TESTE N.º 1 – Proposta de resolução

Caderno 1

1. Opção (B)

Pela lei dos cossenos, sabemos que:

$$4^2 = 2^2 + 3^2 - 2 \times 2 \times 3 \times \cos(F\hat{B}D)$$

Logo:

$$16 = 4 + 9 - 12\cos\left(F\hat{B}D\right) \Leftrightarrow 12\cos\left(F\hat{B}D\right) = -3$$
$$\Leftrightarrow \cos\left(F\hat{B}D\right) = -\frac{1}{4}$$
$$\Leftrightarrow F\hat{B}D = \arccos\left(-\frac{1}{4}\right)$$

ou seja:

$$F\hat{B}D \approx 1{,}823$$

Então:

$$C\hat{B}A \approx \pi - 1{,}823 \approx 1{,}319$$

Novamente, pela lei dos cossenos, vem que:

$$\overline{AC}^2 = 6^2 + 4^2 - 2 \times 6 \times 4 \times \cos(C\hat{B}A)$$

Logo:

$$\overline{AC}^2 = 36 + 16 - 48\cos(1{,}319)$$

ou seja:

$$\overline{AC}^2 = 40{,}041$$

Daqui se conclui que:

$$\overline{AC} \approx 6{,}3$$

2. Consideremos o triângulo [*ADC*]:

$A\hat{D}C = 180° - 50° = 130°$

$A\hat{C}D = 180° - 130° - 30° = 20°$

Aplicando a lei dos senos ao triângulo [*ADC*], vem que:

$$\frac{\operatorname{sen}(20°)}{4} = \frac{\operatorname{sen}(30°)}{\overline{DC}}$$

Logo:

$$\overline{DC} = \frac{4 \times \frac{1}{2}}{\operatorname{sen}(20°)}$$

isto é:

$$\overline{DC} = \frac{2}{\text{sen}(20°)}$$

ou seja:

$$\overline{DC} \approx 5{,}84761$$

Aplicando a lei dos senos ao triângulo [*DBC*], vem que:

$$\frac{\text{sen}(90°)}{\overline{DC}} = \frac{\text{sen}(50°)}{\overline{BC}}$$

ou seja:

$$\frac{1}{5{,}84761} = \frac{\text{sen}(50°)}{\overline{BC}}$$

isto é:

$$\overline{BC} = \text{sen}(50°) \times 5{,}84761$$

Daqui se conclui que:

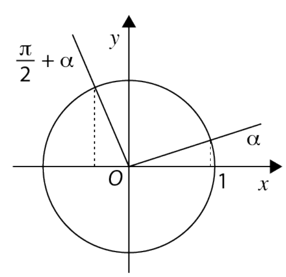
$$\overline{BC} \approx 4{,}47953$$

Assim, a área do triângulo [*ADC*] é igual a:

$$\frac{\overline{AD} \times \overline{BC}}{2} = \frac{4 \times 4{,}47953}{2} \approx 8{,}959 \text{ u.a.}$$

3. Opção (B)

Sabemos que $\cos\alpha = -\frac{3}{4}$.



Logo:

- $\text{sen}\left(\frac{\pi}{2} + \alpha\right) = \cos\alpha = -\frac{3}{4}$

  $\frac{\pi}{2} + \alpha$ não é solução da equação $\text{sen}x = \frac{3}{4}$.

- $\text{sen}\left(\frac{3\pi}{2} - \alpha\right) = -\cos\alpha = \frac{3}{4}$

  $\frac{3\pi}{2} - \alpha$ é solução da equação $\text{sen}x = \frac{3}{4}$.

- $\text{sen}(\pi + \alpha) = -\text{sen}\alpha$ e $\text{sen}^2\alpha = 1 - \frac{9}{16} \Leftrightarrow \text{sen}^2\alpha = \frac{7}{16} \Leftrightarrow \text{sen}\alpha = \pm\frac{\sqrt{7}}{4}$

  $\pi + \alpha$ não é solução da equação $\text{sen}x = \frac{3}{4}$.

- $\text{sen}(2\pi - \alpha) = -\text{sen}\alpha$

  $2\pi - \alpha$ não é solução da equação $\text{sen}x = \frac{3}{4}$.

4.

4.1. Sabemos que $A(\cos\alpha, \text{sen}\,\alpha)$ e $B(1, \text{tg}\,\alpha)$ e que $\cos\alpha > 0$, $\text{sen}\,\alpha < 0$ e $\text{tg}\,\alpha < 0$

A área do trapézio [*ABCD*] é igual a:

$$\frac{\overline{CB}+\overline{DA}}{2} \times \overline{DC} = \frac{1+\cos\alpha}{2} \times (|\text{tg}\alpha| - |\text{sen}\alpha|) =$$

$$= \frac{1+\cos\alpha}{2} \times (-\text{tg}\alpha + \text{sen}\alpha) =$$

$$= -\frac{1}{2} \times (1+\cos\alpha)(\text{tg}\alpha - \text{sen}\alpha) =$$

$$= -\frac{1}{2} \times (\text{tg}\alpha - \text{sen}\alpha + \cos\alpha\text{tg}\alpha - \text{sen}\alpha\cos\alpha) =$$

$$= -\frac{1}{2} \times (\text{tg}\alpha - \text{sen}\alpha + \text{sen}\alpha - \text{sen}\alpha\cos\alpha) =$$

$$= -\frac{1}{2} \times \left(\frac{\text{sen}\alpha}{\cos\alpha} - \frac{\text{sen}\alpha\cos^2\alpha}{\cos\alpha}\right) =$$

$$= -\frac{1}{2} \times \text{tg}\alpha(1-\cos^2\alpha) =$$

$$= -\frac{1}{2} \times \text{tg}\alpha\,\text{sen}^2\alpha$$

4.2. A área do setor circular de ângulo ao centro EOA é igual a $-\frac{\alpha}{2}$.

Pretendemos, então, determinar o(s) valor(es) de α para o(s) qual(is) $-\frac{1}{2}\text{tg}\alpha\,\text{sen}^2\alpha = -\frac{\alpha}{2}$.

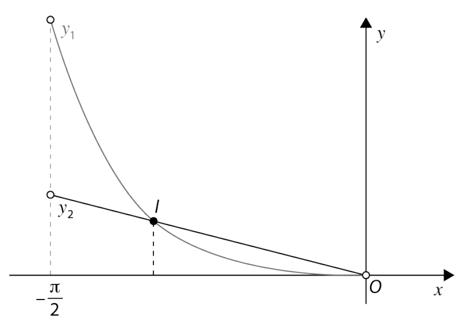
Recorrendo às capacidades gráficas da calculadora, vamos determinar o valor pretendido:

$y_1 = -\frac{1}{2}\text{tg}\alpha\,\text{sen}^2\alpha$

$y_2 = -\frac{\alpha}{2}$

$I(a, b)$

$a \approx -0{,}96$



O valor pretendido com aproximação às centésimas é $-0{,}96$.

4.3. Sabemos que $\text{sen}\left(-\frac{\pi}{2}+\beta\right) = -\frac{2}{3}$, logo$-\cos\beta = -\frac{2}{3} \Leftrightarrow \cos\beta = \frac{2}{3}$.

Pela Fórmula Fundamental da Trigonometria, tem-se que $\text{sen}^2\beta + \cos^2\beta = 1$.

Logo:

$$\text{sen}^2\beta = 1 - \frac{4}{9}$$

ou seja:

$$\text{sen}^2\beta = \frac{5}{9} \Leftrightarrow \text{sen}\beta = \pm\frac{\sqrt{5}}{3}$$

Como $-\frac{\pi}{2} < \beta < 0$, então $\text{sen}\beta = -\frac{\sqrt{5}}{3}$.

Logo:

$$\text{tg}\beta = \frac{-\frac{\sqrt{5}}{3}}{\frac{2}{3}} = -\frac{\sqrt{5}}{2}$$

Daqui se conclui que $A(\beta) = -\frac{1}{2} \times \left(-\frac{\sqrt{5}}{2}\right) \times \frac{5}{9} = \frac{5\sqrt{5}}{36}$.

Caderno 2

5. Opção (C)

Se $\theta \in \left]-\frac{3\pi}{2}, -\pi\right[$, então θ pertence ao 2.º quadrante. Logo, $\text{sen}\theta > 0, \cos\theta < 0$ e $\text{tg}\theta < 0$.

Assim, concluímos que:

- $\text{sen}\theta - \cos\theta > 0$
- $\text{sen}\theta - \text{tg}\theta > 0$
- $\cos\theta + \text{tg}\theta < 0$
- $tg\ \theta \times \cos\theta + \text{sen}\theta > 0$

6. $\text{sen}^2\left(\frac{\pi}{9}\right) - \text{sen}\left(\frac{7\pi}{2}\right) + \cos(2018\pi) - 3\text{tg}\left(\frac{11\pi}{6}\right) + \cos^2\left(\frac{3\pi}{4}\right) + \cos^2\left(-\frac{\pi}{9}\right) =$

$$= \underbrace{\text{sen}^2\left(\frac{\pi}{9}\right) + \cos^2\left(\frac{\pi}{9}\right)}_{1} - \text{sen}\left(\frac{3\pi}{2}\right) + \cos(0) - 3\text{tg}\left(-\frac{\pi}{6}\right) + \cos^2\left(\frac{3\pi}{4}\right) =$$

$$= 1 - (-1) + 1 - 3 \times \left(-\frac{\sqrt{3}}{3}\right) + \left(-\frac{\sqrt{2}}{2}\right)^2 =$$

$$= 3 + \sqrt{3} + \frac{1}{2} =$$

$$= \frac{7}{2} + \sqrt{3}$$

7.

7.1. $f(x) = \frac{2\text{sen}x+2}{\cos x+1} + 4\cos x - 4 =$

$$= \frac{2\text{sen}x+2+4(\cos x-1)(\cos x+1)}{\cos x+1} =$$

$$= \frac{2\text{sen}x+2+4(\cos^2 x-1)}{\cos x+1} =$$

$$= \frac{2\text{sen}x+2+4(-\text{sen}^2 x)}{\cos x+1} =$$

$$= \frac{-4\text{sen}^2 x+2\text{sen}x+2}{\cos x+1}$$

7.2. Seja x pertencente ao domínio de f:

$f(x)=0 \Leftrightarrow -4\text{sen}^2 x+2\text{sen}x+2=0$

$\Leftrightarrow \text{sen}x=\frac{-2\pm\sqrt{4-4\times(-4)\times 2}}{-8}$

$\Leftrightarrow \text{sen}x=\frac{-2\pm 6}{-8}$

$\Leftrightarrow \text{sen}x=1 \quad \vee \quad \text{sen}x=-\frac{1}{2}$

$\Leftrightarrow x=\frac{\pi}{2}+2k\pi \quad \vee \quad x=-\frac{\pi}{6}+2k\pi \quad \vee \quad x=\frac{7\pi}{6}+2k\pi, k\in\mathbb{Z}$

Em $]-\pi,3\pi[\ \backslash\{\pi\}$: $x=-\frac{5\pi}{6}$ ou $x=-\frac{\pi}{6}$ ou $x=\frac{\pi}{2}$ ou $x=\frac{7\pi}{6}$ ou $x=\frac{11\pi}{6}$ ou $x=\frac{5\pi}{2}$

8. $\text{sen}x+2\cos x=1 \Rightarrow (\text{sen}x+2\cos x)^2=1^2$

$\Leftrightarrow \text{sen}^2x+4\text{sen}x\cos x+4\cos^2x=1$

$\Leftrightarrow \underbrace{\text{sen}^2x+\cos^2x}_{1}+4\text{sen}x\cos x+3\cos^2x=1$

$\Leftrightarrow 4\text{sen}x\cos x+3\cos^2x=0$

$\Leftrightarrow \cos x(4\text{sen}x+3\cos x)=0$

$\Leftrightarrow \underbrace{\cos x=0}_{\text{condição impossível em } \mathbb{R}\backslash\left\{x:x=\frac{\pi}{2}+k\pi,k\in\mathbb{Z}\right\}} \quad \vee \quad 4\text{sen}x+3\cos x=0$

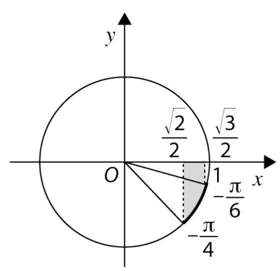
Como $x\neq\frac{\pi}{2}+k\pi, k\in\mathbb{Z}$, então $\cos x\neq 0$.

Logo, $4\text{sen}x+3\cos x=0$.

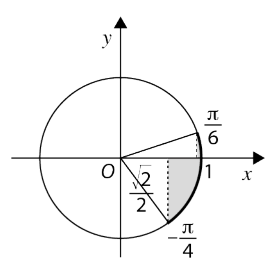
Ora $4\text{sen}x+3\cos x=0 \Leftrightarrow 4\text{sen}x=-3\cos x \Leftrightarrow \frac{\text{sen}x}{\cos x}=-\frac{3}{4} \Leftrightarrow \text{tg}x=-\frac{3}{4}$

9. Opção (B)

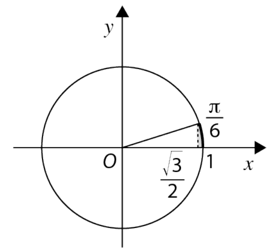
Se $-\frac{\pi}{4}\leq x\leq-\frac{\pi}{6}$, então $\frac{\sqrt{2}}{2}\leq\cos x\leq\frac{\sqrt{3}}{2}$



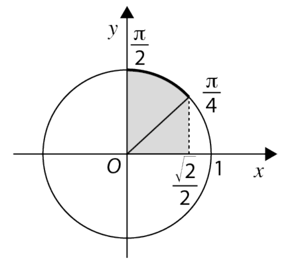
Se $-\frac{\pi}{4}\leq x\leq\frac{\pi}{6}$, então $\frac{\sqrt{2}}{2}\leq\cos x\leq 1$



Se $0 \leq x \leq \frac{\pi}{6}$, então $\frac{\sqrt{3}}{2} \leq \cos x \leq 1$



Se $\frac{\pi}{4} \leq x \leq \frac{\pi}{2}$, então $0 \leq \cos x \leq \frac{\sqrt{2}}{2}$



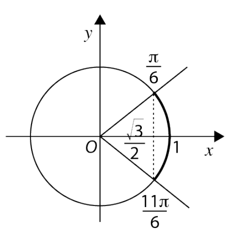
10. Opção (C)

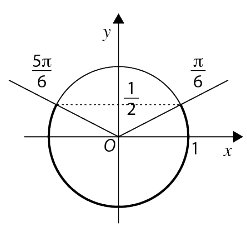
$$\text{arcsen}\left(-\frac{\sqrt{3}}{2}\right) + \text{arccos}\left(-\frac{1}{2}\right) = -\frac{\pi}{3} + \frac{2\pi}{3} = \frac{\pi}{3}$$

11. $\cos x > \frac{\sqrt{3}}{2} \ \wedge \ \text{sen}x \leq \frac{1}{2} \ \wedge \ 0 \leq x \leq 2\pi$

$\Leftrightarrow \left(0 \leq x < \frac{\pi}{6} \ \vee \ \frac{11\pi}{6} < x \leq 2\pi\right) \ \wedge \ \left(0 \leq x \leq \frac{\pi}{6} \ \vee \frac{5\pi}{6} \leq x \leq 2\pi\right)$

$\Leftrightarrow 0 \leq x < \frac{\pi}{6} \ \vee \ \frac{11\pi}{6} < x \leq 2\pi$





C.S. $= \left[0, \frac{\pi}{6}\right[\cup \left]\frac{11\pi}{6}, 2\pi\right]$