

Teaching algebra and calculus using Socratic Dialogue based on Eulerian Sequencing

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Abstract

Modern undergraduates join University courses with poorer background than in the past. University tutors spend more and more time delivering various stepping-stone classes. When doing so, most rely on traditional methods of delivery. However, such methods do not work well when dealing with large groups of undergraduates who have limited background, limited memory, limited proficiency in explanatory reasoning, limited confidence, limited interest in the subject, limited study skills and on top of that, limited time to cover a large amount of material, all aggravated by limited contact with teachers. Yet, these disadvantages can be overcome when dealing with adult learners. In this paper we describe a teaching approach based on the teacher-guided Socratic dialogue, which aims to uncover learner difficulties and reinforce their basic understanding through Eulerian sequencing. Our aim is to present several specific techniques tried in practice and initial evidence that when taught this way ordinary adult learners can achieve relatively deep learning of mathematics – and remarkably quickly.

INTRODUCTION

It is well documented that these days students are entering Universities with more widely diverse educational backgrounds than in the past. For example, in the UK, whilst traditional students have three or more A-levels, widening participation students may have equivalent qualifications from vocational or access courses or BTEC's [13]. Evidence has indicated that many of these students experience difficulties with the transition to university and with progression generally [5]. On top of that, some traditional students meet the necessary minimum requirements but not necessarily have the required knowledge.

As the result, Universities spend considerable resources delivering summer schools, access, foundation and stepping-stone courses. Most teachers, who themselves had been taught in a traditional way, rely on traditional methods of delivery. However, such methods had been developed for the learners with good memory, good pattern recognition abilities, high confidence and considerable time to practice. These suppositions are particularly unrealistic when dealing with undergraduates who have limited background, limited memory, limited proficiency in explanatory reasoning, limited confidence, limited interest in the subject, limited study skills and on top of that, limited time to cover a large amount of material, all aggravated by limited contact with teachers.

Yet, these disadvantages can be overcome when dealing with adult learners. Our aim is to describe in some detail ESD, a teacher-guided methodology based on Socratic (but not sarcastic!) Dialogue, which aims to uncover learner difficulties and reinforce their basic understanding through Eulerian Sequencing, a systematic approach to teaching mathematics as a language that allows students to analyse (sequence) mathematical expressions and thus find the relevant solution algorithms (sequences of solution steps). We present evidence that taught this way adult learners can achieve relatively deep learning of mathematics – and remarkably quickly.

THE ESD TEACHING METHODOLOGY

As mentioned in the Introduction, ESD is a teaching approach involving students in a Socratic Dialogue based on Eulerian sequencing. We will discuss these educational toolkits in turn.

SOCRATIC DIALOGUE

Originally, the concept of Socratic dialogue had been associated with literary works developed in Greece at the turn of the fourth century BC, preserved in the Plato dialogues, in which characters used the question and answer method to discuss moral and philosophical problems, though at least one dialogue centred on mathematics: “Now, my young friend, tell me what is the object of mathematics? What does a mathematician study?” [17]. Towards the end of the last century, the method was extended by such educationalists as Collins [4], who introduced it into a general pedagogical discourse, and Hake [12], who revolutionised the teaching of undergraduate physics, allowing ordinary learners to master Newtonian mechanics.

The main aim of Socratic dialogues is to achieve insights, through a Socratic dialogue teachers can learn to teach, and learners can learn to understand what they are taught. Six facilitator measures for achieving the aim are suggested in [11]:

- holding back one’s own opinion
- eliciting learners opinions
- working towards common understanding
- ensuring that the group focuses on the current question
- striving for consensus
- recognising and using fruitful questions and starting points.

The secondary aim is developing soft skills, such as working in a team and participating in a professional debate.

In ESD methodology Socratic dialogue is adapted to teaching mathematics to large classes of ordinary learners (it has been tried on classes of up to 100 students). In this context the “dialogues” typically involve two speakers at any one time, usually student and teacher, sometimes two students, with the teacher leading and structuring the exchange. The emphasis is on the fact that for learners to achieve insights into the subject the teachers have to achieve insight into the students’ thought processes. Also, students have to learn to verbalise these processes and allow others to do the same.

Standard **Teacher Reservations** expressed when faced with the idea of a teacher guided dialogue are

1. Students feel intimidated when asked questions in class.
2. Socrates new that the answers to his questions existed and he knew the prior level achieved by his students.
3. A dialogue can work only in small groups.
4. Only some students will participate.
5. Students’ attention will wander.
6. A dialogue should be student driven and not teacher driven.
7. It is difficult to take into account cultural differences.
8. If given an opportunity to opt out of the dialogue most will.
9. You can get off course.

10. Students learn best in groups rather than when guided by a teacher.
11. Students have different learning styles.
12. Teachers have to be trained to conduct Socratic dialogues.

These reservations can be dispelled using the following **seven golden rules of Socratic dialogue**:

1. Students should know what to expect.

Experience has shown that students respond to Socratic dialogue well if the teacher makes an introductory speech along the following lines:

“The lectures will be conducted in an interactive manner: it is very important for you to keep asking me questions and I would be asking you questions all the time. If I was in your place I would be terrified! Yet, it is very important for you to learn to debate in front of many people. When you become engineers you will have to do this a lot. Also, it is important for both you and me to have a feedback: I need to know what’s in your heads, how fast or slow I should go and you need to know whether you are really achieving understanding.”

It is very important to keep repeating elements of this speech throughout the academic year as the need arises – partly, because not all students hear what the teacher says every time and partly, because some students might have missed the introductory lecture.

2. Students should be given a choice.

At the end of the introductory speech, and throughout the year, as the need arises, it is important to keep saying

“I believe that you would all benefit from participating in a dialogue. However, if one day you feel that - for whatever reason - you do not want to, just tell me so before class. No questions asked!”

3. Heckling and spurious student-student dialogues should be actively discouraged.

Sometimes students laugh at wrong answers given by others and sometimes they just talk among themselves instead of participating in guided dialogues. Both behaviours should be discouraged by pointing out that neither is professional and that it is very important to learn to work as a team. To stop uncivil laughter students have to be explained that everyone should feel free to express their opinions, however, premature, because discussing incorrect answers can be just as revealing and educational as discussing correct answers. Everyone can learn from their own mistakes and mistakes of others. To stop spurious discussions, students have to be explained that when they talk to each other in a large room, all that the others hear is noise and such discussions are never encouraged into during professional meetings, however relevant their topic.

Experience shows that students respond positively when exposed to rules of professional behaviour – if these are elaborated in a polite and respectful manner.

4. Student questions should be actively encouraged.

The teacher should finish every presentation of a small piece of theory or example with the words “Any questions?” If none are coming it is important to keep repeating something like “Remember, if you do not understand something, it is more than likely that at least

30% of other students do not understand this either. By asking questions you learn better and you help others to learn. There is no reason to be afraid to ask questions in this room.”

5. Teacher questions should be kept very simple.

At this level, the aim of Socratic dialogue is to ascertain that students understand basic concepts and logical steps, simultaneously uncovering and filling in gaps in their background knowledge. Many important examples of how this can be achieved are considered in the next section.

6. There should be a high degree of student engagement.

ESD relies on three major teacher moves:

- a) asking “Who thinks they know the answer to this question?”, making mental note of the number of hands raised and picking one student to talk to, preferably the one who didn’t volunteer to answer. The teacher soon learns which student has problem with which topic, so their progress can be monitored.
- b) offering an answer and asking “Who thinks the suggested answer is correct?”, “Who thinks it is incorrect?”, “Who does not know?” It is important to keep encouraging the “do not know” answer as professional. It is also important to comment on the correct/incorrect answer distribution (I often said in class “In the first year the majority is always wrong”, and this comment was always received with an appreciative laughter).
- c) suggesting students offer their answers, without raising their hands, writing all answers on the white board and then taking a vote on which is correct. At the end of this exercise it is important to discuss what is wrong with each wrong answer.

It is interesting to compare hand voting with electronic voting. The electronic voting has an advantage of producing precise statistics on the voting behaviour, which can be useful for teacher reports. It is also much easier to implement due to its anonymity. It is preferred by shy students, particularly, female. However, in my view, educationalists have to strive to instil professional behaviours rather than pander to student insecurities. Experience shows that creating friendly atmosphere where everyone is happy to contribute is possible and leads to deeper learning. It remains to be proven that it develops good debating habits allowing students to participate in later more stressful discussions.

7. A teacher should maintain psychological control.

In author’s experience, about 50 % of students have an immediate positive reaction to the interactive atmosphere of ESD classroom. Others need to get used to it. There is always a minority who feel threatened by it. It is extremely important for a teacher to

- a) always have a calm and friendly demeanour,
- b) NEVER allow bickering in class. If a student does not want to answer, asks “Why do you pick on me?” or voices any other complaint, immediately stop talking to him/her, make a pacifying joke and move on,
- c) always wear pink to class! This has a strong pacifying effect of its own.

Rules 1, 2, 3, 5 and 7 deal with Teacher Reservation 1. Field notes and audio recordings of lectures conducted by independent researchers (Crisan and Lerman) who

attended the author's classes confirm that students do not feel threatened by this strategy even if they do not always have an answer to the question asked. They feel comfortable attempting to answer any of the questions posed [8]. Rule 5 also deals with Teacher Reservation 2.

Following Rule 6 it is possible to overturn Teacher Reservations 3, 4 and 5. The video of the author's lecture to be found on her website [8] shows her interacting with the class of 50 students. ESD was also practised in a class of a 100 which presented no more problems than a class of 50. When talked to most students naturally remain focused, because they want to be able to answer as many questions as possible and avoid "looking silly" in front of their peers.

Following Rule 4 a teacher can allow student initiative and deal with Teacher Reservation 6. The reservation comes from those who broadly follow Piaget's rather than Vygotsky's approach [16]. Accepting the constructivist argument that every learner has to construct his or her own cognitive structure through their own effort and commitment, the question is how this can be achieved. The Piaget purists believe in "discovery learning". This can work in other disciplines or when learners have considerable time at their disposal or when dealing with extra-ordinary learners with high leaning skills. However, it is unrealistic to expect that ordinary learners of mathematics who have limited time to master the subject can generate enough questions of quality that can lead to significant learning enhancement. No student is going to ask to teach them to remove brackets or discuss what is meant by the word *constant*!

Teacher Reservations 7, 8 and 9 are understandable but the proposed approach was practiced in a Central London University which as such hosts an extremely diverse student population and most students responded to guided teaching in a positive manner. Very few asked to be excused from the dialogue and this happened very rarely. It was easy to keep the class on course. Only a couple of times a year was there a need to say something like "We are running out of time and have to stop this discussion now. However, there will be further opportunities to return to this point during tutorials and revision classes."

Teacher Reservation 10 again is understandable, but in the author's experience, ordinary learners benefit most when exercising individual responsibility for learning. When working in groups, the ideas of the more active students may dominate the group's conclusions. This is particularly evident in mathematics classrooms: unless actively engaged, slow learners fail to make the necessary connections [1], [20].

Similarly, Reservation 11 can be refuted by reference to recent pedagogical experiments that have shown that teaching by taking into account learning styles leads to no significant improvement - see recent articles on neuromyths, such as [18]. Teacher Reservation 12 is valid and one of the aims of this paper is to promote understanding that University teachers need this specific type of training.

EULERIAN SEQUENCING

While Socratic dialogue assures continual learner engagement and provides an immediate feedback to both teacher and learner, systematic approach to teaching mathematical abstractions to ordinary learners can be traced back to Euler. He believed that even those who "with regard to" their "capacity" are "not above mediocrity" (to borrow the archaic phrase of the original editors of his Treatise on Algebra [7]) can be taught and enjoy basic mathematics if this is done "with perspicuity", exposing them to elements of "that admirable system of mathematical logic and language which at once teaches the rules of just inference and furnishes an instrument for prosecuting deductions" that, in the words of his English translator Francis Horner, Euler used in "applying all this to solving physical problems". For this reason the second building block of ESD, a systematic teaching of mathematics as a language, is called "Eulerian sequencing" (we thank Ruth Brown of

LSBU for this happy term). The sequencing is exercised in making explicit primary structures of mathematical expressions and in ordering solution steps. It is supported by employing such scaffolding device as Decision Trees. It is in a perfect accord with recent pedagogical findings which suggest that “The amount learned is proportional to the number of self-explanations that a student generates”, self-explanations being comments on a solution step” that contain... domain-relevant information over and above what was stated in the description of the step” [3]. To quote [6] “there are two general sources for self-explanations: The first is deduction from laws, rules, concepts and definitions acquired earlier, usually by simply instantiating a general principle, concept, or procedure with information relevant to the solution step. The second explanation is generalisation and extension of the step.” Such construction of the content of the solution step yields new general knowledge that helps complete the students' otherwise incomplete understanding of the domain principles and concepts.

Discussion of the Eulerian Sequencing

Standard **Teacher Reservations related** to the idea of Eulerian sequencing are as follows:

1. Ordinary learners cannot master abstract concepts
2. Students learn only by doing a huge number of exercises
3. Different learners have different learning styles, there can be no one good teaching method

To counter Teacher Reservations 1 and 2 let us first introduce **seven golden rules of Eulerian sequencing**:

1. Do concept mapping

As suggested above, at the beginning of each lecture, students should be invited to do informal concept mapping by asking them, “What are the main concepts in Algebra?”, “What are the main concepts in Calculus?”, “What types of variables do we study in Algebra?”, “What type of functions do we study in Calculus?”, “What operations on variables do we study in Algebra?” “What operations on functions do we study in Calculus?” and “Why do we study Algebra?”, “Why do we study Calculus?”

2. Ask questions aimed at reinforcing concepts

“What does the word term mean? “, “What does the word *sum* mean? “, “What does the word *factor* mean? “, “What does the word *product* mean? “, “What do we mean by a *constant*?”, “What do we mean by a *variable*’?” “What does the word *unction* mean?”

3. Ask questions aimed at reinforcing knowledge of methods and rules

Typical examples are “What is the Smile rule”, “What is a factoring rule?”, “What is the order of operations?”, “What is the product rule?”

4. Ask probing questions

Typical examples are “What methods of turning a product into a sum have we already learned?”, “When factoring a sum, how do we find the first term, second term *etc.*?” “What are elementary operations on functions?”

5. Ask questions aimed at sequencing expressions

These questions are aimed at teaching students to itemise mathematical expressions the way a mathematician does. The examples are: “What is the first factor in this product?”, “What is the second factor in this product?” “What is the differentiation variable in this problem?”, “What is the function you are asked to differentiate?”, “What is the constant factor in this term?”

6. Ask questions aimed at teaching reasoning

These are the questions of the Why? and What? type, such as “Why do you suggest to use the product rule?”, “What algebraic rule can be used to turn a product into a sum?”

7. Ask questions aimed at sequencing a solution

An ordinary learner is often baffled as to how start the solution process or how to proceed. The typical sequencing questions are “What is the next question to ask yourself?”, “What step should you perform now?” This stage of teaching can be alleviated by employing Decision Trees.

Students raise seven standard **Issues** with regard to Eulerian sequencing:

1. I was taught differently before
2. I have got used to a different language
3. I cannot use Decision Trees
4. I need more worked examples
5. I need more tests
6. I have time for nothing but training to exams. My colleagues at work cannot answer your questions, so there is no need for me to answer them.
7. I studied hard but there was no connection between what I studied and exam questions

Student Issues 1 – 5 are related to Teacher Reservation 1. Student Issues 6, 7 are related to Teacher Reservation 2. Student Issues can be dealt with respectively further by the following explanations:

1. Human being have been designed to follow the first authority figure they encounter. Most prefer the first set of opinions and methods they have been exposed to. Education is about comparing different opinions and methods and choosing the ones that suite you best
2. When learning a technical subject it is important to use precise language. Maths school language is very imprecise.
3. Flow charts, sequential instructions, algorithmic approaches are routinely used by engineering students and practitioners. If a student finds them unintuitive this is the argument for and not against the necessity to train him/her to approach a problem in a systematic manner.
4. If students understand what they are doing they need fewer exercises. If students need more exercise to boost their confidence there are many library and internet resources they can use for this purpose.
5. If students understand what they are doing they need fewer trial tests. If students need more tests to boost their confidence there are many internet resources they can use for this purpose.
6. Same as 4 and 5.

7. The purpose of ESD is to teach a student to recognise a familiar pattern in unfamiliar pictures. Education of engineers is not the same as training of technicians. Students are admonished to stop learning problems by heart and concentrate on solution processes.

All SDES students approached by independent researchers [7] agreed that they needed time to ‘get used’ to ESD teaching, making statements like *“I was not used to explaining the mathematics”*. In the past they just “did it”, without much verbalising or questioning as to ‘why’ and ‘how’. One student with a secure mathematics background said that he found ESD teaching very different to how he was taught at the pre-university level. However, *“once you get used to the approach, it is OK; it is mainly the same thing but presented differently”*. All students agreed that this approach *“forced you to think, to really understand the mathematics”*. This evidence is supported further e.g. by [6] where impact of dialogue and deep-level-reasoning questions is discussed. The authors describe two studies involving undergraduate students and write:

“In Experiment 1, participants learned material by interacting with or by viewing one of four vicarious learning conditions: a noninteractive recorded version of the AutoTutor dialogues, a dialogue with a deep-level-reasoning question preceding each sentence, a dialogue with a deep-level-reasoning question preceding half of the sentences, or a monologue. Learners in the condition where a deep-level-reasoning question preceded each sentence significantly outperformed those in the other four conditions. Experiment 2 included the same interactive and non-interactive recorded condition, along with 2 vicarious learning conditions involving deep-level-reasoning questions. Both deep-level-reasoning-question conditions significantly outperformed the other conditions “

Both students with weak backgrounds and those who can perform most mathematical manipulations when they join the University find appeal in understanding what these manipulations mean.

Evidence of success

The approach had been practiced by the author from 1993 till 2009 with many different student intakes and the results are pretty consistent. The unit provided students with the mathematical tools and methods needed in all other engineering programmes, namely, the basics of algebra of numbers, including complex numbers, and calculus. Here is some evidence to consider:

1. The University where ESD was practiced has been a UK University of widening participation which means that only 15 % of students had A levels, the rest – various other qualifications that are considered to be equivalent but are somewhat what less academic and more than 50 % of students are mature, some – over 40 years old. When I started the minimum requirement was C in A level maths or equivalent, although later this was lowered down to E. However, some students had a higher grade (up to B) and some mature students were accepted even without reaching E. Also, up to 26% of students could be dyslexic, dyspraxic or dyscalculic [16], most without being aware of their condition. Full time students come from disadvantages backgrounds, often have legal, health, family and financial problems, which means that some work full-time too. Some are given bursaries, which occasionally attracts a person not really ready to commit to education. Part-time students are usually experienced and mature family people with better backgrounds and good at time management.

2. Originally the author only taught Electrical and Electronics Engineering students but later those were also joined by Mechanical and Chemical Engineering students. Civil Engineering students were taught for one year but found the curriculum too involved.

3. The unit had been taught over two semesters and delivered as a two-hour lecture every week. In addition, two-hour tutorials took place once a week (one-hour tutorials for part-time students). The lecture classes varied from 40 to 130 strong and the tutorial classes from 20 to 40. The curriculum is presented in Figure 1.

4. The final mark was composed of two components: a Summer exam contributed 50 % of the final mark and in-course assessment – another 50 %; in its turn, the latter was composed of three components, a Winter Phase Test performed under examination conditions that contributed 30% of the final mark, the Semester I logbook and Semester II logbook, each contributing 10 % of the final mark. Thus, 80 % of the mark was obtained under examination conditions.

Let us discuss the in-course assessment in more detail. Logbooks were introduced to entice students to tutorials. Originally their attendance was poor and when questioned on the reasons they opined that there was no point to attending tutorials because these were not assessed. Students were asked to put their homework in their logbooks as well as their tutorial and revision work (carried out in preparation to test or exam). At the end of each tutorial tutors were asked to initial both the tutorial work and the corresponding homework. Signing the logbooks gave tutors an opportunity to teach students elements of time management and give them a quick feedback on their effort both at home and in class. A particular attention was paid to whether students just attempted to do homework exercises at home or studied their lectures first according to instructions on how to do this given in their study skills documents. If a piece of homework was not done in time or if a tutorial was missed the “out of time” entries were not marked, so as to alleviate the “post mortem” to be conducted if a student underperformed in test or exam. The logbooks were submitted immediately after test/exam and marked taking into account tutorial attendance (as evidenced by tutor initials), quantity of homework, quality of logbook and quantity of revision work. The mark was finalised only after marking the test/exam paper to make certain that the in-course assessment has never brought the overall mark down: The in-course assessment was designed as a mark for effort rather than achievement and if a logbook did not look impressive but the test/exam mark was good the effort was considered to be “adequate to the task”.

Results of Phase Test were discussed in the first week of Semester II. Correct solutions were presented together with the list of common mistakes. Students were then asked to revise their test papers indicating which of the common mistakes did they make during the test. They were given a week for this task and if done well this “post mortem” could bring the marks under 30 % up by up to 10% extra.

5. Both Phase Tests and Summer Exams were composed to contain problems that looked unfamiliar to students. Students were prepared for this psychologically; it was often emphasised that the aim of teaching engineers was to ascertain that they recognised familiar patterns in unfamiliar pictures. This strategy is quite different to a wide spread practice of teaching to test. Comparison of these two approaches to assessment lies outside the scope of this paper. The philosophy behind question composition mentioned only to help the reader to assess the strength of evidence. In the past few years the line managers insisted that the exam questions were scaffolded suggesting solution steps which might weaken the evidence somewhat.

6. It has been shown time and again [8], working with similar student intakes, that if both lectures and tutorials had been delivered via simple exposition promoting the traditional learning by rote the failure rate at the first attempt at exam would be 50% (could be 70%!) and if the methodology had been used to deliver both lectures and tutorials the failure rate would fall down to 30% (*ibid.*). If lectures were delivered using ESD but tutorials were not the pass rate could fall by up to 10%. For part-time students who attended lectures and seminars delivered using ESD the pass rate after the Summer exam was about 90%.

Semester I Topics	Semester II Topics	Weeks
Introduction and algebra (elementary operations)	Complex Numbers (addition, Argand diagram)	1
Algebra (solving equations)	Complex Numbers (multiplication, exp form)	2
Algebra (powers, roots and logs)	Complex numbers (roots, loci)	3
Functions of real variable: diagrammatic representation	Sketching by simple transformations, completing the square	4
Functions of real variable: graphs, polynomials	Sketching by analysis	5
Functions of real variable: exp & log functions	Sketching by analysis (ctd)	6
Functions of real variable: trig & hyperbolic fns	Taylor and Maclaurin series, L'Hospital's rules	7
Sequences, series and limits	Integration methods, partial fractions	8
Continuity and derivatives	Applications of integration	9
Differentiation table, rules & decision tree	Ordinary differential equations	10
Integration of functions (definite integrals)	Revision	11
Integration of functions (indefinite integrals)		

FIGURE 1. A possible curriculum

CONCLUSIONS

Promoting the teacher guided Socratic dialogue based on Eulerian sequencing, the ESD methodology is adapted to modern times to teach mathematics to large groups of engineering undergraduates, with very poor mathematical background, about 26% of whom may be dyslexic, dyspraxic or suffer from dyscalculia. There is lot of evidence that the approach works: It comes from the author's practice who for 16 years taught *Introductory Engineering Mathematics* and coordinated teaching this subject to the first year engineering undergraduates of a UK University of Widening Participation.

The methodology ensures that students develop correct study skills, are taught rather than trained, master algorithmic and iterative approaches to problem solving and last but not least, learn the art of technical debate. Last but not least, the methodology allows teachers to discover common problems and misconceptions that are not necessarily always the same. Thus, not only does ESD allow learners to enhance their learning, it also allows teachers to enhance their teaching.

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