters to the next phase, i.e., the steady turning phase.

(3) Steady turning phase

At this phase, $\delta = \delta_0$; $\dot{v} = 0, \dot{r} = 0$; $v = v_0, r = r_0$, where v_0 and r_0 are the steady sway velocity and the steady yaw rate. It results from Equation (5.40)

$$\begin{aligned} &-Y_v v_0 + (mU - Y_r)r_0 = Y_\delta \delta_0, \\ &-N_v v_0 + (mx_G U - N_r)r_0 = N_\delta \delta_0. \end{aligned} \tag{5.45}$$

Solving Equation (5.45) for v_0 and r_0, we obtain

$$\begin{aligned} v_0 &= \frac{-(mU - Y_r)N_\delta + (mx_G U - N_r)Y_\delta}{-Y_v(mx_G U - N_r) + N_v(mU - Y_r)}\delta_0. \\ r_0 &= \frac{-Y_v N_\delta + N_v Y_\delta}{-Y_v(mx_G U - N_r) + N_v(mU - Y_r)}\delta_0. \end{aligned} \tag{5.46}$$

Noting that the denominator in Equation (5.46) is the coefficient C discussed in subsection §5.3, we obtain the steady turning diameter as follows

$$D_0 = 2R = 2\frac{V_0}{r_0} = 2\frac{V_0}{\delta_0}\left(\frac{C}{-Y_v N_\delta + N_v Y_\delta}\right), \tag{5.47}$$

where $R = \dfrac{V_0}{r_0}$ is the steady turning radius, $V_0 = \sqrt{U^2 + v_0^2}$ is the steady ship speed, see Figure 5.9.

The non-dimensional steady turning diameter is given by

$$D_0' = \frac{D_0}{L} = \frac{2R}{L} = 2\frac{V_0}{r_0 L} = \frac{2}{r_0'}, \tag{5.48}$$

where, with $V_0 = \sqrt{U^2 + v_0^2} \approx U$,

$$r_0' = \frac{r_0 L}{V_0} = \frac{L}{R} = \frac{-Y_v' N_\delta' + N_v' Y_\delta'}{C'} \delta_0. \tag{5.49}$$

Since from Equation (5.46)

$$r_0 = \frac{-Y_v N_\delta + N_v Y_\delta}{C} \delta_0, \tag{5.50}$$

we can see that at a certain rudder angle, a positive C with larger absolute value, which means a better dynamical stability, will result in a smaller steady yaw rate and hence a larger steady turning diameter. This indicates that a better dynamical stability will usually accompany a worse turning ability, and vice versa.

Moreover, since Y_v is negative with large absolute value and $N_\delta > 0$, we have $-Y_v N_\delta > 0$; and since $Y_\delta < 0$ and N_v is small with uncertain sign, it follows that $N_v Y_\delta$ is small with uncertain sign. Hence, $-Y_v N_\delta + N_v Y_\delta$ has the same sign as $-Y_v N_\delta$ has. That means, $-Y_v N_\delta + N_v Y_\delta > 0$.

Therefore, if $C > 0$ (ship is dynamically stable), for a positive rudder angle (rudder to starboard), we have $r_0 > 0$ (turning to starboard); whereas for a negative rudder angle (rudder to port), we have $r_0 < 0$ (turning to port). This situation is what we expect.

On the other hand, if $C < 0$ (ship is dynamically unstable), for a positive rudder angle (rudder to starboard), we have $r_0 < 0$ (turning to port); whereas for a negative rudder angle (rudder to port), we have $r_0 > 0$ (turning to starboard). That means, for a ship which is dynamically unstable, with rudder to starboard, the ship will turn to port; whereas with rudder to port, the ship will turn to starboard. This implies that the ship is uncontrollable.

CHAPTER 6: CONTROL DEVICES（1.5 hours）

Demand: Know the various kinds of control devices; know the geometrical and hydrodynamic characteristics of rudder and master the basic method of rudder design.

§6.1 Kinds of Control Devices

Ship manoeuvrability is the ability of a ship to maintain a steady course or to change the state of motion including the direction and the speed of motion under control actions, which is exerted by a control device. Hence control device is a vital element to ship manoeuvrability.

In general, control devices can be classified into two categories: active control devices and passive control devices. Active control devices convert energy into control forces actively in a more direct manner, whereas passive control devices produce control forces passively by absorbing energy from the ambient water. Passive control devices are usually used for common manoeuvring purpose, whereas active control devices are installed on ships operated at lower speeds or with special manoeuvrability requirements.

The most popular active control device is lateral thruster, which consists of a transverse tunnel with an impeller in the middle and produces a lateral force by jet reaction, see Figure 6.1. It may be installed at the bow or at the stern. A lateral thruster at the bow, the bow thruster, is generally effective at lower ship speeds at which the rudder is least effective. At higher ship speeds, the flow around the hull distorts the outflow of bow thruster; hence the thruster becomes practically ineffective. On the other hand, a lateral thruster at the stern, the stern thruster, is in boundary layer flow and is effective at higher ship speeds.

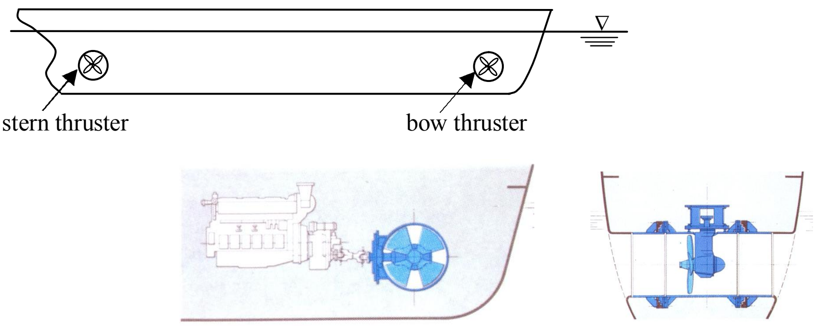
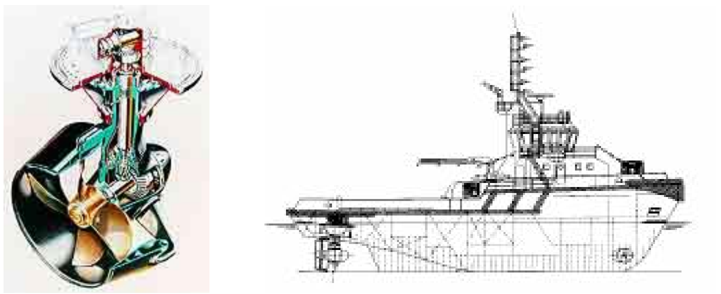


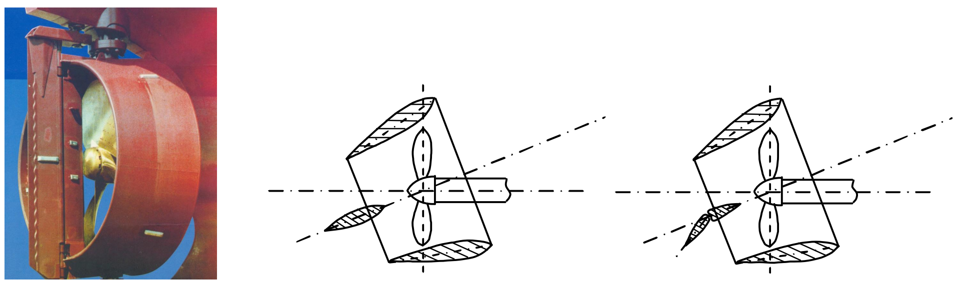
Figure 6.1 Lateral thruster as an active control device

Other active control devices include azimuthing rudder propeller, also called Z-driver, which is a azimuthing ducted propeller hinged in a vertical position and is used as a combined propulsion and steering system, see Figure 6.2, and steering nozzle, which consists of a main propeller and a steering nozzle combined with a fixed or flapped fin and creates steering force by means of a deflected jet, see Figure 6.3.



rudder propeller used on a tug

Figure 6.2 SCHOTTEL rudder propeller



BECKER Kort nozzle (a) with fixed fin (b) with flapped fin

Figure 6.3 Steering nozzle as an active control device

The simplest and most commonly used passive control device is rudder, which can be further divided into two groups, conventional rudder and unconventional rudder. Conventional rudder has simple form and structure and is used for common manoeuvring purpose, see Figure 6.4. Unconventional rudder usually has higher hydrodynamic performance and produces high lift in some active manner. It is derived from conventional rudder and is used for meeting special manoeuvrability requirements.

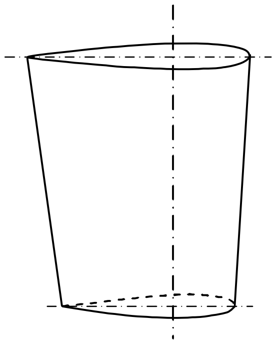
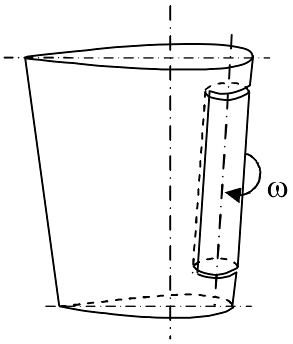
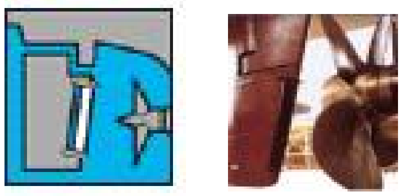



Figure 6.4 Conventional rudder as a passive control device

Unconventional rudders include rotating cylinder rudder, flap rudder, etc.

A rotating cylinder rudder incorporates a vertical rotatable cylinder on the forward edge of the rudder, see Figure 6.5. When the cylinder is rotating, circulation is generated and a lift is produced which is normal to the stream and the cylinder.

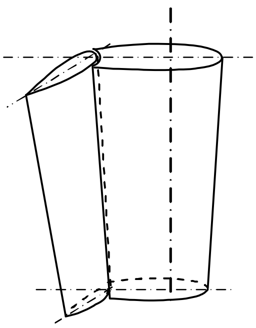
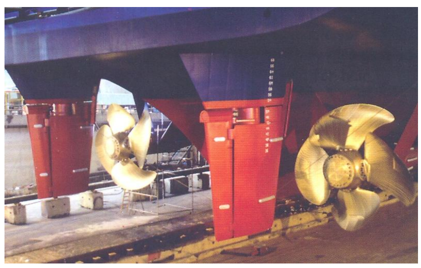




BECKER rotating cylinder rudder

Figure 6.5 Rotating cylinder rudder as a unconventional rudder

A flap rudder is a kind of all-moveable rudder with a tail flap, see Figure 6.6. The flap can turn relative to the main part of the rudder and so make the camber of the rudder variable. In this way higher lift may be produced.

BECKER flap rudder

Figure 6.6 Flap rudder as a unconventional rudder

§6.2 Rudder and Its Design

In this chapter we will only deal with rudders and will concentrate our attention on the conventional rudder and its design.

§6.2.1 Rudder types

The main types of conventional rudder include:

Simple type, rudder post; simple type, fully balanced; balanced, with fixed structure; semi-balanced underhung rudder; spade rudder; see Figure 6.7.

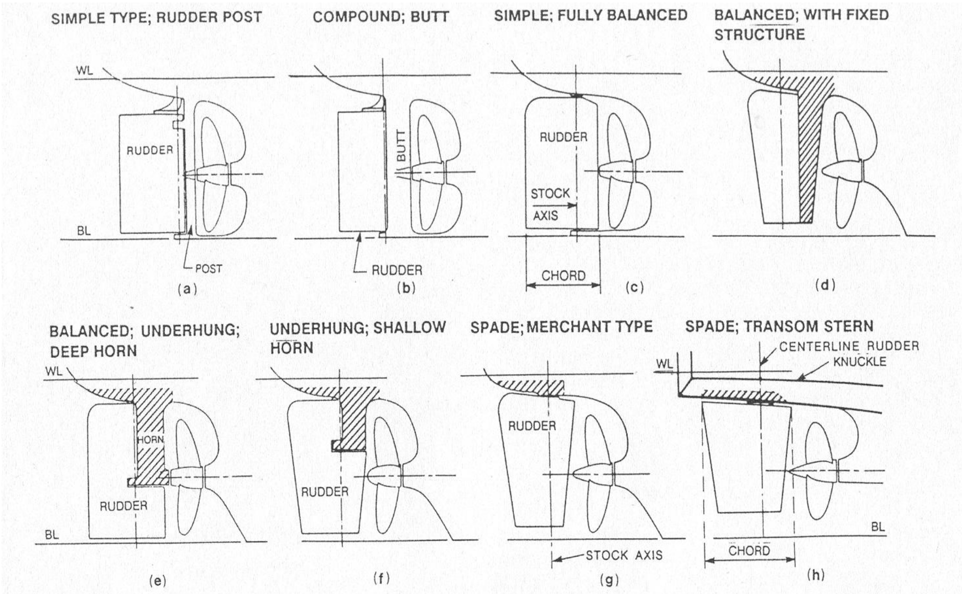
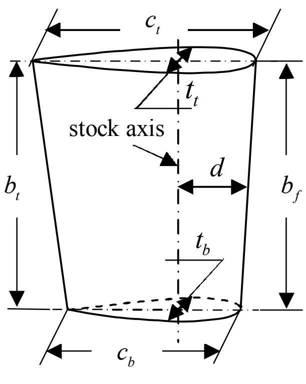


Figure 6.7 Rudder types (from *Principles of Naval Architecture*, Volume III, SNAME, 1989)

§6.2.2 Rudder geometry

A typical all-movable rudder is shown in Figure 6.8. The parameters describing rudder geometry include rudder height (span) b, chord length c, maximal thickness of section t, rudder stock position d, section shape, and rudder area A_R.



b_f rudder height at the leading edge
b_t rudder height at the trailing edge
c_t chord length at the top side
c_b chord length at the bottom side
t_t thickness of section at the top side
t_b thickness of section at the bottom side
d mean distance between the leading edge and the stock axis

Figure 6.8 A typical all-movable conventional rudder

Since most rudders are neither rectangular in shape nor of uniform thickness, the mean values of the geometrical parameters are to be used:

Mean rudder height $\bar{b} = \dfrac{b_t + b_f}{2}$; Mean chord length $\bar{c} = \dfrac{c_t + c_b}{2}$; Mean thickness $\bar{t} = \dfrac{t_t + t_b}{2}$.

The ratio $\bar{b}/\bar{c}$ is the geometric aspect ratio; the ratio $\bar{t}/\bar{c}$ is the thickness-chord ratio. The rudder area can be taken as $A_R = \bar{b} \times \bar{c}$.

§6.2.3 Rudder force in free stream

A rudder with rudder angle (angle of attack) α in free stream of velocity U is considered. A resultant hydrodynamic force $\vec{F}$ acting on the rudder will be induced. This force acts on a single point called the center of pressure, CP. The resultant force can be resolved into a lift component, L, normal to the direction of free stream and a drag component, D, parallel to the direction of the free stream. The resultant force can also be divided into a component normal to the centerplane of the rudder and a component in the centerplane, see Figure 6.9.

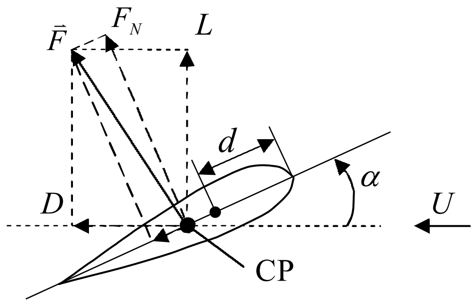


Figure 6.9 Rudder force in free stream

Denoting the component normal to the centerplane as F_N, we have

$$F_N = L\cos\alpha + D\sin\alpha. \tag{6.1}$$

The product of this normal force times the distance of the center of pressure from the axis of the rudder stock gives the hydrodynamic torque about the rudder stock, which is of importance to rudder design.

For comparison of rudder forces and moments acting on rudders of different sizes and operating at different speeds, it is convenient to express the rudder forces and moments in non-dimensional form:

$$\begin{aligned} C_L &= \frac{L}{\frac{1}{2}\rho U^2 A_R}, \quad C_D = \frac{D}{\frac{1}{2}\rho U^2 A_R}; \\ C_N &= \frac{F_N}{\frac{1}{2}\rho U^2 A_R} = C_L\cos\alpha + C_D\sin\alpha. \end{aligned} \tag{6.2}$$

Obviously, rudders are to be designed to give as large lift coefficient C_L as possible, and as small drag coefficient C_D as possible, so that the rudders are of high efficiency in hydrodynamic performance.

§6.2.4 Rudder design

The basic considerations in rudder design include the following aspects:

(1) Rudder type. For the special application purpose, decision is to be made if active or passive control device, conventional or unconventional rudder should be chosen. Also among different types of

conventional rudder one proper type is to be chosen.

(2) The location of rudder. The rudder is usually placed abaft the propeller at the stern. The propeller stream is to be utilized to improve the rudder effectiveness and hence the controllability at slow speed of ship.

(3) Number of rudder. Especially for inland waterway ships, due to the higher requirement of manoeuvrability on one hand and the limitation of water depth on the other hand, usually two or three rudders are needed to give enough rudder area.

(4) Rudder area (rudder size, height). In principle, rudder area should be as large as possible. However, rudder size is usually limited by the space at stern. Moreover, larger rudder area requires more power to be consumed by turning the rudder.

(5) Rudder section shape. Different rudder section shapes have different hydrodynamic characteristics and hence give different rudder effectiveness. Some rudder section shapes with high hydrodynamic performance were proposed based on systematical model tests.

CHAPTER 7: MEASURES TO IMPROVE SHIP MANOEUVRABILITY（0.5 hour）

Demand: Know the effects of ship form and appendages etc. on ship manoeuvrability; know the measures to be taken to improve manoeuvrability; be able to apply the knowledge in practical ship design.

Traditionally, manoeuvrability has received little attention during the ship design stage. Analysis of manoeuvrability is usually performed late in the design, when the principal dimensions and ship form have been determined according to requirements of other hydrodynamic performances. With the adoption of the IMO’s Standards for Ship Manoeuvrability, manoeuvrability has gradually become an important item in the ship design spiral. According to IMO’s standards, ship has to be designed with adequate manoeuvrability. Therefore, it is important to choose the principal dimensions and ship form by making a compromise among ship manoeuvrability and other hydrodynamic performances at the design stage.

To simplify the analysis of the effects of principal dimensions and ship form on ship manoeuvrability, we recall the definitions of manoeuvrability indexes

$$K \approx -\frac{N_\delta}{N_r}, \quad T \approx -\frac{I_z - N_{\dot{r}}}{N_r};$$

the stability criterion

$$C = -Y_v(mx_G U - N_r) + N_v(mU - Y_r) > 0$$

and the steady yaw rate and steady turning diameter given by

$$r_0 = \frac{-Y_v N_\delta + N_v Y_\delta}{C}\delta_0, \quad D_0 = 2\frac{V_0}{r_0}.$$

As discussed in Chapter 5, larger K gives a better turning ability; smaller T gives better dynamic stability on a straight course; larger K and smaller T give a better initial turning ability and course-changing ability. A positive C with larger magnitude gives better dynamic stability on a straight course. Based on this knowledge we can analyze the effects of the hydrodynamic coefficients on manoeuvrability as follows.

Since K is proportional to N_δ, a larger N_δ will give a larger K and hence a better turning ability. This means, the rudder is to be designed to give as larger yaw moment as possible. This can be achieved by designing rudder with high efficiency and larger rudder area. Actually, such a rudder at zero rudder angle will play a roll of stabilizing fin, and hence is also favorable for dynamic stability on a straight course.

Since K and T are inversely proportional to N_r, a larger N_r will give a smaller K and a smaller T, and hence a worse turning ability and a better dynamic stability on a straight course. Therefore, to obtain a better dynamic stability on a straight course, the lateral area of the centerplane of the ship should be as full as possible at both ends of the ship. On the other hand, to obtain a better turning ability, the centerplane of the ship should be full in the midship and be thin at both ends.

Since T is proportional to $I_z - N_{\dot{r}}$, a large $I_z - N_{\dot{r}}$ will give a large T and hence a worse dynamic stability on a straight course. Therefore, the mass distribution of the ship should be concentrated near the midships for a better dynamic stability on a straight course.

Since x_G is small, and Y_v, N_r are negative with large magnitudes, $Y_v N_r$ is positive with large magnitude and $-Y_v(mx_G U - N_r)$ is also positive. Moreover, since N_v, Y_r are small with uncertain sign, $mU >> |Y_r|$, $mU - Y_r$ is positive, the sign of $N_v(mU - Y_r)$ is the same as N_v. If N_v is positive, $N_v(mU - Y_r)$ will be positive, and C will be positive and large. Therefore, to obtain a large and positive C, Y_v and N_r should be negative with as large magnitudes as possible, and N_v should be positive with as large magnitude as possible. To ensure that N_v is positive, a positive lateral velocity v should induce a positive yaw moment N, and a negative lateral velocity v should induce a negative yaw moment N. This means, the lateral area of the centerplane of the ship should be fuller at the stern than at the bow. For this reason, a trim at the stern will result in a better dynamic stability.

Since Y_δ is negative and N_δ is positive with rudder deflection to starboard defined as positive, we can see from Equation (5.50) that a positive N_v will result in a smaller steady yaw rate and hence a larger steady turning diameter, which implies a worse turning stability. In other words, in order to obtain a better turning ability, N_v should be negative. Here we see once again that the course stability and the turning ability are in conflict with each other.

CHAPTER 8: BRIEF INTRODUCTION TO SEAKEEPING（0.5 hour）

Demand: Know the basic concepts of seakeeping and the relation between seakeeping performances and a safe and comfortable navigation; know the ship motions in waves with six degrees of freedom.

The second part of this course, seakeeping, deals with ship motions in ocean waves, which is contrary to ship manoeuvring motions in calm water as discussed previously.

§8.1 Definition of Seakeeping and Its Contents

Seakeeping performance concerns the ability of seagoing ships to carry out a particular mission in a given sea condition, speed and heading. A ship traveling in rough seas will undergo irregular oscillating motions. These motions will not only affect the effectiveness of the ship in attaining its mission, but also may result in danger of structural damage or capsizing. Therefore, good seakeeping performance is vitally important for a safe and effective navigation.

Seakeeping performance includes the ship performance of motions at seas, as well as that of the derived responses, such as:

Slamming: a sudden relative motion between the ship and the ambient water, especially at the bow, see Figure 8.1.

Shipping water on deck: see Figure 8.2.

Propeller racing: propeller out of water and run rapidly, see Figure 8.2.

Speed loss: including (passive) speed loss due to added resistance in waves, increased propeller loading and reduced propulsive efficiency due to propeller racing, and the so-called voluntary speed loss, i.e., to low the speed actively in order to reduce the motions induced by waves.

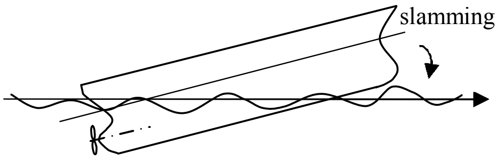


Figure 8.1 Slamming (impact at bow)

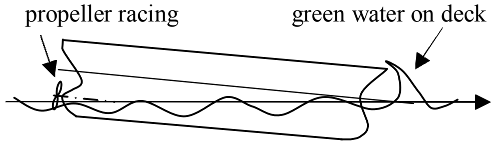


Figure 8.2 Water on deck and propeller racing

§8.2 Ship Motions in Six Degrees of Freedom

A ship floating on the free surface and traveling in rough seas will undergo oscillating motions in six degrees of freedom. These are three oscillating translations and three oscillating rotations, as shown in Figure 8.3. It is worthwhile to compare the manoeuvring motions discussed previously, which are motions in single direction.

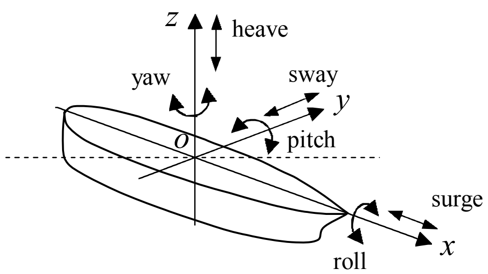


Figure 8.3 Ship motion in six degrees of freedom

CHAPTER 9: OCEAN WAVES AND STATISTIC ANALYSIS（3.5 hours）

Demand: Know the causes, classifications, statistic characteristics and expressions of ocean waves; understand and master the theories of regular and irregular waves; master the basic concepts and methods of spectrum analysis; master the response relation of linear systems; be able to apply the methods of spectrum analysis and frequency response to prediction of the statistic characteristics of ship motion in ocean waves.

§9.1 Brief Introduction

Ocean waves constitute the environment of seagoing ships and are also the cause of ship's oscillating motion at seas.

Ocean waves are characterized by their irregularity, both in time and in space. The ship motions induced by ocean waves are also irregular. However, according to the principle of superposition, the irregular motions of a ship in ocean waves can be described as the linear superposition of the responses of the ship to all the wave components, which are regular and have different lengths, amplitudes and propagating directions, where the amplitudes are assumed as small. For this reason, we will start with the introduction of the regular gravity waves of simple form in this chapter.

§9.2 Regular Waves

One of the regular gravity waves is the cosine wave, whose wave profile (free surface elevation) is given by

$$\zeta(x,t) = \zeta_a \cos(kx - \omega t), \tag{9.1}$$

where ζ_a is the wave amplitude, k is the wave number, and ω is the circular frequency of the wave.

This cosine wave is a harmonic wave propagating in the positive x-direction. Figure 9.1 shows the wave contour at a certain moment t_i. If we observe the free surface elevation at a certain position x_j, we can draw the time history of the free surface elevation as shown in Figure 9.2.

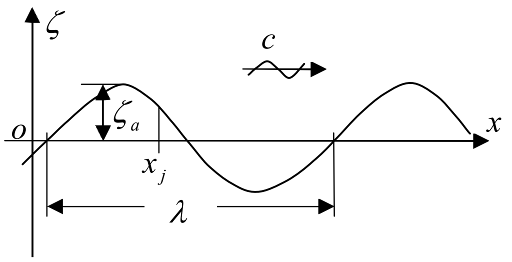


Figure 9.1 Wave contour at time t_i

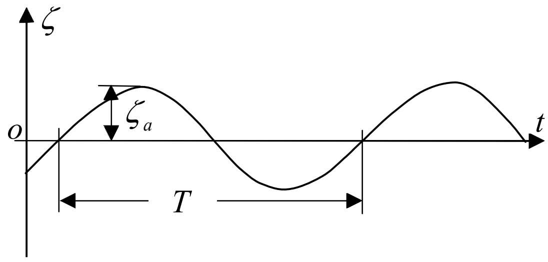


Figure 9.2 Free surface elevation at position x_j

The relations between the wave parameters are as follows

$$k = \frac{2\pi}{\lambda}, \text{ where } \lambda \text{ is the wave length;}$$

$$\omega = \frac{2\pi}{T}, \text{ where } T \text{ is the wave period.}$$

It follows that

$$\frac{\omega}{k} = \frac{\lambda}{T} = c,$$

where c is the wave propagation speed or the phase velocity (It should be pointed out that only the wave form moves with this phase velocity, not the water particles).

The wave contour described by Equation (9.1) is a contour of equal pressure (equal to the atmospheric pressure). It can be shown that the contour of equal pressure at any depth h_0 is also cosine curves with wave amplitude $\zeta_a e^{-kh_0} = \zeta_a e^{-\frac{2\pi}{\lambda}h_0}$, see Figure 9.3.

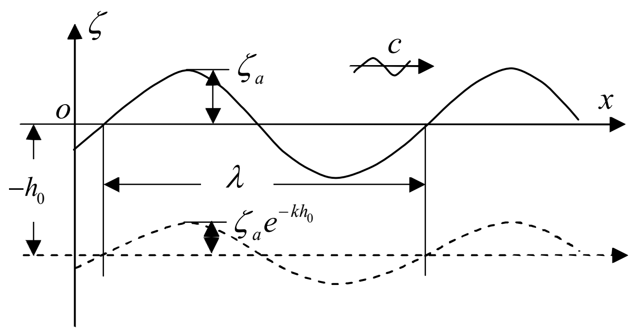


Figure 9.3 Wave contour of equal pressure

The contour of equal pressure at depth h_0 is described by

$$\zeta(x,t) = \zeta_a e^{-\frac{2\pi}{\lambda}h_0} \cos(kx - \omega t). \tag{9.2}$$

That means, the wave amplitude of the contour of equal pressure at $z = -h_0$ decays proportionally to e^{-kh_0}. The pressure at $z = -h_0$ is given by

$$p = -\rho g(z - \zeta) = \rho g h_0 + \rho g \zeta_a e^{-\frac{2\pi}{\lambda}h_0} \cos(kx - \omega t). \tag{9.3}$$

The last term of the right hand side of Equation (9.3) denotes the difference in pressure due to the wave elevation ζ at $z = -h_0$ which decays proportionally to e^{-kh_0}. This difference is due to the so-called Smith effect (Smith W. E., 1883).

§9.3 Basic Theory of Irregular Waves

Ocean waves are characterized by their irregularity, both in time and in space. Mathematically, however, they can be described as a random or stochastic process with statistical steady, or stationary characteristic appearance under short-term statistically stationary conditions. Therefore, probability and statistics theory can be used to analyze the wave characteristics.

According to the principle of superposition, irregular waves can be described as a linear superposition of infinite number of simple, regular harmonic wave components having various amplitudes, lengths, periods or frequencies and directions of propagation. Hence, an irregular wave propagating in the positive x-direction can be written as

$$\zeta(x,t) = \sum_{i=1}^{\infty} \zeta_i = \sum_{i=1}^{\infty} \zeta_{ai} \cos(k_i x - \omega_i t + \varepsilon_i), \tag{9.4}$$

where ζ_{ai}, k_i, ω_i, ε_i are the wave amplitude, wave number, wave frequency and stochastic phase of the ith wave component. Since ε_i is stochastic, the wave contour ζ_i of the i^{th} wave component is stochastic.

If all the regular wave components propagate in the same direction, long-crested waves which are two-dimensional irregular waves, as described by Equation (9.4), are obtained. On the other hand, if the wave components propagate in different directions, short-crested waves which are three-dimensional irregular waves are obtained. The later is the more general case.

§9.4 Theoretical Basic of Spectrum Analysis

The energy in a train of regular waves consists of kinetic energy associated with the orbital motion of water particles and potential energy resulted from the change of water level in wave hollows and crests. Under one wave length λ, the kinetic energy and the potential energy per unit breadth of a wave as described by Equation (9.1) are given by

$$E_k = \frac{1}{4}\rho g \zeta_a^2 \lambda, \quad E_p = \frac{1}{4}\rho g \zeta_a^2 \lambda.$$

The total average energy per unit area of free surface is

$$\overline{E} = \frac{E_k + E_p}{\lambda} = \frac{1}{2}\rho g \zeta_a^2.$$

Since ocean waves can be regarded as the linear superposition of simple, regular harmonic wave components, the energy of ocean waves can be obtained from the summation of energy of the wave components. The total average energy per unit area of free surface for wave components of frequencies (ω_i, $\omega_i + \Delta\omega$) is given by

$$\overline{E} = \sum_{\omega_i}^{\omega_i+\Delta\omega} (\frac{1}{2}\rho g \zeta_{ai}^2) = \frac{1}{2}\rho g \sum_{\omega_i}^{\omega_i+\Delta\omega} \zeta_{ai}^2 .$$

Denoting this energy by $\rho g S_\zeta(\omega_i)\Delta\omega$, where S_ζ is the so-called wave spectrum, we have

$$S_\zeta(\omega_i) = \frac{\frac{1}{2}\sum_{\omega_i}^{\omega_i+\Delta\omega} \zeta_{ai}^2}{\Delta\omega} .$$

Here $S_\zeta(\omega_i)$ denotes the wave energy density at ω_i. S_ζ shows the distribution of energy of the irregular waves among the different regular components having different frequencies.

As $\Delta\omega \to 0$, $(\omega_i, \omega_i + \Delta\omega) \to \omega_i$; Therefore, as $\Delta\omega \to 0$,

$$S_\zeta(\omega_i)d\omega = \frac{1}{2}\zeta_{ai}^2 .$$

The wave spectrum shows clearly which wave components composing the irregular ocean waves are important, see Figure 9.4.

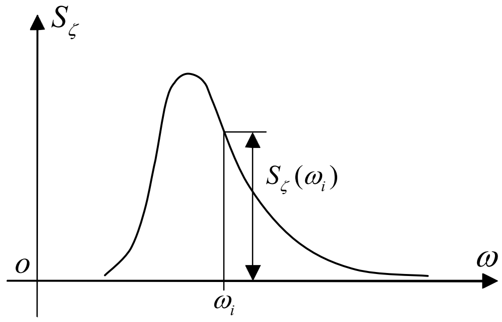


Figure 9.4 S_ζ versus ω curve

It can be shown that the wave spectrum and the variance of the wave contour have a simple relationship. The wave contour of irregular waves is given by Equation (9.4). At the origin $x = 0$, we have the free surface elevation changing with time t as follows

$$\zeta(0,t) = \sum_{i=1}^{\infty} \zeta_i = \sum_{i=1}^{\infty} \zeta_{ai} \cos(\omega_i t - \varepsilon_i) \qquad (9.5)$$

where ε_i is stochastic phase and can take any value among $(0, 2\pi)$; hence, the probability density function for ε_i is $f(\varepsilon_i) = \frac{1}{2\pi}$.

The average value (expectation) of the free surface elevation is

$$M(\zeta) = \sum_{i=1}^{\infty} M(\zeta_i) = \sum_{i=1}^{\infty} \int_0^{2\pi} \zeta_{ai} \cos(\omega_i t - \varepsilon_i) f(\varepsilon_i) d\varepsilon_i = \sum_{i=1}^{\infty} \int_0^{2\pi} \zeta_{ai} \cos(\omega_i t - \varepsilon_i) \frac{1}{2\pi} d\varepsilon_i = 0.$$

The variance, σ_ζ^2, of the free surface elevation is

$$D(\zeta)=\sigma_\zeta^2=\sum_{i=1}^{\infty}D(\zeta_i)=\sum_{i=1}^{\infty}\int_0^{2\pi}[\zeta_{ai}\cos(\omega_i t-\varepsilon_i)-M(\zeta_i)]^2 f(\varepsilon_i)d\varepsilon_i$$

$$=\sum_{i=1}^{\infty}\int_0^{2\pi}\zeta_{ai}^2\cos^2(\omega_i t-\varepsilon_i)\frac{1}{2\pi}d\varepsilon_i=\sum_{i=1}^{\infty}\frac{\zeta_{ai}^2}{2}=\sum_{i=1}^{\infty}S_\zeta(\omega_i)\Delta\omega\Big|_{\Delta\omega\to 0}=\int_0^{\infty}S_\zeta(\omega)d\omega.$$

That means, the area under the curve $S_\zeta(\omega)\sim\omega$ is equal to σ_ζ^2, the variance of the free surface elevation.

In general, we can define the n^th order moment of the area under the curve $S_\zeta(\omega)\sim\omega$ with respect to the vertical axis at $\omega=0$ as follows

$$m_{n\zeta}=\int_0^{\infty}\omega^n S_\zeta(\omega)d\omega. \tag{9.6}$$

For $n=0$, we have $m_{0\zeta}=\int_0^{\infty}\omega^0 S_\zeta(\omega)d\omega=\sigma_\zeta^2$ (variance of ζ);

For $n=2$, we have $m_{2\zeta}=\int_0^{\infty}\omega^2 S_\zeta(\omega)d\omega=\sigma_{\dot\zeta}^2$ (variance of $\dot\zeta$);

For $n=4$, we have $m_{4\zeta}=\int_0^{\infty}\omega^4 S_\zeta(\omega)d\omega=\sigma_{\ddot\zeta}^2$ (variance of $\ddot\zeta$).

The variance σ_ζ^2 can be used to estimate the statistical characteristics of the free surface elevation, for example, the average wave amplitude $\bar\zeta_a=1.25\sigma_\zeta$; the significant wave amplitude $\bar\zeta_{a\frac{1}{3}}=2.00\sigma_\zeta$.

§9.5 Response Relationships of Linear System

In this subsection we will deal with the relationship between the wave spectrum and the response spectrum of ship motions in irregular waves. This relationship can be used to predict the statistical characteristics of ship motions in ocean waves. Under the assumption that the ship in the ocean waves is a linear system which converts the energy from ocean waves into the energy of ship motions, the response spectrum of ship motions can be derived from the wave spectrum.

Ship motions in ocean waves are induced by the ocean waves. Regarding the ship as a linear system, the input to the system is the ocean waves and the output of the system is the ship motions, see Figure 9.5.

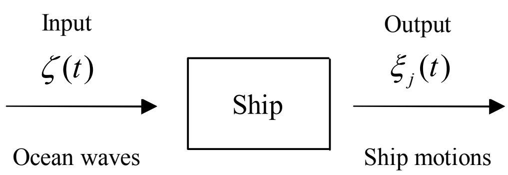


Figure 9.5 Ship in waves as a energy-transferring system

Assuming that the wave amplitudes and the amplitudes of ship motions are small, the principle of superposition holds and the induced irregular ship motions can be regarded as a linear superposition of infinite number of simple, regular motions, each of which are induced by a simple, regular wave component.

Input: $\zeta_i = \zeta_{ai}\cos(k_i x - \omega_i t + \varepsilon_i),$

Output: $\xi_i = L[\zeta_i] = \xi_{ai}\cos(k_i x - \omega_i t + \varepsilon_i + \Delta\varepsilon_i),$

where L is the linear transfer operator, ξ_{ai} is the amplitude of ship motion (it can be surge, sway, heave, roll, pitch or yaw motion), $\Delta\varepsilon_i$ is the phase difference.

Denoting the ratio of amplitudes as

$$Y_{\xi\zeta}(\omega_i) = \frac{\xi_{ai}}{\zeta_{ai}}. \tag{9.7}$$

This ratio is called the transfer function, or the Response Amplitude Operator (RAO).

The ship motion is obtained by linear superposition

$$\xi(t) = \sum_{i=1}^{\infty}\xi_i = \sum_{i=1}^{\infty}\xi_{ai}\cos(k_i x - \omega_i t + \varepsilon_i + \Delta\varepsilon_i) = \sum_{i=1}^{\infty}Y_{\xi\zeta}(\omega_i)\zeta_{ai}\cos(k_i x - \omega_i t + \varepsilon_i + \Delta\varepsilon_i). \tag{9.8}$$

Similar to the definition of wave spectrum, we can define the response spectrum of ship motions as

$$S_{\xi\zeta}(\omega_i)d\omega = \frac{1}{2}\xi_{ai}^2 = \frac{1}{2}\left[Y_{\xi\zeta}(\omega_i)\zeta_{ai}\right]^2 = Y_{\xi\zeta}^2(\omega_i)\frac{1}{2}\zeta_{ai}^2 = Y_{\xi\zeta}^2(\omega_i)S_\zeta(\omega_i)d\omega.$$

It follows that

$$S_{\xi\zeta}(\omega) = Y_{\xi\zeta}^2(\omega)S_\zeta(\omega). \tag{9.9}$$

Equation (9.9) expresses the relationship between the wave spectrum and the response spectrum. It can be used to solve the following three kinds of problems.

(1) The wave spectrum and the response amplitude operator are known, the response spectrum is to be determined in order to predict the statistic characteristics of ship motions. Most problems are of this kind.

(2) The wave spectrum and the response spectrum are known, the response amplitude operator is to be determined. Model tests are usually conducted in model basin for this purpose.

(3) The response spectrum and the response amplitude operator are known for a ship traveling at seas, the wave spectrum of the seas is to be determined.

CHAPTER 10: ROLL MOTION（4 hours）

Demand: Master the basic theory and method for linear roll motion; master the methods of roll tests with model in regular waves, to be able to determine the response amplitude operator for roll motion from the tests; know and master the methods of model tests and empirical formulae for determining the hydrodynamic coefficients of roll motion; know the estimation methods for roll amplitude; know the various kinds of roll control devices and understand the principles of roll control with these devices.

Ship motions in six degrees of freedom can be classified into two categories, i.e., the longitudinal motions in the vertical plane and the transverse motions in the horizontal plane. Surge, heave and pitch belong to the first category, whereas sway, roll and yaw belong to the second category. Among these modes of motion, roll, heave and pitch are most likely to be of large amplitudes due to the existence of the restoring force and moment, and hence are most important for seakeeping performance. In the present chapter we will deal with the roll motion, whereas in the next chapter we will deal with the heave and pitch motions.

§10.1 Linear Theory of Roll Motion

We start the analysis from the roll motion in calm water. A ship floating freely on the free surface is considered. The ship is initially forced to heel with an angle θ_0, and then let to roll freely.

§10.1.1 Roll motion in calm water without damping

First, we consider the simplest case of roll motion in calm water without damping. The moments acting on the ship include the restoring moment

$$M(\theta) = -Dh\theta$$

and the inertial moment[6]

$$M(\ddot{\theta}) = -(I_x + \Delta I_x)\ddot{\theta} = -I_{x1}\ddot{\theta}$$

where D is the displacement, h is the transverse metacentric height ($\overline{GM}$), θ is the rolling angle; $I_{x1} = I_x + \Delta I_x$, I_x is the mass moment of inertia about x axis, ΔI_x is the added mass moment of inertia; see Figure 10.1.

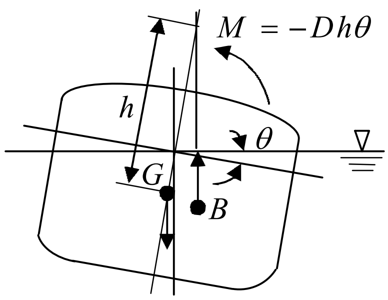


Figure 10.1 Ship in roll motion

The equation of roll motion is

$$-I_{x1}\ddot{\theta} - Dh\theta = 0, \quad \text{or} \quad \ddot{\theta} + \frac{Dh}{I_{x1}}\theta = 0.$$

Since $\frac{Dh}{I_{x1}} > 0$, we denote it as $\frac{Dh}{I_{x1}} = n_\theta^2$ and obtain

$$\ddot{\theta} + n_\theta^2\theta = 0.$$

This is an ordinary differential equation of second order, whose solution is given by

[6] According to Newtonian law of motion, $I_x\ddot{\theta} = -\Delta I_x\ddot{\theta} - Dh\theta.$ Here we denote the term $-(I_x + \Delta I_x)\ddot{\theta}$ as the inertial moment.

$$\theta(t) = c_1 \cos n_\theta t + c_2 \sin n_\theta t,$$

where c_1 and c_2 are the integral constants to be determined by the initial condition.

Here we use the initial condition $\theta = \theta_0$ and $\dot{\theta} = 0$ at $t = 0$, and obtain $c_1 = \theta_0$, $c_2 = 0$. Hence

$$\theta(t) = \theta_0 \cos n_\theta t. \tag{10.1}$$

Equation (10.1) shows that the roll motion in calm water without roll damping is harmonic with a constant amplitude equal to θ_0 and a natural frequency n_θ or a natural period T_θ:

$$n_\theta = \sqrt{\frac{Dh}{I_{x1}}}; \qquad T_\theta = \frac{2\pi}{n_\theta} = 2\pi\sqrt{\frac{I_{x1}}{Dh}}. \tag{10.2}$$

§10.1.2 Roll motion in calm water with damping

Actually, there exist damping forces against the roll motion. Here we consider only the linear component of roll damping moment under the assumption of small roll motion. The linear roll damping moment is expressed by

$$M(\dot{\theta}) = -2N_\theta \dot{\theta},$$

where $N_\theta > 0$ is the damping coefficient. In this case the equation of roll motion becomes

$$-I_{x1}\ddot{\theta} - 2N_\theta\dot{\theta} - Dh\theta = 0, \quad \text{or} \quad \ddot{\theta} + \frac{2N_\theta}{I_{x1}}\dot{\theta} + \frac{Dh}{I_{x1}}\theta = 0.$$

Denoting $\frac{N_\theta}{I_{x1}} = \nu_\theta$ and $\frac{Dh}{I_{x1}} = n_\theta^2$, we obtain the equation of roll motion

$$\ddot{\theta} + 2\nu_\theta\dot{\theta} + n_\theta^2\theta = 0. \tag{10.3}$$

The solution of Equation (10.3) is given by

$$\theta(t) = e^{-\nu_\theta t}(c_1 \cos n_{\theta 1} t + c_2 \sin n_{\theta 1} t), \tag{10.4}$$

where

$$\nu_\theta = \frac{N_\theta}{I_{x1}}, \quad n_{\theta 1} = \sqrt{n_\theta^2 - \nu_\theta^2}, \tag{10.5}$$

c_1 and c_2 are the integral constants to be determined by the initial condition. From the initial condition $\theta = \theta_0$ and $\dot{\theta} = 0$ at $t = 0$, we have

$$c_1 = \theta_0, \quad c_2 = \frac{\nu_\theta}{n_{\theta 1}}\theta_0.$$

Therefore, from Equation (10.4) we obtain

$$\theta(t) = e^{-\nu_\theta t}\left(\theta_0 \cos n_{\theta 1} t + \frac{\nu_\theta}{n_{\theta 1}}\theta_0 \sin n_{\theta 1} t\right). \tag{10.6}$$

Denoting $\theta_0 = \theta_m \cos\varepsilon_\theta$, $\frac{\nu_\theta}{n_{\theta 1}}\theta_0 = \theta_m \sin\varepsilon_\theta$, we can rewrite Equation (10.6) as

$$\theta(t) = \theta_m e^{-\nu_\theta t}\cos(n_{\theta 1}t - \varepsilon_\theta), \tag{10.7}$$

where $\theta_m = \theta_0\sqrt{1+\left(\frac{\nu_\theta}{n_{\theta 1}}\right)^2} = \theta_0\frac{n_\theta}{n_{\theta 1}}$ and $\varepsilon_\theta = \arctan\left(\frac{\nu_\theta}{n_{\theta 1}}\right)$.

From Equation (10.7) it can be seen that the amplitude of roll motion is $\theta_m e^{-\nu_\theta t}$ which decays with $e^{-\nu_\theta t}$. The frequency of roll motion is $n_{\theta 1} = \sqrt{n_\theta^2 - \nu_\theta^2} < n_\theta$; accordingly, the period of roll motion $T_{\theta 1} > T_\theta$. The phase difference of roll motion is ε_θ.

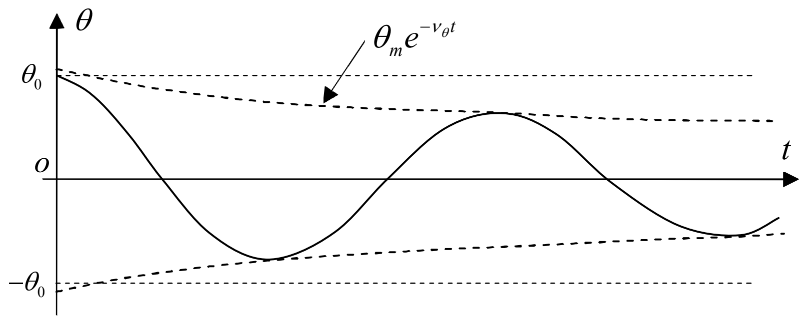


Figure 10.2 Roll motion in calm water with damping

As $t\to\infty$, $e^{-\nu_\theta t}\to 0$, resulting in $\theta\to 0$. For a better motion performance the roll angle should decay to zero as fast as possible. Therefore, ν_θ should be as large as possible. Since $\nu_\theta = N_\theta / I_{x1}$, large N_θ is favorable for good seakeeping performance.

§10.1.3 Roll motion in regular waves

Now we proceed to analysis of linear roll motion in regular waves. We consider only the case of quartering seas, since roll motions are worst for most large ships when they are traveling in quartering seas.

The key point in the analysis is the appropriate expression of the disturbance moment due to the waves. We assume that the roll angle is small, so that the linear theory of roll motion holds.

As shown in Figure 10.3, the wave contour is described by

$$\zeta(y,t) = \zeta_a \cos(ky - \omega t).$$

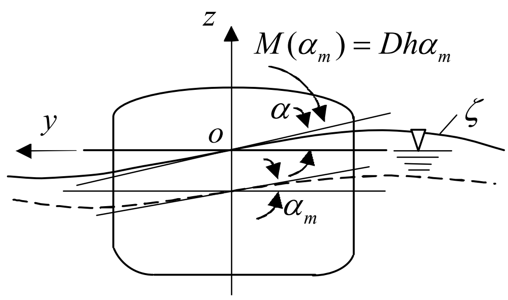


Figure 10.3 Ship in regular waves

The slope of wave contour at the origin $y = 0$ is given by

$$\tan\alpha = \left.\frac{d\zeta}{dy}\right|_{y=0} = -k\zeta_a \sin(-\omega t) = k\zeta_a \sin\omega t .$$

Hence, assuming the free surface elevation is small, we have

$$\tan\alpha \approx \alpha = k\zeta_a \sin\omega t = \alpha_0 \sin\omega t ,$$

where $\alpha_0 = k\zeta_a$ is the amplitude of the wave slope.

Similar to the moment due to the restoring force, we assume that the disturbance moment due to the waves can be expressed in the form

$$M(\alpha_m) = Dh\alpha_m ,$$

where α_m is the effective wave slope which is defined as

$$\alpha_m = \alpha_{m0} \sin\omega t , \quad \text{with} \quad \alpha_{m0} = K_\theta \alpha_0 ;$$

where K_θ is a coefficient depending on the ship form.

Then we obtain the equation of roll motion in regular waves in the form

$$-I_{x1}\ddot{\theta} - 2N_\theta\dot{\theta} - Dh\theta + Dh\alpha_m = 0, \tag{10.8}$$

where the sign of the wave disturbance moment is positive, because it is the moment that makes the ship rolling.

Equation (10.8) can be rewritten as

$$I_{x1}\ddot{\theta} + 2N_\theta\dot{\theta} + Dh\theta = Dh\alpha_{m0} \sin\omega t .$$

Using the definitions of ν_θ and n_θ, we have

$$\ddot{\theta} + 2\nu_\theta\dot{\theta} + n_\theta^2\theta = n_\theta^2\alpha_{m0} \sin\omega t . \tag{10.9}$$

Equation (10.9) is an ordinary differential equation of second order with nonzero right hand side. This equation has the solution in the form

$$\theta(t) = \theta_c + \theta^* ,$$

where θ_c is the solution of Equation (10.3) given by Equation (10.7) and θ^* is a special solution of Equation (10.9) which has the following form

$$\theta^*(t) = \theta_a \sin(\omega t + \varepsilon_{\theta\alpha}) , \tag{10.10}$$

where θ_a is the amplitude of roll motion induced by waves, ω is the frequency of induced roll motion (equal to the wave frequency), $\varepsilon_{\theta\alpha}$ is the phase difference of the roll motion relative to that of the wave slope.

As $t \to \infty$, we have $\theta_c \to 0$ and $\theta \to \theta^*$. Therefore, as $t \to \infty$, we obtain

$$\theta(t) \to \theta^*(t) = \theta_a \sin(\omega t + \varepsilon_{\theta\alpha}) . \tag{10.11}$$

From Equation (10.10) we have

$$\dot{\theta}^*(t) = \omega\theta_a \cos(\omega t + \varepsilon_{\theta\alpha}) \quad \text{and} \quad \ddot{\theta}^*(t) = -\omega^2\theta_a \sin(\omega t + \varepsilon_{\theta\alpha}). \tag{10.12}$$

Since Equation (10.10) is a special solution of Equation (10.9), substituting Equations (10.10) and

(10.12) into Equation (10.9), and comparing the coefficients of $\sin\omega t$ and $\cos\omega t$ on both sides of the equation, we have

$$\begin{aligned}&\theta_a[-2\nu_\theta\omega\sin\varepsilon_{\theta\alpha}+(n_\theta^2-\omega^2)\cos\varepsilon_{\theta\alpha}]=n_\theta^2\alpha_{m0},\\&2\nu_\theta\omega\cos\varepsilon_{\theta\alpha}+(n_\theta^2-\omega^2)\sin\varepsilon_{\theta\alpha}=0.\end{aligned}\tag{10.13}$$

From the second equation of Equation (10.13) we have

$$\tan\varepsilon_{\theta\alpha}=\frac{-2\nu_\theta\omega}{n_\theta^2-\omega^2},\quad\text{or}\quad\varepsilon_{\theta\alpha}=\arctan\left(\frac{-2\nu_\theta\omega}{n_\theta^2-\omega^2}\right);\tag{10.14}$$

$$\sin\varepsilon_{\theta\alpha}=\frac{-2\nu_\theta\omega}{\sqrt{(-2\nu_\theta\omega)^2+(n_\theta^2-\omega^2)^2}};\quad\cos\varepsilon_{\theta\alpha}=\frac{n_\theta^2-\omega^2}{\sqrt{(-2\nu_\theta\omega)^2+(n_\theta^2-\omega^2)^2}}.\tag{10.15}$$

From the first equation of Equation (10.13) we have

$$\theta_a=\frac{n_\theta^2\alpha_{m0}}{(n_\theta^2-\omega^2)\cos\varepsilon_{\theta\alpha}-2\nu_\theta\omega\sin\varepsilon_{\theta\alpha}}=\frac{n_\theta^2\alpha_{m0}}{\sqrt{(n_\theta^2-\omega^2)^2+(-2\nu_\theta\omega)^2}}.\tag{10.16}$$

Denoting $\Lambda_\theta=\dfrac{\omega}{n_\theta}$ and $\mu_\theta=\dfrac{\nu_\theta}{n_\theta}$, where Λ_θ is known as tuning factor, we have

$$\theta_a=\frac{\alpha_{m0}}{\sqrt{(1-\Lambda_\theta^2)^2+4\mu_\theta^2\Lambda_\theta^2}};\quad\varepsilon_{\theta\alpha}=\arctan\left(\frac{-2\mu_\theta\Lambda_\theta}{1-\Lambda_\theta^2}\right).\tag{10.17}$$

We denote the ratio θ_a to α_{m0} as $K_{\theta\alpha}$, that is

$$K_{\theta\alpha}=\frac{\theta_a}{\alpha_{m0}}=\frac{1}{\sqrt{(1-\Lambda_\theta^2)^2+4\mu_\theta^2\Lambda_\theta^2}},\tag{10.18}$$

$K_{\theta\alpha}$ is called magnification factor.

In the following we will discuss some special cases according to Equations (10.17) and (10.18).

(1) As $\Lambda_\theta=\dfrac{\omega}{n_\theta}=\dfrac{T_\theta}{T}\approx 0$ (i.e., $\omega<<n_\theta$ or $T>>T_\theta$), we have $K_{\theta\alpha}\approx 1$ and $\varepsilon_{\theta\alpha}\approx 0$.

In this case, $\theta_a\approx\alpha_{m0}$. The ship has a roll amplitude equal to the amplitude of effective wave slope, whereas the phase difference is zero. This case corresponds to a small ship traveling on large waves, see Figure 10.4.

(2) As $\Lambda_\theta=\dfrac{\omega}{n_\theta}=\dfrac{T_\theta}{T}\to\infty$ (i.e., $\omega>>n_\theta$ or $T<<T_\theta$), we have $K_{\theta\alpha}\to 0$, $\varepsilon_{\theta\alpha}\to -180^\circ$.

In this case, $\theta_a\approx 0$. The ship has a roll angle approximately equal to zero. This case corresponds to a big ship traveling on small waves. The ship responds hardly to waves, see Figure 10.5.

(3) As $\Lambda_\theta=\dfrac{\omega}{n_\theta}=\dfrac{T_\theta}{T}\approx 1$ (i.e., $\omega\approx n_\theta$ or $T\approx T_\theta$), we have $K_{\theta\alpha}\approx\dfrac{1}{2\mu_\theta}$, $\varepsilon_{\theta\alpha}\approx -90^\circ$.

In this case, $\theta_a\approx\dfrac{\alpha_{m0}}{2\mu_\theta}$. Since μ_θ is usually very small, θ_a will be very large. This corresponds to the case of resonance. Therefore, as $\Lambda_\theta \approx 1$ ($\omega \approx n_\theta$ or $T \approx T_\theta$), the so-called resonance will occur. This situation should be avoided.

Actually, the roll amplitude is very large not only when $\Lambda_\theta \approx 1$; in the range of $0.7 < \Lambda_\theta < 1.3$, i.e., in the so-called resonance zone, the roll amplitude is also large. Hence it should be avoided that a ship travels in the range of $0.7 < \Lambda_\theta < 1.3$.

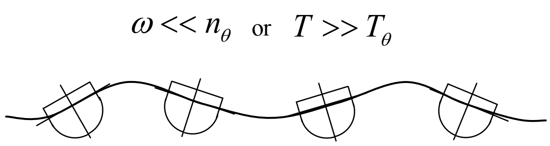


Figure 10.4 Small ship on large waves

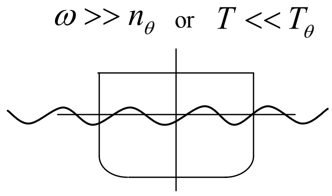


Figure 10.5 Large ship on small waves

From Equation (10.18) it can be seen that in order to reduce the roll response to wave disturbance, μ_θ should be as large as possible. Since

$$\mu_\theta = \frac{\nu_\theta}{n_\theta} = \frac{N_\theta}{n_\theta I_{x1}} = \frac{N_\theta}{I_{x1}}\sqrt{\frac{I_{x1}}{Dh}} = \frac{N_\theta}{\sqrt{DhI_{x1}}},$$

it is obvious that large N_θ, small h and small I_{x1} will result in large μ_θ, and hence small roll amplitude in waves.

Small h will not only result in large μ_θ, but also large natural period T_θ. Therefore, the favorableness of small h for seakeeping performance is double-fold. That means, from the point of view of seakeeping, h should be as small as possible. However, from the point of view of ship hydrostatic performance, transverse stability may be damnified, if h is too small. Therefore, h should be properly chosen by making a compromise between seakeeping performance and hydrostatic transverse stability. In principle, h should be as small as possible under the precondition of an enough transverse stability.

§10.1.4 Roll motion in irregular waves

Further we can analyze the statistical characteristics of linear roll motion of ships in irregular waves. The transfer function, or the response amplitude operator (RAO), for roll motion is

$$Y_{\theta\zeta}(\omega) = \frac{\theta_a}{\zeta_a}.$$

Since $\alpha_0 = k\zeta_a$, $\alpha_{m0} = K_\theta \alpha_0$ and $\theta_a = K_{\theta\alpha}\alpha_{m0}$, we have

$$Y_{\theta\zeta}(\omega) = \frac{\theta_a}{\zeta_a} = \frac{k}{\alpha_0} K_{\theta\alpha}\alpha_{m0} = kK_{\theta\alpha}K_\theta. \tag{10.19}$$

For deep water $k = \dfrac{\omega^2}{g}$, we have

$$Y_{\theta\zeta}(\omega) = \frac{\omega^2}{g} K_{\theta\alpha}K_\theta. \tag{10.20}$$

Equations (10.19) and (10.20) show clearly the relationship between the response amplitude operator (RAO) and the magnification factor.

If we know the wave spectrum $S_{\zeta}(\omega)$, once we have determined the transfer function $Y_{\theta\zeta}(\omega)$, we can obtain the response spectrum of roll motion according to Equation (9.9) and then predict the statistical characteristics of ship roll motion in ocean waves.

§10.2 Roll Test

Model test is an important method for investigating seakeeping performance. Since the hydrodynamic phenomena in roll motion are too complicated, especially when nonlinear effects are dominating, pure theoretical or numerical methods usually can not give results accurate enough, and model test is regarded as the most reliable method to predict roll motion.

Roll tests with scale models are usually carried out in towing tank or seakeeping model basin. There are mainly three kinds of roll tests

(1) Roll test in calm water. The purpose is to determine the natural period, as well as hydrodynamic coefficients, especially the damping coefficients.
(2) Roll test in regular waves. The purpose is to determine the transfer function of roll motion.
(3) Roll test in irregular waves. The purpose is to determine the transfer function of roll motion or wave spectrum.

§10.3 Determining the Hydrodynamic Coefficients of Roll Motion

Hydrodynamic coefficients of roll motion can be determined by empirical formulae or by model tests in model basin. In the following the method of model test for determining the parameters and damping coefficients of roll motion will be briefly introduced.

Roll test in calm water to determine the natural period and the damping coefficients:

Roll test with scale model in towing tank or model basin is conducted in calm water. The roll angle is recorded, as shown in Figure 10.6.

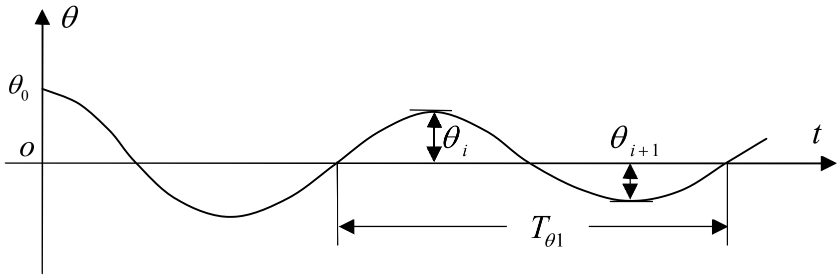


Figure 10.6 Time history of the roll angle in roll test

From the record the amplitude θ_i and the roll period $T_{\theta 1}$ are measured. $T_{\theta 1}$ is defined as

$$T_{\theta 1} = \frac{2\pi}{n_{\theta 1}} = \frac{2\pi}{\sqrt{n_\theta^2 - \nu_\theta^2}} = \frac{2\pi}{n_\theta \sqrt{1-\mu_\theta^2}} = \frac{T_\theta}{\sqrt{1-\mu_\theta^2}} .$$

Since μ_θ is very small, $T_{\theta 1}$ is approximately equal to T_θ. The natural period T_θ is an important parameter evaluating the seakeeping performance.

Denoting $\Delta\theta_i = \theta_i - \theta_{i+1}$, $\theta_{mi} = (\theta_i + \theta_{i+1})/2$, we can draw the $\Delta\theta$ versus θ_m curve as shown in

Figure 10.7. We express this relationship as

$$\Delta\theta = a\theta_m + b\theta_m^2, \tag{10.21}$$

where the coefficients a and b can be determined by curve fitting technique.

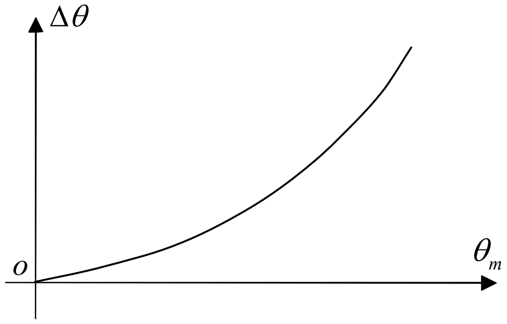


Figure 10.7 $\Delta\theta$ versus θ_m curve

Corresponding to the expression of Equation (10.21), the damping moment can be expressed as

$$M(\dot{\theta}) = -2N_\theta\dot{\theta} - W\left|\dot{\theta}\right|\dot{\theta}, \tag{10.22}$$

where N_θ and W are the coefficients of linear and nonlinear components of damping moment, respectively.

According to the relation between the work done by the damping moment and the change of roll motion energy, we can prove that there exist relations between the coefficients a, b and the coefficients N_θ, W as follows

$$2N_\theta = \frac{2Dh}{n_\theta \pi} a \quad \text{and} \quad W = \frac{3Dh}{4n_\theta^2} b;$$

or in reverse,

$$a = \frac{n_\theta \pi}{Dh} N_\theta = \frac{T_\theta}{2I_{x1}} N_\theta \quad \text{and} \quad b = \frac{4n_\theta^2}{3Dh} W = \frac{4}{3I_{x1}} W.$$

From these relations we can determine N_θ and W from a and b.

§10.4 Estimation of Roll Amplitude

As roll amplitude increases, the nonlinear component of damping moment becomes dominating. In this case the damping moment can be expressed as

$$M(\dot{\theta}) = -W\left|\dot{\theta}\right|\dot{\theta}.$$

Correspondingly, the $\Delta\theta$ versus θ_m curve can be expressed in the form

$$\Delta\theta = B\theta_m^2.$$

Letting the work done by the nonlinear damping moment equal to that done by an equivalent linear damping moment, we can derive the equivalent linear damping coefficient as

$$2\bar{N}_\theta = \frac{8}{3\pi}\theta_m \omega W = \frac{2}{\pi}\theta_m \omega I_{x1} B.$$

Accordingly,

$$2\bar{\mu}_\theta = 2\frac{\bar{\nu}_\theta}{n_\theta} = 2\frac{\bar{N}_\theta}{n_\theta I_{x1}} = \frac{2}{\pi}\frac{\theta_m \omega I_{x1}}{n_\theta I_{x1}}B = \frac{2}{\pi}\frac{\omega}{n_\theta}\theta_m B.$$

When resonance occurs, we have $\frac{\omega}{n_\theta} = 1$. Then the maximal roll amplitude can be estimated as

$$\theta_a = \frac{\alpha_{m0}}{2\bar{\mu}_\theta} = \frac{\alpha_{m0}}{\frac{2}{\pi}\theta_m B} \approx \frac{\alpha_{m0}}{\frac{2}{\pi}\theta_a B}, \quad \text{or} \quad \theta_a = \sqrt{\frac{\pi\alpha_{m0}}{2B}}.$$

§10.5 Roll Control Devices

For motions with effects of restoring forces or moments, such as heave, roll and pitch, resonance may occur when the natural frequency equal to the wave frequency. At resonant frequency the magnification factor for roll motion can be very large due to very small roll damping. This implies severe roll motion which has great negative effects on seakeeping performances. For example, large roll motions may cause the goods shifting, resulting in loss of transverse stability; or it may deteriorate the working condition of the crew. Therefore, it is necessary for us to take measures to reduce or to control the roll motion response to wave disturbance. For this purpose anti-rolling devices, or the so-called roll stabilizers, can be used. In the following the anti-rolling devices in common usage are introduced.

Anti-rolling devices can be classified into three groups, the passive, the controlled-passive and the active ones.

The most commonly used passive anti-rolling device is bilge keel, which is mounted on the ship hull near the bilge and is used to increase the roll damping, see Figure 10.8. Bilge keel is the simplest anti-rolling device and most of seagoing ships are equipped with it.

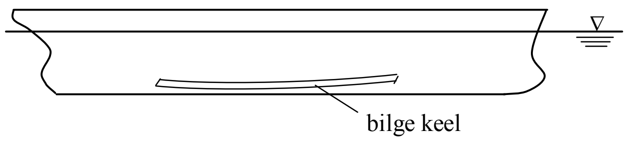


Figure 10.8 Bilge keel as passive anti-rolling device

The passive anti-rolling tank is another one of passive anti-rolling devices. This includes free surface tank and U-tube tank, see Figure 10.9. With careful design the fluid motion in these tanks causes a moment which counteracts part of the disturbance moment by waves.

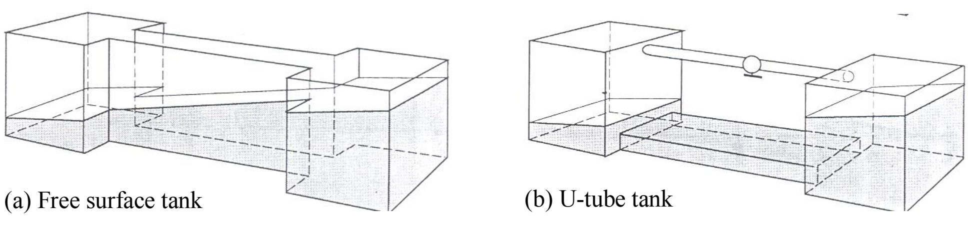


Figure 10.9 Passive anti-rolling tanks

The controlled-passive roll tank is a controlled-passive anti-rolling devices where the fluid motion in the tank is controlled in some manner.

The most popular active anti-rolling device is the stabilizing fins which are mounted on both sides of ship hull, see Figure 10.10. This kind of anti-rolling device is generally used on warships or luxury liners which have special requirements of roll performance. When the fins are set at opposite angles of attack, the lifts caused by the flows over them form a moment about the longitudinal axis. The angles of attack can be controlled to create a moment against the roll motion. When not being used to reduce roll motion the fins are usually retracted to avoid the drag acting on the fins.

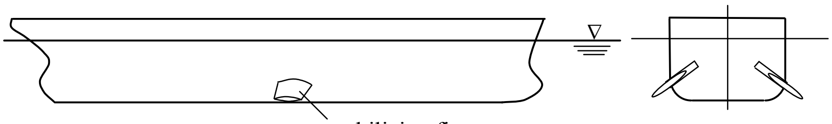


Figure 10.10 Stabilizing fins as active anti-rolling device

The active tank stabilizer is another kind of active anti-rolling device. This device uses a flow pump to force the water in the tank to flow from one side of the ship to another side to create actively a moment against roll motion.

CHAPTER 11: PITCH AND HEAVE MOTIONS（4 hours）

Demand: Know and master the theoretical and numerical methods for analysis of ship motions; know the method of spectrum analysis and the methods of model test for determining the transfer function for pitch and heave motions; know the various kinds of derived responses due to ship motions; know the basic concepts of ship motion in oblique waves.

Pitch and heave motions are longitudinal motions in vertical plane. These motions are affected by restoring force and moment, and may respond with large amplitudes when resonance occurs. Although the responses of pitch and heave motions are not so serious as that of roll motion, the derived responses due to these motions have remarkable negative effects on the seakeeping performance.

§11.1 Theoretical Computation for Seakeeping Problems

§11.1.1 Brief introduction

For the longitudinal motions the viscous effects are usually small and can be neglected. For this reason, hydrodynamic characteristics of pitch and heave motions are usually investigated by theoretical and numerical methods based on potential theory.

Traditional theoretical and numerical methods include the strip theory and slender-body theory. Based on the assumption that the ship is slender, i.e., the longitudinal dimension of the ship is much larger than the transverse and vertical dimensions, the three-dimensional flow around the ship is transformed into two-dimensional flow around the sections of the ship, which simplifies the problem greatly.

More advanced theoretical and numerical method is the three-dimensional method such as Boundary Element Method (BEM), or the so-called panel method, which is firstly applied for linear motions of small amplitude with linearized boundary conditions on the free surface and on the mean wetted body surface, then for nonlinear motion of large amplitude with nonlinear boundary conditions on the free surface and body surface. The unsteady problem of ship motion is traditionally solved by using three-dimensional method in frequency domain; more actually it is solved in time domain, allowing to predict the instantaneous ship motion with large amplitudes.

§11.1.2 Coordinate systems

A ship traveling in forward motion with a constant speed U at seas is considered. At the same time the ship undergoes an oscillating motion of small amplitude in six degrees of freedom about its mean position.

The j-th mode of motion is denoted by ξ_j, where $j=1,2,3$ correspond to translating motions surge, sway and heave, respectively; and $j=4,5,6$ correspond to rotating motions roll, pitch and yaw, respectively.

For seakeeping problem, three right-handed coordinate systems are adopted, see Figure 11.1:

(1) Earth-fixed coordinate system $o_0-x_0y_0z_0$. A frame of reference fixed on the earth, with x_0-axis pointing to the direction of ship forward motion and z_0-axis pointing upwards.

(2) Moving coordinate system $o'-x'y'z'$. A frame of reference moving with the speed U. At the initial moment the moving coordinate system coincides with the earth-fixed coordinate system.

(3) Body-fixed coordinate system $o-xyz$. A frame of reference fixed on the ship and moving together with the ship. This coordinate system has oscillating motions in six degrees of freedom relative to the moving coordinate system.

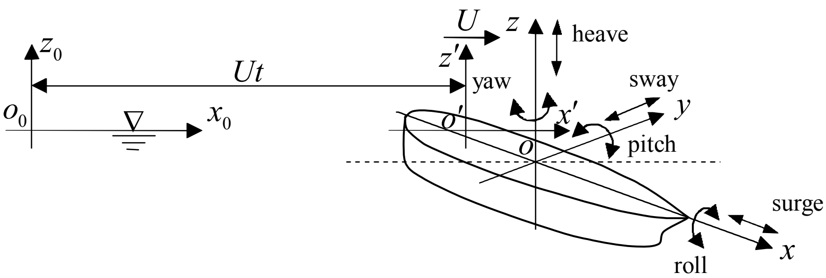


Figure 11.1 Coordinate systems used in ship motion problem

§11.1.3 Forces acting on the ship

Here we will only consider the cases in linear theory, where the amplitudes of waves and the ship motions are assumed to be small.

The forces acting on a ship traveling at seas include:

(1) Hydrostatic restoring forces

The i-th component of hydrostatic restoring forces (net hydrostatic forces) is given by

$$F_{Ci}=-\sum_{j=1}^{6}C_{ij}\xi_j,\quad (i=1,2,\cdots,6) \tag{11.1}$$

where ξ_j is the j-th mode of motion; C_{ij} is the hydrostatic restoring force coefficient which corresponds to the i-th component of hydrostatic forces induced by the unit j-th mode of motion; the minus sign in Equation (11.1) means the force is against the motion.

Assuming that the ship is symmetrical about its longitudinal centerplane, the hydrostatic restoring force coefficient C_{ij} is given by

$$C_{ij}=\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho g A_w & 0 & -\rho g A_w x_f & 0 \\ 0 & 0 & 0 & \rho g\left(S_{22}+\nabla z_B\right)-mgz_G & 0 & 0 \\ 0 & 0 & -\rho g A_w x_f & 0 & \rho g\left(S_{11}+\nabla z_B\right)-mgz_G & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

where ρ is the mass density of fluid; g is the gravitational acceleration; A_w is the area of water-plane; x_f is the longitudinal coordinate of the centre of flotation; ∇ is the displacement; m is the mass of the ship; z_B and z_G are the vertical coordinates of the centre of buoyancy and the centre of gravity, respectively. S_{11} and S_{22} are given by

$$S_{11}=\iint_{A_w} x^2 dxdy=\int_L 2yx^2 dx\,;\quad S_{22}=\iint_{A_w} y^2 dxdy=\int_L \frac{2}{3}y^3 dx\,,$$

where $y(x)>0$ is the semi-breadth of water-plane.

It should be noted that the matrix of C_{ij} is symmetrical, which means $C_{ij}=C_{ji}$. C_{ij} has to be positive to ensure the ship has hydrostatic stability.

(2) Hydrodynamic forces

Hydrodynamic forces are composed of two parts: the radiation forces due to the oscillating motions of the ship and the wave exciting forces due to the diffracted waves and the incident waves:

The i-th component of radiation forces is given by

$$F_{Ri}=-\sum_{j=1}^{6}(A_{ij}\ddot{\xi}_j+B_{ij}\dot{\xi}_j)\,,\quad (i=1,2,\cdots,6) \tag{11.2}$$

where A_{ij} is the added mass coefficient, B_{ij} is the damping coefficient.

The i-th component of wave exciting forces is

$$F_{Ei}=F_i^D+F_i^I\,,\quad (i=1,2,\cdots,6) \tag{11.3}$$

where F_i^D is the diffraction exciting forces due to the diffracted waves and F_i^I is the so-called Froude-Krylov exciting forces induced by the incident waves[7].

§11.1.4 Linearized equations of ship motion

From the Newtonian law of motion we obtain the linearized equations of ship motion as follows

[7] The concept of approximate exciting forces due to incident waves was first introduced in the classical work on ship roll motion done by Froude in 1861 and generalized to ship motions in six degrees of freedom by Krylov in 1898.

$$\sum_{j=1}^{6} M_{ij}\ddot{\xi}_j = F_{Ci} + F_{Ri} + F_i^D + F_i^I , \quad (i = 1, 2, \cdots, 6) \tag{11.4}$$

where M_{ij} is the mass matrix (inertia matrix) given by

$$M_{ij} = \begin{pmatrix} m & 0 & 0 & 0 & mz_G & 0 \\ 0 & m & 0 & -mz_G & 0 & mx_G \\ 0 & 0 & m & 0 & -mx_G & 0 \\ 0 & -mz_G & 0 & I_x & 0 & -I_{xz} \\ mz_G & 0 & -mx_G & 0 & I_y & 0 \\ 0 & mx_G & 0 & -I_{xz} & 0 & I_z \end{pmatrix}.$$

It can be seen that M_{ij} is a symmetric matrix, i.e., $M_{ij} = M_{ji}$.

Substituting the expressions of forces, Equations (11.1) ~ (11.3), into the equations of motion, Equation (11.4), we obtain

$$\sum_{j=1}^{6} [(M_{ij} + A_{ij})\ddot{\xi}_j + B_{ij}\dot{\xi}_j + C_{ij}\xi_j] = F_i^D + F_i^I , \quad (i = 1, 2, \cdots, 6) \tag{11.5}$$

M_{ij}, C_{ij} and F_i^I can be calculated directly, whereas the added mass coefficient A_{ij} and the damping coefficient B_{ij} are to be determined by solving the radiation problem; the diffraction exciting forces F_i^D are to be determined by solving the diffraction problem.

§11.2 Pitch and Heave in Irregular Waves

The statistical characteristics of pitch and heave motion responses to irregular waves can be estimated by spectrum analysis. The key point is to determine the response spectrums or the response amplitude operators (RAO).

For pitch motion:

$$Y_{\phi\zeta}(\omega) = \frac{\phi_a}{\zeta_a}, \quad S_{\phi\zeta}(\omega) = Y_{\phi\zeta}^2(\omega) S_\zeta(\omega);$$

where ϕ_a is the pitch amplitude.

For heave motion:

$$Y_{Z\zeta}(\omega) = \frac{Z_a}{\zeta_a}, \quad S_{Z\zeta}(\omega) = Y_{Z\zeta}^2(\omega) S_\zeta(\omega)$$

where Z_a is the pitch amplitude.

$Y_{\phi\zeta}(\omega)$ and $Y_{Z\zeta}(\omega)$ can be obtained from model tests or theoretical and numerical methods. After that, the response spectrums of pitch and heave motions can be determined and the statistical characteristics of pitch and heave motions can be estimated by spectrum analysis.

§11.3 Derived Responses due to Ship Motion

Derived responses from motions of notable amplitude in six degrees of freedom have important and negative effects on seakeeping performance. These responses are mostly due to the pitch and heave motions and include:

(1) Local motions, i.e., vertical or horizontal motions, velocities and accelerations at special points;
(2) Relative motions between a location in the ship and the encountered waves;
(3) Shipping water on deck (green water on deck);
(4) Slamming;
(5) Yawing and broaching-to;
(6) Added resistance and powering in waves;
(7) Wave bending moments and loads on ship.

All these responses are usually related to nonlinear effects.

§11.4 Ship Motion in Oblique Waves

Considering that a ship travels with a constant forward speed U at an angle μ to regular waves, as shown in Figure 11.2. $\mu = 0^\circ$ corresponds to following sea, $\mu = 90^\circ$ to beam sea and $\mu = 180^\circ$ to head sea.

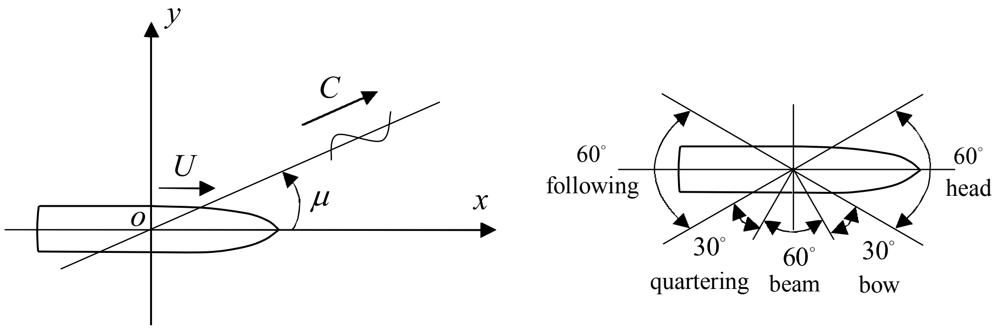


Figure 11.2 Ship traveling at an angle to waves

The wave velocity relative to the ship is

$$C_e = C - U\cos\mu .$$

The wave period relative to the ship is

$$T_e = \frac{\lambda}{C_e} = \frac{\lambda}{C - U\cos\mu} = \frac{\frac{\lambda}{C}}{1 - \frac{U}{C}\cos\mu} = \frac{T}{1 - \frac{U}{C}\cos\mu} .$$

T_e is called the period of encounter. The corresponding frequency of encounter is defined as

$$\omega_e = \frac{2\pi}{T_e} = \frac{2\pi}{\lambda}(C - U\cos\mu) = \frac{2\pi}{\lambda}C - \frac{2\pi}{\lambda}U\cos\mu = \frac{2\pi}{T} - kU\cos\mu = \omega - kU\cos\mu .$$

For deep water $k = \omega^2 / g$, we obtain

$$\omega_e = \frac{2\pi}{T_e} = \omega - \frac{\omega^2}{g}U\cos\mu . \tag{11.6}$$

Therefore, if a ship travels with a forward speed U at an angle μ to regular waves, the effective wave frequency is the frequency of encounter. In all formulas containing the wave frequency ω, ω should be replaced by the frequency of encounter ω_e. Correspondingly, T should be replaced by T_e. For example, when roll resonance occurs, we have $T_e = T_\theta$ or $\omega_e = n_\theta$. Therefore, when

$$T_e = \frac{\lambda}{C - U\cos\mu} = T_\theta \quad \text{or} \quad \omega_e = \omega - \frac{\omega^2}{g} U\cos\mu = n_\theta, \tag{11.7}$$

roll resonance will occur.

Noting that the relations among the wave parameters are

$$C = \frac{\lambda}{T},\ T = \frac{2\pi}{\omega} \text{ and } k = \frac{2\pi}{\lambda} \quad \Rightarrow \quad C = \frac{\omega}{k};$$

In deep water $k = \omega^2 / g$, resulting in

$$C = \frac{\omega}{k} = \frac{g}{\omega} \quad \Rightarrow \quad \lambda = C \cdot T = \frac{gT}{\omega} = \frac{gT^2}{2\pi} \quad \Rightarrow \quad T = \sqrt{\frac{2\pi}{g}\lambda} \text{ and } \lambda = \frac{2\pi}{g}C^2.$$

If a ship travels in deep water on regular waves of wave length λ, wave velocity C and wave frequency ω, the ship velocity U and the angle μ can be chosen according to Equation (11.7) to avoid roll resonance.

CHAPTER 12: DESIGN OF SEAKEEPING PERFORMANCES AND FULL-SCALE TRIALS（2 hours）

Demand: Know the various kinds of seakeeping performance indexes to be considered in ship design; know the effects of ship principal dimensions and ship form on seakeeping performances; to be able to apply the related knowledge in ship design to improve the seakeeping performances; know the methods of full-scale trails of seakeeping and know how to analyze the results.

§12.1 Seakeeping Performance Index

Seakeeping performance of a ship depends not only on its form and the principal dimensions, but also on the environment in which it is operated. In general, the methodology for assessing seakeeping performance depends on four factors:

(1) Mission. Missions are assigned to the ship, which may be subdivided into three categories:

(a) Port-to-port transportation of goods or people;

(b) Military missions carried out entirely at sea;

(c) Commercial missions at sea, such as fishing, oil drilling, etc.

(2) Environment. Waves, wind speed, which are defined as a function of both location and time, and quantified by sea state numbers. Significant wave height, $H_{1/3}$, and modal wave period, T_m, are the two parameters currently used to characterize the sea state.

(3) Ship responses. Ship motion responses are described as a function of sea state numbers; ship velocity and ship heading angle relative to waves will affect ship motion performance and determine the maximum attainable speed in waves.

(4) Seakeeping performance criteria. This is a key element in developing a methodology for assessing ship operational performance at seas; and these criteria determine whether or not a mission can be carried out. Examples of seakeeping performance criteria are as follows:

(a) Absolute motion amplitudes: Roll angle, pitch angle, vertical displacement of points on flight deck, etc;

(b) Absolute velocities and accelerations: Vertical acceleration, lateral acceleration, etc;

(c) Motions relative to sea: Frequency of slamming, frequency of emergence of a sonar dome, frequency of deck wetness; probability of propeller emergence.

Prescribed limiting values are given for these criteria to make certain if the seakeeping performance is acceptable or unacceptable.

§12.2 Effects of Ship Principal Dimensions and Form on Seakeeping Performances

Theory shows that the effects of longitudinal motions, i.e., surge, heave and pitch, and transverse motions, i.e., sway, roll and yaw, can be considered separately. For this reason, the effects of ship principal dimensions and form on the seakeeping performance will be discussed separately for the two kinds of motion,

§12.2.1 Factors affecting pitch and heave motions

In general, for conventional mono-hull ships in head seas, the longer the ship, the less the average wave excitation; that means, longer ship length is favorable.

As the draft increases, slamming is reduced.

With increasing breadth-to-length ratio, there is a distinct increase in damping. Therefore, from the point of view of roll motion response, larger breadth-to-length ratio is favorable.

With increasing length -to-draft ratio, the relative bow motion is reduced. Hence larger length -to-draft ratio is favorable for green water on deck.

§12.2.2 Factors affecting roll motion

The first consideration in design to reduce roll motion is to introduce artificial damping by using anti-roll device like bilge keels which are the most simplest and yet effective roll control device.

Since resonance effects are very important in rolling, it is desirable to design for a natural period that avoids resonance zone.

As discussed in Chapter 10, smaller transverse metacentric height is favorable for roll motion performance.

Besides the anti-roll devices introduced in Chapter 6, rudders can also be used as roll control devices, although they are mainly used for changing the course and controlling the heading of the ship. This application has the advantage of making use of an existing system, thus reducing the cost.

§12.3 Full-scale Trials of Seakeeping

To be completed.

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