Mathematical Model for Manoeuvring Ship Motion (MMG Model)

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SUMMARY

For the numerical simulation of manoeuvring ship motion, many kind of mathematical model has been proposed, and developed with computing tools. Each mathematical model has merits for each simulation purpose. Simple rudder to yaw response model is also useful for real time simulation and control.

In this paper, the author reviews the mathematical model for manoeuvring ship motion, particularly MMG model that is now widely used for many kinds of simulation.

1. INTRODUCTION

When the hydrodynamic forces during manoeuvring can be provided for each time step, no mathematical model is required. In such case, only the equation of motion is used for simulations. Recent computer Fluid Dynamics (CFD) techniques gradually makes it possible to do so, however, these simulation techniques have not been completed yet.

In order to describe the hydrodynamic forces for each time step, mathematical models are usually used. However, the expression is not so simple because of the existence of unsteady contributions. Consequently, the expression of hydrodynamic forces in the mathematical model has been assumed that they just depend on velocity and acceleration components. That is well known "quasi-steady approach". In the simulation of manouvring ship motion, the following equation of motion for 3 degrees of freedom is generally used.

$$\left.\begin{aligned} m(\dot{u}_G - v_G r) &= X_G = X_{GA} + X_{GS} \\ m(\dot{v}_G + u_G r) &= Y_G = Y_{GA} + Y_{GS} \\ I_{zz}\dot{r} &= N_G = N_{GA} + N_{GS} \end{aligned}\right\} \qquad (1)$$

or

$$\left.\begin{aligned} m(\dot{u} - vr - x_G r^2) &= X_M = X_A + X_S \\ m(\dot{v} + x_G\dot{r} + ur) &= Y_M = Y_A + Y_S \\ I_{zz}\dot{r} + m x_G(\dot{v} + ur) &= N_M = N_{GA} + N_{GS} \end{aligned}\right\} \qquad (2)$$

where, m : mass of ship

I_{ZZ} : moment of inertia of ship in yaw motion

Eq.(1) is the equation of motion with the center of gravity of ship: G, and the notation of u_G,v_G and r are velocity components at center of gravity of ship, X_G, Y_G and N_G represent the hydrodynamic forces and moment acting on G. Meanwhile, eq.(2) is the motion of equation referred to mid-ship: M, and the notation of u, v and r are velocity components at mid-ship, X_M, Y_M and N_M acting on M. x_G represents the location of G in x-axis direction. In each equation, the suffix "A" represents forces and moment by acceleration components such as $\dot{u}, \dot{v}, \dot{r}$, and the suffix "S" by velocity components u, v, r including rudder angle δ and propeller revolution n.

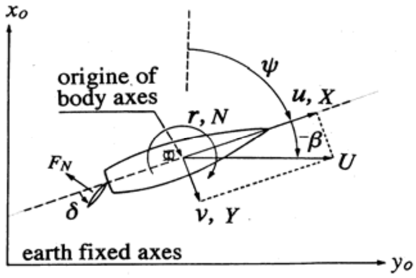

Fig.1 Co-ordinate system

$$\left.\begin{aligned} X_A &= f_{AX}(\dot{u}) = X_{\dot{u}}\dot{u} \\ Y_A &= f_{AY}(\dot{v}, \dot{r}) = Y_{\dot{v}}\dot{v} + Y_{\dot{r}}\dot{r} \\ N_A &= f_{AN}(\dot{v}, \dot{r}) = N_{\dot{v}}\dot{v} + N_{\dot{r}}\dot{r} \\ \\ X_S &= f_{SX}(u, v, r, \delta, n) \\ Y_S &= f_{SY}(u, v, r, \delta, n) \\ N_S &= f_{SN}(u, v, r, \delta, n) \end{aligned}\right\} \qquad (3)$$

For the expression of these hydrodynamic forces and moment, some polynomial functions with acceleration and velocity components are used. The coefficients of them correspond to hydrodynamic derivatives and can be obtained from,

1) Captive model test such as oblique towing test (OTT), rotating arm test (RAT), circular motion test (CMT) and planar motion mechanism (PMM) test.
2) Numeric calculation
3) Identification to the free-model tests or full- scale trials
4) Database of hydrodynamic derivatives

2. TOTAL FORCE MODEL

As mentioned above, steady hydrodynamic forces X_S, Y_S and moment N_S are the functions of u, v, r, rudder angle δ and propeller revolution n. In the total force model, these functions are described as the following polynomials using Tylor expansion, for example.

$$\left.\begin{aligned}X_S &= X_{vv}v^2 + X_{vr}vr + X_{rr}r^2 \\ &\quad + X_0u^2 + X_{un}un + X_{nn}n^2 + X_{\delta\delta}\delta^2 \\ Y_S &= Y_v v + (Y_r - m_x)r + Y_{vvv}v^3 + Y_{vvr}v^2r + Y_{vrr}vr^2 + Y_{rrr}r^3 \\ &\quad + Y_\delta\delta + Y_{\delta v}\delta v + Y_{\delta r}\delta r + Y_{\delta n}\delta n \\ N_S &= N_v v + N_r r + N_{vvv}v^3 + N_{vvr}v^2r + N_{vrr}vr^2 + N_{rrr}r^3 \\ &\quad + N_\delta\delta + N_{\delta v}\delta v + N_{\delta r}\delta r + N_{\delta n}\delta n\end{aligned}\right\} \tag{4}$$

These total force models can easily express the steady hydrodynamic forces, and has been widely used [1]. The coefficients in eq.(4) are called hydrodynamic derivatives, and can be obtained by some model tests using scaled model. These mathematical models can be usually applied for the simulation of the specified ship.

3. RUDDER TO YAW RESPONSE MODEL

Above mathematical model can be reduced as the following equation eliminating the non-linear terms when ship motion is small.

$$\left.\begin{aligned}X_S &= X_0u^2 \\ Y_S &= Y_v v + (Y_r - m_x)r + Y_\delta\delta \\ N_S &= N_v v + N_r r + N_\delta\delta\end{aligned}\right\} \tag{5}$$

The eq.(3) is also reduced as the followings,

$$\left.\begin{aligned}X_A &= -m_x\dot{u} \\ Y_A &= -m_y\dot{v} \\ N_A &= -J_{zz}\dot{r}\end{aligned}\right\} \tag{6}$$

,where m_x, m_y and J_{zz} are the added mass and moment of inertia.

From eq.(5) and (6), the following response model of rudder to yaw can be introduced under the constant ship's speed. This is well-known Nomoto's formula [2]

$$T_1T_2\ddot{r} + (T_1 + T_2)\dot{r} + r = K(\delta + T_3\dot{\delta}) \tag{7}$$

Coefficients in the above equation: T_1T_2 , (T_1+T_2) , T_3 and K are as the followings.

$$\begin{aligned}T_1T_2 &= (I_{zz}+J_{zz})(m+m_y)/D \\ T_1+T_2 &= -\{(I_{zz}+J_{zz})Y_v + (m+m_y)N_r\}/D \\ KT_3 &= N_\delta(m+m_y)/D \\ K &= -(Y_\delta N_v + N_\delta Y_v)/D \\ D &= N_rY_v + \{(m+m_x)U - Y_r\}N_v\end{aligned}$$

Although the eq.(7) express only the small steering motion, it is very simple and useful particularly for ship control field such as autopilot.

These models including the total force model, however, can not simulate the ship motion even when hull form is the same but rudder or propeller is just modified from original one, because almost every coefficient in eq.(4) or eq.(5) becomes different from the original one. Therefore, another model tests with altered rudder or propeller are required for obtaining the coefficients, which consumes a lot of cost and time for the manouevring simulations.

4. MMG MODEL

Japanese research group named Manoeuvring Mathematical Modeling Group (MMG) proposed a new concept of mathematical model during 1976-1980 [3][4]. This model is called as MMG model. It consists of the individual open water characteristics of hull, propeller, and rudder, and the interaction effect between them. The insistence of the model is put on that it has physical meaning as much as possible and is constructed as simple as possible. This concept is demonstrated by the following schematic diagram.

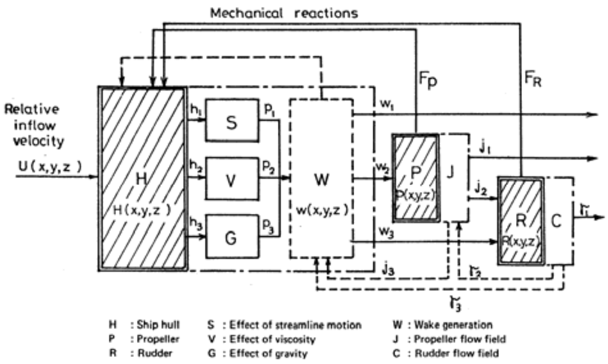

Fig.2 Schematic diagram of MMG model [3].

The expression of steady forces and moment can be described separating into the following components from above the viewpoint of the physical meaning.

$$\left.\begin{aligned} X_S &= X_H + X_R + X_P \\ Y_S &= Y_H + Y_R + Y_P \\ N_S &= N_H + N_R + N_P \end{aligned}\right\} \quad (8)$$

,where the subscripts H, P and R refer to hull, propeller and rudder respectively.

4.1 Forces and Moment Acting on Hull

X_H, Y_H and N_H are approximated by the following non-dimensional polynomials of β and r' like the total force model. This expression is just one example and many kinds of polynomials have been proposed.

$$\left.\begin{aligned} &X_H = (\rho/2)LdU^2 \\ &\times\left\{X'_0 + X'_{\beta\beta}\beta^2 + \left(X'_{\beta r} - m'_y\right)\beta r' + X'_{rr}r'^2 + X'_{\beta\beta\beta\beta}\beta^4\right\} \\ &Y_H = (\rho/2)LdU^2 \\ &\times\left\{Y'_\beta\beta + \left(Y'_r - m'_x\right)r' + Y'_{\beta\beta\beta}\beta^3 + Y'_{\beta\beta r}\beta^2 r' + Y'_{\beta rr}\beta r'^2 + Y'_{rrr}r'^3\right\} \\ &N_H = (\rho/2)L^2dU^2 \\ &\times\left\{N'_\beta\beta + N'_r r' + N'_{\beta\beta\beta}\beta^3 + N'_{\beta\beta r}\beta^2 r' + N'_{\beta rr}\beta r'^2 + N'_{rrr}r'^3\right\} \end{aligned}\right\} \quad (9)$$

Drift angle: β and non-dimensional turning rate: r' are expressed as β=-sin^{-1}(v/U), r' =r(L/U). The notations of u and v are velocity components and U is the resultant velocity at the mid-ship. In Fig.3, an example of measured hull force and moment coefficients by CMT is shown. Forces and moment are made non-dimensional by $(\rho/2)LdU^2$ and $(\rho/2)L^2dU^2$ respectively and plotted against drift angle. The several curves in each figures show the analyzed characteristics using eq.(9) with the parameter of r'.

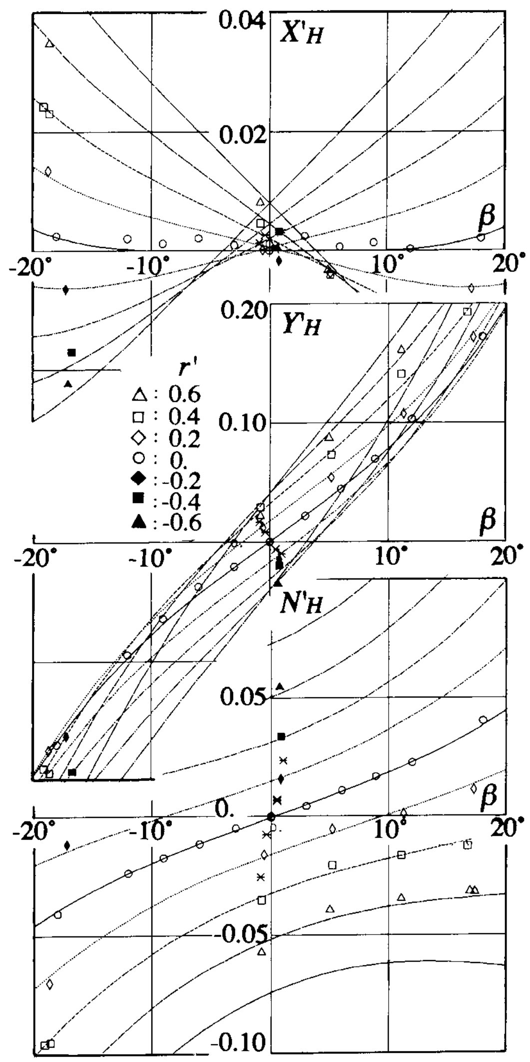

Fig.3 Hull force and moment coefficients measured by CMT [5].

4.2 Force and Moment Induced by Propeller

X_P , Y_P , N_P are expressed as the following formulas.

$$\left.\begin{aligned} X_P &= (1-t)\rho K_T D_P^4 n^2 \\ Y_P &= 0 \\ N_P &= 0 \end{aligned}\right\} \quad (10)$$

, where K_T is the thrust coefficient of propeller and it is described as the function of advance constant of propeller J.

$$K_T = a_0 + a_1 J + a_2 J^2 \quad (11)$$

D_P represents the propeller diameter and (1-t) is the thrust deduction factor that is the interaction between hull and propeller.

4.3 Force and Moment Induced by Rudder

X_R, Y_R, N_R are expressed as the following formulas taking account of the interactions between hull and rudder as shown in Fig.4.

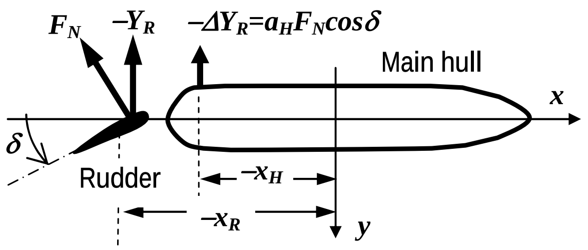

Fig.4 Schematic diagram of rudder force and hull rudder interaction.

$$\left.\begin{aligned} X_R &= -(1-t_R)F_N \sin\delta \\ Y_R &= -(1+a_H)F_N \cos\delta \\ N_R &= -(x_R + a_H x_H)F_N \cos\delta \end{aligned}\right\} \tag{11}$$

where, δ is rudder angle, x_R represents the location of rudder (=-L/2), and t_R, a_H, and x_H are the interactive force coefficients between hull and rudder.

F_N is rudder normal force and can be described as the following.

$$F_N = \frac{\rho}{2} A_R f_\alpha U_R^{\ 2} \sin\alpha_R \tag{12}$$

where, A_R is rudder area. f_α is the graduent of the lift coefficient of ruder, and can be approximated as the function of rudder aspect ratio Λ. The following is well-known Fujii's prediction formula.

$$f_\alpha = 6.13\Lambda/(2.25+\Lambda) \tag{13}$$

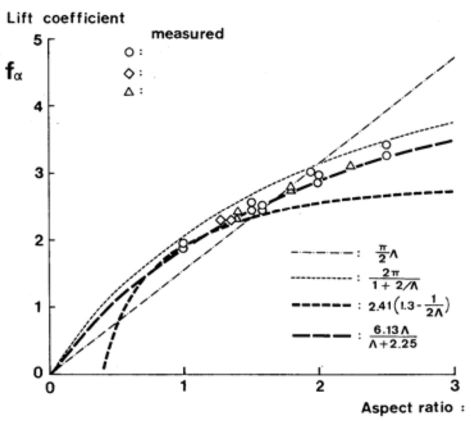

Fig.5 Graduent of the lift coefficient of ruder, observed and estimated [4].

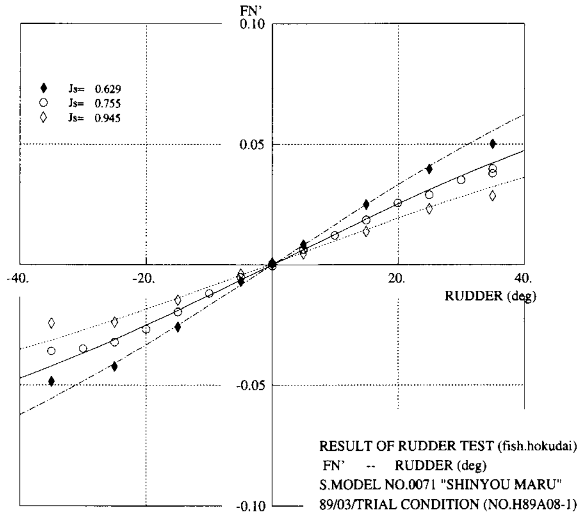

Fig.6 An example of rudder normal force for various propeller thrust [5].

As the ruder normal force is strongly affected by the propeller stream particularly in case that the rudder is located just behind the propeller. An example of measured rudder normal force is shown in Fig.6 for different 3 propeller thrust, where it can be seen that the stronger propeller thrust makes the larger rudder normal force.
In MMG model, the effect of propeller stream is included by the londituginal inflow velocity of rudder u_R . It can be described as the followings.

$$u_R = \varepsilon(1-w)u\sqrt{\eta\left\{1+\kappa\left(\sqrt{1+8K_T/\pi J^2}-1\right)\right\}^2 + (1-\eta)} \tag{14}$$

,where $\varepsilon = u_{R0}/u_P = (1-w_R)/(1-w)$

$\kappa = k_x/\varepsilon$

$\eta = D_P/H$

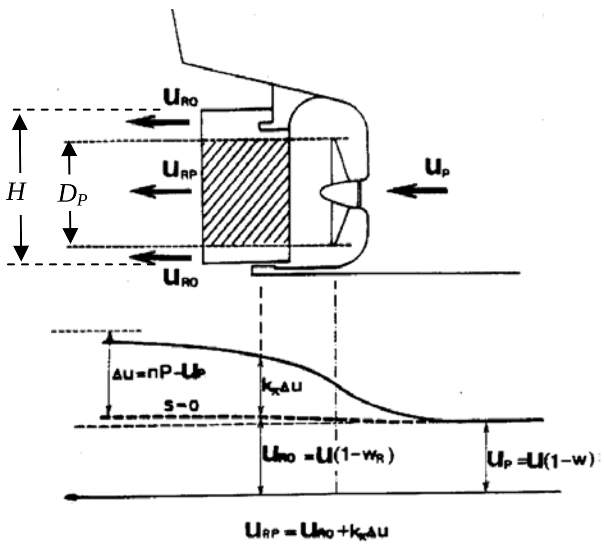

Fig.7 Schematic diagram of longitudinal inflow velocity of rudder[4].

An example of measured longitudinal inflow velocity is shown in Fig.8, where it can be seen that the increase of u_R when propeller stream (=propeller slip ratio in this figure) becomes high.

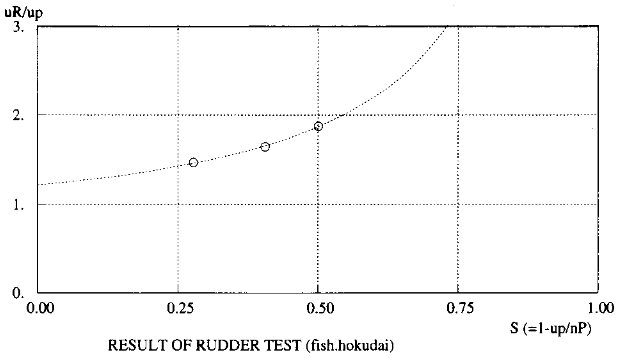

Fig. 8 u_R for various propeller slip ratio measured by rudder tests [5].

Meanwhile, the lateral inflow velocity of rudder v_R is described as eq.(15) according to the following scheme in Fig.9.

$$\beta_R = \gamma_R\left(\beta - r\left(l_R/L\right)\right)$$

or $$v_R = \gamma_R\left(v + r l_R\right) \tag{15}$$

, where γ_R represents the flow-straitening factor by ship hull. Since the lateral inflow angle is reduced by ship motion v and r, the rudder normal force produces the damping force and moment depending on v and r. This effect is called as the course-stabilizing factor of rudder. Measured lateral inflow velocity is shown in Fig.10, where the predicted characteristic by eq.(15) is compared.

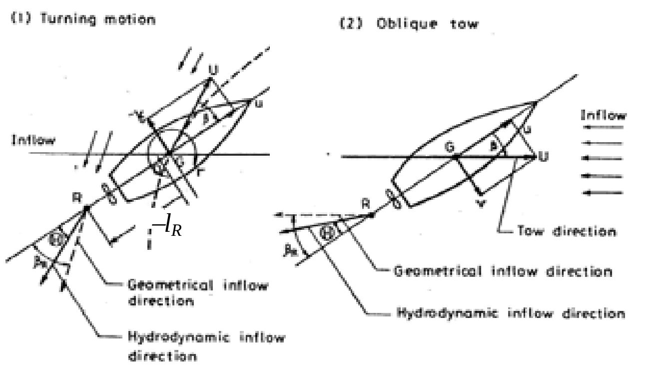

Fig.9 Schematic diagram of lateral inflow velocity of rudder [3].

Taking account these interactions among hull, propeller and rudder, inflow velocity U_R and angle α_R in eq.(12) can be calculated as the following.

$$\left.\begin{aligned} U_R &= \sqrt{u_R^{\ 2} + v_R^{\ 2}} \\ \alpha_R &= \delta - \tan^{-1}\left(\frac{-v_R}{u_R}\right) \end{aligned}\right\} \tag{16}$$

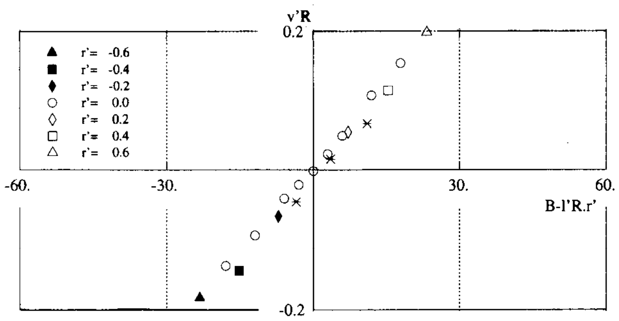

Fig.10 v'_R measured by CMT or oblique towing test with rudder angle [5]

5. SIMULATION OF MOTION

Using the above-mentioned MMG model, manoeuvring ship motions can be predicted by the computer simulation. Simulated ship is the scaled model of fishery training ship [5]. The principal particulars are listed in Table 1. Hydrodynamic derivatives and coefficients for the simulation are listed in Table 2. Simulated ship motions are shown in Fig.11 - Fig.13.

Table 1 Principal particulars of ship model.

Hull			
length	L	(m)	2.480
beam	B	(m)	0.496
draft	d	(m)	0.183
trim		(m)	0.083
Rudder			
area ratio(A_R/Ld)			1/37.5
aspect ratio	Λ		1.801
Propeller			
diameter	D_P	(m)	0.135
pitch ratio	p		0.775
turning direction			right

Table 2 Hydrodynamic derivatives and coefficients.

X_H		Y_H		N_H	
m'_x	-0.0182	m'_y	-0.0182	J'_{ZZ}	-0.0055
X'_0	-0.0196	Y'_β	0.3979	N'_β	0.0992
$X'_{\beta\beta}$	-0.0082	Y'_r-m_x	0.0918	N'_r	-0.0579
$X'_{\beta r}$-m_y	-0.1446	$Y'_{\beta\beta\beta}$	1.6016	$N'_{\beta\beta\beta}$	0.1439
X'_{rr}	0.0125	$Y'_{\beta\beta r}$	-0.2953	$N'_{\beta\beta r}$	-0.3574
$X'_{\beta\beta\beta\beta}$	0.3190	$Y'_{\beta rr}$	0.4140	$N'_{\beta rr}$	0.0183
		Y'_{rrr}	-0.0496	N'_{rrr}	-0.0207
a_0	0.3560	a_1	-0.2930	a_2	-0.1675
1-w	0.61	ε	1.137	1-t_R	0.709
.1-t	0.73	κ	0.551	a_H	0.28
		η	0.914	x_H/L	-0.377
		γ_R	0.467		
		l_R	-0.670		

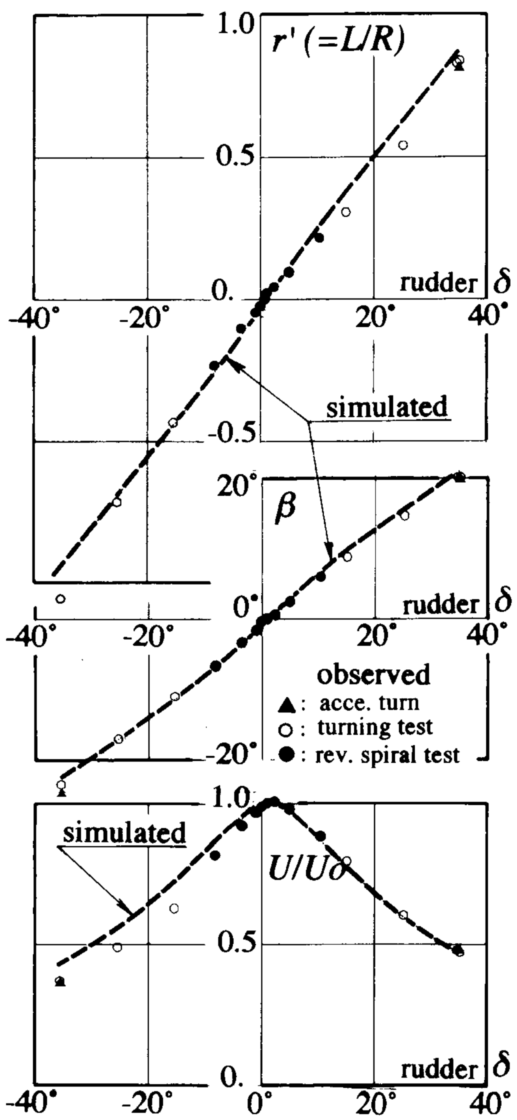

Fig.11 Simulated steady turning performance [5].

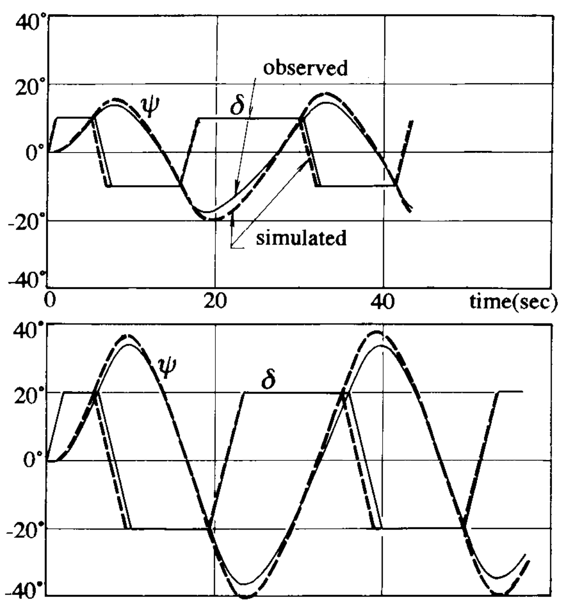

Fig.12 Simulated Z-manoeuvres [5]

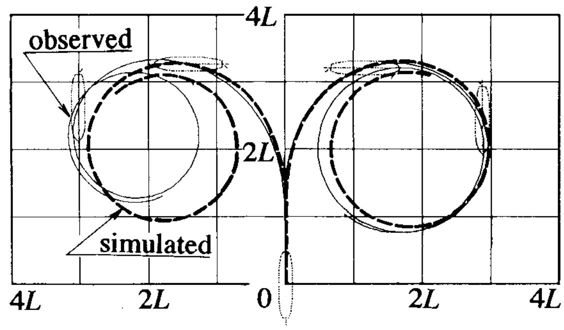

Fig.13 Simulated turning trajectories [5]

6. IN ACTUAL SEA CONDITIONS

Above mentioned mathematical model and simulations are under the calm and deep water condition. In the actual navigation, there are many kinds of environmental forces such as wind, wave and so on. In such simulations, environmental forces can be just added in eq.(8) of MMG model. Ship-ship interaction force can be also taken into account, and then the manoeuvring ship motion can be simulated.

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