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PREDICTING METHOD OF NATURAL FREQUENCY FOR SHIP'S OVERALL VERTICAL VIBRATION

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Summary

At ship design stage, empirical formulas are generally used to predict the overall vertical vibration natural frequency of ship to avoid the harmful resonance against main excitation. Nowadays, with the development of modern large-scale ships, the existing empirical formulas are becoming unpractical, some corrections shall be applied on these classical formulas or a new formula is to be derived for the prediction of vertical vibration frequencies of ship. In this paper, a new empirical formula is given for predicting the natural frequency of ship overall vertical vibration. Based on the Timoshenko beam theory, the formula is derived by introducing by shear coefficient to correction shear uniform distribution hypothesis for a thin-walled box beam with free-free boundary condition. This new formula is obtained by statistical analysis for large amounts of measured natural frequency results of the ship overall vertical vibration. The proposed prediction method in this paper was used to predict the natural frequencies of six ships. The predicted natural frequencies are consistent with that by measurements. The comparison with measurements show that the formula proposed in this article is more feasible to use, and also provide a new method to the prediction of natural frequency of the ship overall vertical vibration.

Key words: ship; overall vertical vibration; natural frequency; shear coefficient

1. Introduction

Ship is a kind of complex elastic structures. During ship sailing operation, it will be encountered with the external exciting forces from main engine, propellers and wave loads. These incentives will cause the hull vibration issue and even ship structural damage. The excessive vibration may lead to structure fatigue failure, affect the shipping efficiency, human-being health, and the service life of equipment [1]. So, to forecast the ship hull natural frequencies accurately at design stage is very important for avoiding harmful vibration especially the overall vertical vibration.

In general, empirical formulas are widely and efficiently used to predict the ship-hull vertical vibration at the design stage. While at detail design stage, the 1D or 3D FEM method is to be used for the more accurate vibration analysis. 1D FEM can predict the lower order natural frequencies of the hull vibration easily and accurately. For the higher order hull vibration, the 3D FEM shall be applied to get more accurate results. However, the 3D FEM method will take lots of time to build the 3D whole ship FE model. Compared to 1D and 3D FEM methods, the empirical formula method is simple and practical. Many professionals have done a lot of work in the study of empirical formulas for prediction of the natural frequency of ship's overall vertical vibration [2-7]. Schlick formula and Todd formula are earliest used as the empirical formulas, and presently the Kumai formula recommended by DET NORSKE VERITAS (DNV), the formula recommended by Japanese shipbuilding design, the formula specified in the ship industry standard of the People's Republic of China and etc. are widely used to predict the natural frequencies of ship in the shipbuilding industry.

In recent years, the focus of ship's overall vibration study is mainly centered on the ship's overall vibration analysis by 3D FEM methods [8-9] and computation of added mass in ship vibration analysis. Specifically, Ivo SENJANOVIĆ et al [10-12] systematically study on the coupled horizontal and torsional vibration of the ship hull with large hatch openings such as container ship et al; Josip BAŠIĆ and Joško PARUNOV [13] make an in-depth study on the calculation method of added mass in ship vibration analysis and compare the result of each calculation method. Due to the practical measured data of ship's overall vibration is less, the study on calculation formula of ships natural frequency of vibration is rarely.

At present, the empirical formulas for predicting ships natural frequency of vibration are proposed in the sixties-nineties of the twentieth century. With the development of modern large-scale ships, the aforementioned empirical formulas are gradually becoming unpractical and unsatisfactory due to the change of dimensions and arrangements of new ships. In this paper, we collect some recent vibration test data of large ship and combine with the previous practical measured data to propose a new empirical formula. Based on the Timoshenko beam theory, the formula is derived by introducing by shear coefficient to correction shear uniform distribution hypothesis and obtained by statistical analysis method to get the ships natural frequency of overall vertical vibration.

2. Formula Derivation for Natural Frequency of Bending Vibration of Thin-walled Uniform Beam

The bending vibration equation of thin-walled uniform beam in consideration of the effect of shear deformation and moment of inertia is[1]:

$$EI\frac{\partial^4 y}{\partial x^4}+\rho A\frac{\partial^2 y}{\partial t^2}-\rho I\left(1+\frac{E}{kG}\right)\frac{\partial^4 y}{\partial x^2\partial t^2}+\frac{\rho^2 I}{kG}\frac{\partial^4 y}{\partial t^4}=0 \tag{1}$$

where I is beam section moment of inertia. E is elastic modulus. G is shear modulus of material. A is cross-section area of beam. ρ is density of the material. k coefficient related to the cross-sectional shape which had been sufficient studied by G.R.Cowper[14].

Timoshenko beam theory assumes that when the beam bending deformation occurs, the cross-section perpendicular to the middle section originally remain as the plane; so, the assumption of uniform distribution of shear stress and shear strain on the section can be introduced. Actually, Shear stress and shear strain on the cross section is not evenly distributed. In ref.[14] G.R.Cowper has carried on the correction of Timoshenko beam theory, derived equation of shear coefficient, and numerical solution of series of beam cross section is given. Among them calculation formula of straight thin-walled box beam shear coefficient is:

$$k=\frac{10(1+\nu)(1+3m)^2}{(12+72m+150m^2+90m^3)+\nu(11+66m+135m^2+90m^3)+10n^2[(3+\nu)m+3m^2]} \tag{2}$$

where $m=Bt_1/Ht$, $n=B/H$. B is the height of beam. H is the width of beam. t_1 is the thickness of upper and lower wing plate. t is the thickness of web. ν is poisson ratio.

For simply supported boundary conditions at both ends, the solution of equation (1) is:

$$y=A_j\sin\frac{j\pi x}{l}\sin(\omega_j t+\varphi_j)$$

Bring it into Eq.(1), the equation can be re-written as follows:

$$EI\left(\frac{j\pi}{l}\right)^4-\rho A\omega_j^2-\rho I\left(1+\frac{E}{kG}\right)\left(\frac{j\pi}{l}\right)^2\omega_j^2+\frac{\rho^2 I}{kG}\omega_j^4=0 \tag{3}$$

Compared with the first item, the last item is a coupling term of shear deformation and moment of inertia is small amount, which can be neglected.

$$EI\left(\frac{j\pi}{l}\right)^4-\rho A\omega_j^2-\rho I\left(1+\frac{E}{kG}\right)\left(\frac{j\pi}{l}\right)^2\omega_j^2=0 \tag{4}$$

For simply supported boundary conditions at both ends, Equation (4) has its analytical solution, whose bending vibration natural frequency of simply supported beam is[1]:

$$\omega=\frac{j^2\pi^2}{L^2}\sqrt{\frac{EI}{\rho A}}\sqrt{\frac{1}{1+\frac{j^2\pi^2 I}{L^2 A}(1+\frac{E}{kG})}} \tag{5}$$

where L is the length of beam. j is vibration order number.

For simply supported at both ends boundary conditions, bending vibration natural frequency of Euler beam is[15]:

$$\omega=\frac{j^2\pi^2}{L^2}\sqrt{\frac{EI}{\rho A}} \tag{6}$$

For free at two ends boundary conditions, Euler beam natural frequency of bending vibration is:

$$\omega=\frac{(j+0.5)^2\pi^2}{L^2}\sqrt{\frac{EI}{\rho A}} \tag{7}$$

According to Eqs.(5)—(7), in this paper approximate formula for natural frequency of bending vibration of free at two ends Timoshenko beam considering the effect of shear deformation and moment of inertia are given:

$$\omega=\frac{(j+0.5)^2\pi^2}{L^2}\sqrt{\frac{EI}{\rho A}}\sqrt{\frac{1}{1+\frac{(j+0.5)^2\pi^2 I}{L^2 A}(1+\frac{E}{kG})}} \tag{8}$$

3. Calculation of Natural Frequency for Uniform Thin-walled Box Beam

In order to verify the accuracy of the proposed formula for natural frequency of bending vibration uniform section beam, three methods were used respectively: one-dimensional Timoshenko beam model, 3-D space beam model and formula solution (8). The vertical bending vibration natural frequency of uniform thin-walled box beam model is calculated with the free-free boundary condition of two ends. The calculated results of various methods are listed in Table 1.The uniform thin-walled box beam model is shown in Fig.1, where the length of beam $L = 180\text{m}$, beam width $B = 24\text{m}$, beam height $H = 16\text{m}$, thickness $t = 0.013\text{m}$, density of the material $\rho = 7800\,\text{kg}/\text{m}^3$, elastic modulus $E = 2.07\times10^{11}\,\text{N}/\text{m}^2$, poisson ratio $\nu = 0.3$.

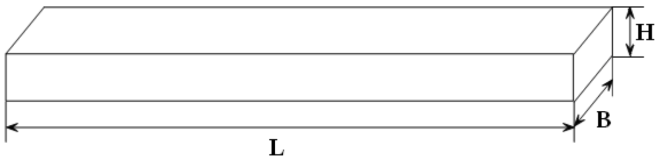

Fig. 1 Uniform thin-walled box beam model

Table 1 Comparison of nature frequencies of thin-walled box beam

Order	One-dimensional Timoshenko beam model /Hz	3-D space beam model /Hz	Formula (8) /Hz
1	3.63	3.51	3.41
2	8.55	8.19	7.92
3	14.14	13.48	12.98
4	19.82	18.83	18.11
5	25.50	24.12	23.20
6	31.16	29.23	28.23

The comparison results as indicated in Table 1 show that the predicted natural frequencies from three methods are similar. The 3-D space beam model get more accurate results cause the space beam models based on the Timoshenko beam theory are used. For one dimension Timoshenko beam model, lower-order frequencies are accurate, but error will increase for higher-order vibration. The proposed formula by introducing shear coefficient k will correct Timoshenko beam theory shear uniform distribution hypothesis, the result is much closer to that by 3-D space beam model. Using the proposed formula derived in this article, the natural frequency of bending vibration for free-free boundary condition uniform section beam can be calculated accurately.

4. Calculation Formula of Natural Frequency of Ship's Vertical Vibration

In general, the ship structure is considered as the variable cross-section beam floating freely in the water for the prediction of its overall vertical natural frequencies. Its natural frequencies usually depend on their stiffness and mass of ship structure (including the added mass of surrounding water). When the principal dimensions and weight of ship is determined, the natural frequency of vertical vibration ship will be governed by geometrical characteristics of parallel middle sections (moment of inertia and shear area). As the variable cross-section beam, the mass and moment of inertia are nonuniform distributed along the ship length. So the natural frequency of vibration can not be directly calculated by the formula (8). According to the structure characteristics of the ship and the formula (8), calculation formula of natural frequency of overall vibration of the ship in consideration of the effect of shear deformation and moment of inertia is:

$$f_n = A_n \sqrt{\frac{I_V}{\Delta_V L^3 (1+\alpha)}} \tag{9}$$

where A_n is natural frequency coefficient of vertical vibration of ship. L is length. I_V is midship section moment of inertia for horizontal axis. α is shear and rotational inertia influence coefficient, $\alpha = \frac{I}{A}(1+\frac{E}{kG})(\frac{(j+0.5)\pi}{L})^2$, k is shear coefficient, obtained by the formula (2). Δ_V is mass of ship structure including added mass, $\Delta_V = (1+\tau)\Delta$, τ is added mass coefficient,using Todd formula calculate added mass coefficient in vertical vibration, $\tau = (0.2+\frac{B}{3d})$, Δ is ship displacement, t .

Considering the superstructure are short but high of large oil tanker, bulk carrier and container ship, the impact on the overall vertical vibration due to superstructure can be ignored. Based on the over 90 ships' measured data which has been collected by author for years, the value of coefficient A_n can be obtained by least square regression. The value of coefficient A_n are shown in Table 2.

Table 2 Coefficient A_n of natural frequency of the ship overall vertical vibration

Ship type	First order coefficient	Second order coefficient	Third order coefficient
Bulk carrier	0.667×10^5	0.155×10^6	0.255×10^6
Tanker	0.690×10^5	0.165×10^6	0.30×10^6
Container ship	0.655×10^5	0.150×10^6	0.250×10^6

At the very beginning design stage of ship, the moment of inertia of midship section is often lacking. Therefore, approximate calculation formula of the moment of inertia in this article is proposed:

$$I = cBD^2L \tag{10}$$

where L is length. B is breadth. D is depth. By statistical analysis of over ninety different type ships, the coefficient c can be taken as $c = 1.07\times10^{-4}$; For container ship and large opening bulk carrier, c can be taken as 0.95×10^{-4} because their the moment of inertia are relatively lower. When the length, breadth and depth were designed, the moment of inertia of midship section can be estimated.

For the approximate prediction, the effect term of shear and moment of inertia $\sqrt{1/(1+\alpha)}$ in formula (9) will be taken an approximate value, Through statistical analysis of over ninety different type ships, the effect can be taken as 0.909, 0.781 and 0.671 respectively for the first three orders vertical vibration.

Therefore, formula (9) can be transformed into the following form:

$$f_n = A_n \sqrt{\frac{I}{\Delta_V L^3 (1+\alpha)}} \approx A_n \times \sqrt{\frac{cBD^2L}{\Delta_V L^3(1+\alpha)}} = C_n \times \sqrt{\frac{BD^2}{\Delta_V L^2}}$$

Namely simplified formula suitable for initial design stage of ship is as follows:

$$f_n = C_n \times \sqrt{\frac{BD^2}{\Delta_V L^2}} \tag{11}$$

where coefficient C_n value are shown in Table 3.

Table 3 Coefficient C_n of natural frequency of the ship overall vertical vibration

Ship type	First order coefficient	Second order coefficient	Third order coefficient
Bulk carrier	620.6	1212.2	1735.6
Tanker	639.5	1373.8	2082.7
Container ship	560.6	1111.7	1591.8

5. Calculation Example

In order to validate the correctness of the proposed formula, six ships including 2 bulk carriers, 3 tankers and a container ship was taken for example, formula (9), formula (11) and other approximate formulas were used to calculate the first three orders natural frequency of ship. While the formulas recommended by DNV[16], Japanese shipbuilding design handbook [17] and the ship industry standard of the People's Republic of China[18] are used to calculate the natural frequencies of the first three orders vertical vibration. One of the formula recommended in Japanese shipbuilding design handbook is in consideration of the effect of shear deformation and moment of inertia of ship, while the other one ignore such effect. Because only the calculation methods of added mass of water for tanker and cargo ship were given by Japanese shipbuilding design handbook, the Todd formula will be used to calculate the added mass of water for container ship in this paper. Approximate formulas are as follows:

(1) The formula recommended by DNV is(DNV):

$$N_{2V} = 1.61 \times 10^6 \sqrt{\frac{I_V}{\Delta_V L^3}} \tag{12}$$

where N_{2V} is natural frequency of first order vibration. $\Delta_V = (1.2 + \frac{B}{3d})\Delta$, Δ is the displacement of ship, kg.

For high order vibration: $N_{nV} = N_{2V}(n-1)^{\mu_V}$.

where N_{nV} is natural frequency of high order vibration. for different types ships, the values of coefficient μ_V is different, $\mu_V = 1.02$ for tankers, $\mu_V = 1.0$ for bulk carriers.

(2) Recommended formulas in Japanese shipbuilding design handbook

Formula without considering of the effect of shear deformation and moment of inertia (Japan1):

$$\begin{aligned} N_{2V} &= 27.1 \times 10^5 \sqrt{\frac{I_V}{\Delta_V L^3}} + 14.5 \\ N_{3V} &= 38.8 \times 10^5 \sqrt{\frac{I_V}{\Delta_V L^3}} + 58.5 \end{aligned} \tag{13}$$

where for cargo ship $\Delta_V = \{1+0.3(\frac{B}{T}) - 0.033(\frac{B}{T})^2\}\times\Delta$,

for tankers $\Delta_V = \{1+0.4(\frac{B}{T}) - 0.035(\frac{B}{T})^2\}\times\Delta$.

Formula considering of the effect of shear deformation and moment of inertia(Japan2):

$$N_{nV} = c_n\sqrt{\frac{I_V}{\Delta_V L^3[1+(\alpha+\beta)n^2\pi^2]}} \tag{14}$$

where $c_2 = 4\times10^6$, $c_3 = 9.2\times10^6$. coefficient $\alpha = \frac{EI_V}{GA_S L^2}$, A_S is effective shear area. coefficient $\beta = \frac{r_0^2}{L^2}$, r_0 turning radius.

(3) The formula specified in the ship industry standard of the People's Republic of China(China)

for two and three node vertical vibration:

$$f_{iv} = K_{ib}(A_{iv}K_{iv}E_{iv}\sqrt{\frac{I_V}{\Delta_V L^3}} + B_{iv}) \tag{15}$$

where f_{iv} is natural frequency of i node vertical vibration of hull girder. A_{iv} and B_{iv} is regression coefficient. K_{ib} is reduction coefficient of bending stiffness for vertical vibration of hull girder. K_{iv} is influence coefficient for natural frequency of vertical vibration caused by the change of moment of inertia of section along the ship length. E_{iv} is influence coefficient of superstructure; calculation method of each parameters see references[18].

Formula of natural frequency of four and five nodes vertical vibration:

$$f_{iv} = A_{iv}\sqrt{\frac{A_{ov}}{\Delta_V L}} + B_{iv} \tag{16}$$

where A_{ov} is shear area of midship section for vertical vibration of hull girder.

The related parameters of six ships are shown in Table 4. The calculation results and measured values are shown in Table 5.

Table 4 related parameters of six ships

Ship type	Length /m	Breadth /m	Depth /m	Moment of inertia /m^4	Displacement /t
Bulk carrier	183.0	27.4	14.8	115.46	19900.0
Bulk carrier	144.8	20.4	12.2	50.23	19720.0
Tanker	255.8	37.1	18.4	403.97	102810.0
Tanker	218.0	32.9	15.6	212.03	66289.0
Tanker	320.0	58.0	31.0	1454.62	342506.7
Container ship	319.0	42.8	24.6	549.60	136701.0

Table 5 Natural frequency of the ship overall vertical vibration

Number	Order	Natural frequency / Hz						
		DNV	Japan 1	Japan 2	China	Formula(9)	Formula(11)	Measured value
1	1	0.905	1.321	1.301	1.038	1.065	1.076	1.083
	2	1.810	2.508	2.515	2.061	2.134	2.102	2.167
	3	2.715	3.962	3.903	2.892	2.993	3.010	3.017
2	1	1.045	1.301	1.270	1.181	1.214	1.192	1.217
	2	2.091	2.480	2.445	2.300	2.419	2.328	2.450
	3	3.136	3.903	3.809	3.277	3.357	3.333	3.517
3	1	0.536	0.752	0.637	0.736	0.659	0.600	0.672
	2	1.087	1.706	1.258	1.407	1.334	1.290	1.350
	3	1.644	2.307	1.956	2.219	2.151	1.955	/
4	1	0.610	0.824	0.726	0.803	0.803	0.695	0.813
	2	1.237	1.809	1.433	1.561	1.575	1.494	1.600
	3	1.871	2.528	2.229	2.382	2.464	2.265	2.467
5	1	0.403	0.624	0.460	0.533	0.483	0.560	0.490
	2	0.817	1.523	0.889	0.959	1.058	1.203	0.970
	3	1.235	1.914	1.413	1.822	1.738	1.824	1.710
6	1	0.384	0.640	*	0.574	0.450	0.518	0.460
	2	*	*	*	1.096	0.918	1.028	0.950
	3	*	*	*	1.795	1.345	1.472	1.510

Note: /: lack of measured value, *: The approximate formulas are not available for such type of ships.

Calculation results of each formulas compared with measured values, are expressed in the form of bar-chart, as shown in Fig.2.

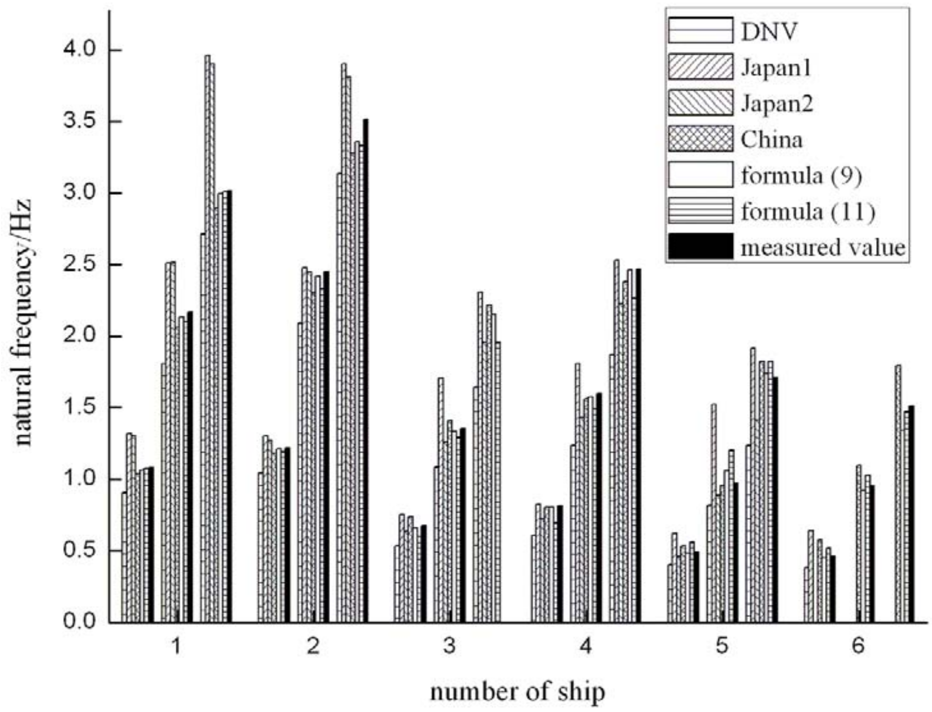

Fig. 2 The comparison of vertical vibration's nature frequency

According to the comparisons of these results, calculation results by DNV formula are lower than the measured values. the results of 1st formula in Japanese shipbuilding design handbook are higher than the measured values, another formula considering of the effect of shear deformation and moment of inertia, calculation result is lower for tanker, while the difference is little comparing with the first formula for bulk carrier, that is because the adding mass from water in the formula is special for cargo ship design of Japanese shipbuilding design handbook. It is not suitable for bulk carrier which can not be use directly for bulk carrier design. For the formula which is brought out by the ship industry standard of the People's Republic of China, the error is relatively smaller, but predict formula are not good enough for the container ship design. The calculation results of formula (9) proposed in this article is much closer to experiments measured values. Formula (9) can get better results for large ships than the others. The proposed formula(11) in this article can also calculate the natural frequency of overall vertical vibration accurately. At the beginning design stage of ship, the natural frequency of overall vertical vibration can be estimated quickly by using formula(11).

6. Conclusion

In this paper, a new formula for prediction of natural frequency of the ship overall vertical vibration was deduced. Based on the Timoshenko beam theory, the formula for natural frequency of vibrating thin-walled beam with free-free boundary conditions is derived by introducing shear coefficient to correct shear uniform distribution hypothesis. According to large amounts of measured values different types of ships, a new formula for prediction of natural frequency of the ship overall vertical vibration is obtained by statistical analysis method. Considering the lack of the section moment of inertia in early stage of ship design, so an approximate calculation formula of moment of inertia is also given in this paper. The simplified formula for natural frequency of the ship overall vertical vibration is derived. Taking six types of ships for example, the frequency results calculated by the proposed method in this paper are closer to the measured values. It has proved that the proposed method in this article is effective, and provides a new tool for prediction of ship overall vertical natural frequency. Only several types of ships are calculated because of the limitation of the accumulated data. In the future, more and more data of different types of ship will be collected, and more influential elements are needed to be considered to make the prediction results more accurate, and make the application range more wider.

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