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ORIGINAL ARTICLE

Integrating technology into mathematics teaching at the university level

Zsolt Lavicza

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Abstract The emergence of new computing technologies in the second half of the twentieth century brought about new potentials and promised the rapid transformation of the teaching and learning of mathematics. However, despite the vast investments in technology resources for schools and universities, the realities of schooling and the complexities of technology-equipped environments resulted in a much slower integration process than was predicted in the 1980s. Hence researchers, together with teachers and mathematicians, began examining and reflecting on various aspects of technology-assisted teaching and learning and on the causes of slow technology integration. Studies highlighted that as technology becomes increasingly available in schools, teachers' beliefs and conceptions about technology use in teaching are key factors for understanding the slowness of technology integration. In this paper, I outline the shift of research focus from learning and technology environment-related issues to teachers' beliefs and conceptions. In addition, I highlight that over the past two decades a considerable imbalance has developed in favour of school-level research against university-level research. However, several changes in universities, such as students declining mathematical preparedness and demands from other sciences and employers, necessitate closer attention to university-level research. Thus, I outline some results of my study that aimed to reflect on the paucity of research and examined the current extend of technology use, particularly Computer Algebra Systems (CAS) at universities, mathematicians' views about the role of CAS in tertiary mathematics teaching, and the factors influencing technology integration. I argue that due to mathematicians' extensive use of CAS in their research and teaching, documenting their teaching practices and carrying out research at this level would not only be beneficial at the university level but also contribute to our understanding of technology integration at all levels.

Keywords Technology · Review · University level · Mathematicians

Z. Lavicza (✉)
Faculty of Education, University of Cambridge,
Cambridge, UK
e-mail: zl221@cam.ac.uk

1 Introduction

In this paper, I will review aspects of technology integration into mathematics education in relation to the university-level teaching. I organised this review chronologically to highlight the major shifts in research foci and exemplify these shifts through International Commission for Mathematics Instruction (ICMI) studies and conferences. Firstly, I will illustrate that the emergence of new technologies presented high hopes for their rapid integration into classrooms, but recent studies suggest that this integration is considerably slower than expected. A number of researchers attempted to examine the reasons behind this slowness, and gradually, the focus of their investigations shifted from access problems and the examination of student learning in technology-equipped environments towards understanding teachers' beliefs and thinking about technology use in teaching. Secondly, I will highlight that although initially most studies focused on university-level teaching a considerable imbalance has developed in favour of school-level research over the past decades. At the same time, universities have started to face new challenges due to the declining mathematical preparedness of students enrolling to higher education and the demand posed by the emergence of new technologies. Therefore, I argue that conducting educational research at the university level is even timelier than ever. In addition, understanding technology integration without knowledge about the tertiary level is clearly incomplete. Thus, in the third part of the paper, I outline some results of a study that I carried out to examine the extent of technology use, in particular attention to Computer Algebra Systems (CAS), by mathematicians, their beliefs about technology, and to explore the factors that influence CAS integration into teaching tertiary mathematics teaching.

2 The overview of technology integration into mathematics education

2.1 High hopes and the reality of slow technology integration

Computers were originally invented to enhance and accelerate tedious mathematical operations. Beginning in the mid-twentieth century, technological innovations quickly expanded the use of computers, first in scientific work and then in everyday life. As predicted, computers began to transform both mathematics and mathematics teaching (Kaput 1992). By the late 1980s, practices in various fields of mathematics were fundamentally transformed, largely because of the availability of technology. Moreover, technology gave birth to nurtured new scientific fields such as computational mathematics, statistics, and computer science (Dubinsky 1996; Strasser 2008). Computers and calculators started to become increasingly used, initially for demonstrating mathematical concepts by teachers in classrooms, and later when technology became more accessible for students' personal use within classrooms, and eventually at home (Wong 2003). The emergence of various technologies offered new potentials and promised spectacular possibilities to change and revitalise mathematics education at all levels. In 1989, the National Council of Teachers of Mathematics in the United States outlined the goals that teaching with technology should realise in mathematics education (NCTM 1989). These ambitious goals were later reformulated in the so-called *Technology Principle* in the new *Principles and Standards for School Mathematics* (NCTM 2000), envisaging a universal access to technologies and the benefits of technology use.

The potential benefits of technology integration presented stellar prospects for mathematics teaching and learning. However, the use of technologies in schools, because of their high cost and limited capabilities, was not extensive in the 1980s (Kaput 1992; Paola 2008), but many thought (rightly) that this was only a temporary situation. As a reaction, in many countries, governments channelled extensive funding for technology to equip schools and universities with up-to-date hardware and software (Hawkridge 1990). Nevertheless, resolving access to technology was not a sufficient condition for extensive technology integration, because the complexities of these new learning environments were more taxing than originally expected (Laborde 2001; Ruthven 2008). Rubin (1999) described these issues in his paper suggesting that technology integration has: "rich potential, significant obstacles, and important concerns" (p. 19). The mathematics education research community reacted speedily to the emerging difficulties and developed research agendas to examine the challenges posed by use of technology in education. Several research projects were set up to investigate various aspects of technology integration into the teaching and learning of mathematics. Even the ICMI devoted its first large-scale study to technology in mathematics teaching and learning, titled as "The Influence of Computers and Informatics on Mathematics and its Teaching" (Churchhouse et al. 1986).

Despite the difficulties articulated by several of its authors, the ICMI-1 study presented an optimistic future for the integration of technology into mathematics education. Some years later, due to increasing access to both computers and calculators, Kaput (1992) predicted that technology would become rapidly integrated into all levels of education. Steen (1988) illustrated the atmosphere of the time:

> Anyone who presumes to describe the roles of technology in mathematics education faces challenges akin to describing a newly active volcano—the mathematics mountain is changing before our eyes, with myriad forces operating on it and within it simultaneously. Many of these forces contain a technological component. (p. 612)

Over the next decade, building on the work of technology pioneers research on technology issues related to mathematics education expanded rapidly, as illustrated by a review of publications on this topic (Lagrange, Artigue, Laborde, and Trouche 2003). Later, a combination of more affordable technology, further technological advancements, and increased government and school investments on Information and Communication Technologies (ICT) led to technology becoming more accessible, particularly in well-off countries (Mullis et al. 2000, 2004; Ofsted 2005; Ruthven and Hennessy 2002). However, the accumulated evidence of the last 15 years indicates that the promise of rapid expansion of technology use in education has not been realised and technology still plays a marginal role in mathematics teaching and learning (Becker 2001; Cuban, Kirkpatrick, and Peck 2001; Ruthven 2008). Furthermore, Laborde's (2008) recent review of the proceedings of the past three International Congresses on Mathematics Education (ICME), held every 4 years since 1969, indicated a slight decline in the proportion of papers related explicitly to technology.

In spite of the recognised importance of technology use in mathematics education, the initial enthusiasm for its potentials seemed to have weakened during the past few years (Laborde 2008). This loss of enthusiasm, however, has not categorically degraded the perceived potentials of technology in education, though it may be the result of a natural cycle in the adoption of new technologies in education (Ruthven 2007b, 2008). Ruthven (2007b) explained the rise and fall of enthusiasm towards new technologies during the process of their integration into education. Based on historical examples, this cycle begins with an initial enthusiasm for the new technology and causes a surge of interest, but over time difficulties arise resulting uncertainty and aversion. In the next two sections, I will briefly highlight some of the benefits and difficulties examined by researchers followed by the increased interest towards the role of the teaching in technology integration. It is important to stress that the issues in this list refer to the developments in the past two decades, but Ruthven (2008) highlights that the pace of changes of digital technologies is so rapid that research agendas are changing more rapidly than any time before.

2.2 Shift of attention to the teacher

The review of the literature of technology-related research papers at the end of the 1990s by Lagrange et al. (2003) highlighted that while the majority of research projects focused on various aspects of students' learning in technology environments little attention had been paid to teachers and teaching. This trend was later confirmed by Laborde's (2008) review of ICME conferences. It was becoming more clear that teachers play a key role in technology integration and examining their beliefs about technology and technology-assisted teaching is important for the understanding of technology integration into mathematics teaching. Thus, this shift of focus contributed to the increased attention of recent studies on teachers and on providing explanations for various aspects of the slow pace of technology integration (e.g. Artigue 2005; Laborde 2001; Ruthven 2008; Stacey, Kendal, and Pierce 2002; Monaghan 2004). Results of these studies revealed a variety of factors that influence technology integration. Among these were external factors such as accessibility of computers, technological imperfections, and policy decisions. However, evidence has accumulated that teachers' conceptions, beliefs, and knowledge are crucial factors in technology integration. In addition, accounting for social and cultural influences on classroom environments was essential to be able to describe the complexity of their conceptions.

2.3 Shift of attention from the university to the school level

Another research trend, shifting attention from university to school level, can be observed by reviewing the publications and reports of ICMI studies and ICME conferences as well as relevant "meta-analyses" of publications. The first ICMI study in 1985 was almost exclusively concerned with the integration of technology into university-level mathematics (Churchhouse et al. 1986). Despite difficulties articulated by several of its authors, the study presented an optimistic future for technology integration into mathematics education (e.g. Tall and West 1986; Seidman and Rice 1986; Mascarello and Winkelmann 1986). Fifteen years later, the ICMI-11 study reported on the use of technology in a variety of mathematics courses taught in universities (Holton 2001) and other papers described the ways in which technology could be used to enhance students' learning and the impact of technology on classroom communication (King, Hillel, and Artigue 2001). Overall, ICMI-11 suggested that despite the optimism of ICMI-1, the integration of technology was not so widespread and its use was only 'cosmetic' (Hillel 2001). The systematic review of university-level technology literature by Thomas and Holton (2003) provided a thorough summary of the chronological phases of technology integration into undergraduate curricula. But, at that time, it was clear that the majority of educational research activity shifted to the pre-university level (Lagrange et al., 2003; Thomas and Holton 2003). In addition, according to these studies, similar to the school level, issues of the 'teacher dimension' in relation to technology use were lightly researched in the university context. There were only a handful of studies that investigated this issue. In the 17th ICMI study, *Technology Revisited*, only two university-related research papers were reported (Buteau & Muller, 2006; Lavicza, 2006). At the ICME-10 and ICME-11 (e.g. Jarvis, Lavicza, and Buteau, 2008) conferences as well as at the ICMI 100th Anniversary Symposium, there was little representation of university-level research, not counting for teacher education studies. However, several specialised conferences and professional organisations developed around the issues of university-level mathematics teaching and technology integration. For instance, the Research on Undergraduate Mathematics Education (RUME) Special Interest Group of the Mathematical Association of America organises groups, research, and conferences in relation to university-level education in the US; in the UK, the Mathematics, Statistics, and Operation Research Network (MSOR) engage mathematicians in similar activities; the annual

International Conference on Technology in Collegiate Mathematics (ICTCM) offers a forum for mathematicians to showcase their teaching innovations; and the bi-annual International Conference on the Teaching of Mathematics—at the Undergraduate Level (ICTMT) attracts research reports on studies at the university level (Buteau, Lavicza, Jarvis, and Marshall, 2009). Despite this development of activities, most reports are practitioners' accounts on innovations. There are studies and they encourage mathematicians to engage in educational research and collaborate with educational researchers (Challis and Gretton 2002; Sultan and Artzt 2005), though such research activity is still limited compared to that conducted at the school level.

2.4 Increasing importance of university-level research

During the past two decades, university-level mathematics teaching has encountered new challenges which are contributing to the changes in higher education teaching practices. Firstly, the increased enrolment in universities, the lower student interest in STEM[1] subjects in schools, and difficulties in school-level education resulted in a decline in mathematical preparedness of students entering universities causing major challenges (National Science Board 2007). Particularly, there has been substantial anxiety about the decline of students' quantitative skills and increased difficulties with their mathematical work. This development could cause considerable difficulties in the education and the economy of developing countries (Peelo and Whitehead 2006). In the past two decades, several reports were published suggesting the decline in students' mathematical abilities (London Mathematical Society 1995; HEFCE 2005; Savage and Hawkes 2000; National Science Board 2007). A number of these reports used the word 'crisis' to describe the situation and even the media began to pay attention to this problem (BBC 2006, 2007). Most of these academic, governmental, and media reports rationalised the need for mathematical skills because of the necessity of mathematics for economic development and for supporting learning in science and engineering disciplines.

Secondly, the emergence of new technologies available for teaching opened new perspectives and intensified demands for the changes in teaching practices. In addition, mathematics departments responded to the challenge posed with the emergence of new technologies and to the demand of other disciplines and society (Savage and Hawkes 2000; National Science Board 2007). Furthermore, policy makers are urging universities to nurture mathematically skilled individuals, who can function well in the modern society, and adequately satisfy the needs of employers, particularly in science and engineering (Savage and Hawkes 2000)

2.5 Limited research on technology use of mathematicians

The observed weaknesses in students' mathematical preparedness and the availability of technology prompted numerous mathematicians to experiment with innovative teaching and a number of them have turned their attention to pedagogical issues (Buteau et al. 2009). Moreover, in many cases, the integration of technology into undergraduate teaching is seen as way to revitalise teaching and assist students to raise their level of mathematical understanding (Devlin 1997). Although university-level mathematics teaching is undergoing considerable changes and is in need of assistance, little attention has been paid to teaching issues at this level by the educational research community. In particular, little is known about the current extent of technology use and mathematicians' practices in university teaching (Lavicza, 2007).

As discussed earlier, at the school level, national and international surveys regularly reassessed the state of technology usage in schools, though according to my knowledge, no such review has been conducted at the university level. Hosein (2005) conducted a study surveying 311 (77 responses) lecturers about their use of mathematical software while teaching *linear programming* for undergraduate students in the English speaking countries: Australia, New Zealand, UK and US. Hosein, Aczel and Clow (2006) reported that the use of software during teaching was not extensive; however, software was associated with most of the courses. They described that academics working in applied disciplines tended to use software more extensively than their colleagues in pure disciplines, but their study was quite subject-specific, hindering the possibility to draw inferences on technology use in mathematics teaching.

3 Examining technology (CAS) integration by mathematicians study

To offer an overview of technology use in mathematics teaching at the university level, I carried out a study on mathematicians. I particularly focused on mathematicians' use of CAS, their views on the role of CAS in the mathematics teaching, and the factors influencing CAS integration into university mathematics teaching. In this section, I will briefly outline this study and some of the principle findings (more detailed descriptions of the methodology and results can be found in Lavicza, 2007, 2008a, b).

[1] STEM: Science, Technology, Engineering, and Mathematics.

3.1 Aims, methods, and data analysis

This study aimed (1) to examine the current extent of technology use in universities; (2) to uncover mathematicians' views on the role of technology in mathematics literacy and curricula; and (3) to explore the factors influencing technology integration into mathematics teaching and learning at universities.

To be able to address the aims of the study, I decided to take an international comparative stance. The participating countries, Hungary, United Kingdom, and United States, represent a variety of cultural and economic considerations. According to the international comparative research literature (e.g. Atweh, Clarkson, & Nebres, 2003), comparing similar and dissimilar cultures is beneficial and we can learn from practices utilised in a variety of countries. Obviously, my selection has also been influenced by my personal and professional background, my familiarity with the higher education systems in these countries, and the extensive literature of teacher conceptions both locally and comparatively.

The design of the study followed a two-phase mixed methods approach (Johnson and Onwuegbuzie 2004). The first qualitative phase of the study comprised interviews, class observations, and the review of curriculum materials of 22 mathematicians in Hungary (HU), the United Kingdom (UK), and the United States (US). Based on the findings of this phase, I developed an on-line questionnaire, which was sent to 4,500 mathematicians in the participating countries. The selection of participants utilised a stratified sampling scheme considering the size, type, and specialisation of universities in different countries. For example, in the US, the Carnagie Classification of universities was followed and sampling method of the Conference Board of the Mathematical Sciences (CBMS) was utilised (Lutzer, Maxwell, & Rodi, 2002).

The development of the questionnaire took a period of 7 months due to several rounds of piloting and modifications. The final version of the questionnaire included 35 main questions and many of these involved sub-questions/statements (the full questionnaire can be found in Appendix). The questionnaire contained six sections: mathematicians' (1) introduction and description of the study; (2) background information; (3) views on CAS in mathematics teaching and learning; (4) access to technology and training; (5) own use of CAS in teaching; and (6) comments and further participation in the study. The questionnaire items were mainly developed based on the first phase of the study and feedback from piloting. However, in Sect. 2, mathematicians' views on technology were greatly influenced by the socio-cultural theories utilised by Ruthven and Hennessy (2002) and Becker (2001). After examining the option of five- or six-point Likert scales, I decided to utilise a five-point Likert scale (from strongly disagree to strongly agree) in this part of the questionnaire to offer a 'Neutral' option for participants. In addition, for those questions required active use of CAS in teaching or research, a 'Not Applicable' (N/A) option was given to avoid missing values for such questions. The statistical analysis later indicated the robustness of the developed instrument.

The second phase of the study concentrated solely on a particular technology application, CAS,[2] because CAS is one the most widely used mathematical software packages in university mathematics. Furthermore, the review of all kinds of technology would have been unfeasible for such a questionnaire study.

Interview data were transcribed and analysed utilising HyperResearch software. Details of this analysis are reported in Lavicza (2007). The questionnaire data were organised, cleaned and imported into SPSS software and various descriptive and inferential statistical techniques were performed on it. In addition, Structural Equation Modelling techniques were applied, with AMOS software, to uncover influences on CAS integration.

3.2 Results

3.2.1 Mathematicians' interest towards technology integration and pedagogical issues

Ultimately, 1,103 mathematicians responded to the questionnaire, which constitutes an unexpectedly high 25% response rate.[3] In addition to responses to closed questionnaire items, mathematicians wrote approximately total of 150 pages for the optional open questions and sent approximately 600 e-mails many of which included relevant comments. Furthermore, 297 mathematicians volunteered to participate in future technology-related studies.

The high response rate and the generally positive feedback suggest that mathematicians are possibly interested in learning about technology applications in mathematics teaching and many of them are open to discuss educational issues. However, there is a perception among mathematics educators that mathematicians are difficult to approach with educational issues and often unsympathetic to educational research(ers) (personal communication).[4] Nonetheless, it is

[2] CAS: Any software package that is capable to perform numeric and symbolic computations and visualise mathematical expressions. Examples: Derive, GeoGebra, Maple, Mathematica, MuPad, Matlab (included), etc.

[3] Response rates by country: 521 US (20%), 347 UK (25.2%), and 235 HU (46.35%).

[4] This statement is based on discussions with a number of mathematicians and mathematics education researchers. Particularly, I would like to mention long discussions with Professor Leone Burton who was a prominent researcher of mathematicians and she cautioned me about many aspects of my research.

encouraging to note that a large number of mathematicians were receptive to participate and collaborate in educational research. This increased attention by mathematicians offers an opportunity to develop collaborative relationships and find how to best enhance the use of technology in mathematics teaching.

Engagement in such relationships could ease one of the important concerns of participants that they did not feel support and encouragement from their colleagues and departments to use CAS in their teaching (61 and 54, respectively, responders commented about these issues in their written responses). Moreover, the additional time required to develop technology-enhanced teaching materials was not acknowledged and appreciated in many departments (45 comments). Certainly, this problem was transcended by technology enthusiasts, but it may be a substantial barrier for many mathematicians who are open to experiment with CAS in their teaching. Unsupportive environments discourage people from working on teaching innovations and make it difficult to change mathematicians' views about using CAS in teaching. However, engaging mathematicians and administrators in a dialogue about the role of technology in teaching and various technology-assisted innovative practices could initiate changes in mathematics departments and appreciation (time or monetary) on such teaching activities (Allen et al. 1999).

3.2.2 Extensive use of CAS in research

The analysis of the data revealed that more than two-thirds of participants reported at least occasional[5] use of CAS in their own mathematical research. This percentage is considerably high even when considering biases due to the 25% response rate of the questionnaire. It is likely that those mathematicians who have strong opinions about CAS use were overrepresented among the responders. After accounting for this possible bias, by considering the worst-case scenario that all CAS users replied to the questionnaire, the number of mathematicians who acquired knowledge using CAS for their research is still high. This knowledge then can readily be utilised for CAS-assisted teaching. In fact, the statistical models developed for evaluating influences on CAS integration into teaching revealed that CAS use in research is by far the most important factor on mathematicians' decisions to employ CAS in their teaching practices. In relation to school-level studies, the proficiency in the use of a software package offers an advantage to mathematicians over teachers as they often do not require initial training for software before beginning to use it in their teaching.

[5] I defined occasional use on a percentage scale and it means less than 50% of use in their classes.

3.2.3 Extensive use of CAS in teaching

More than half of participants reported CAS use in teaching. Even if the overrepresentation of CAS users in the sample is taken into account, the level of CAS use at universities is comparable and even higher with the level of pervasive computer use in grade-8 classes from the countries that participated in the recent TIMSS study, which was reported as occurring in only 3–5% of the classes (Mullis et al. 2004). It can be also noted that besides the use of CAS, mathematicians in their written responses reported to employ other kinds of technologies, such as online homework and assessment systems, spreadsheets, and specialised software packages, in their teaching to be able to compare to general technology use at the school level. Therefore, it can be argued that the extent of technology use at universities is substantial and it suggests that mathematicians have developed an extensive array of technology-assisted teaching materials and pedagogical approaches. Hence, documenting, examining, and accumulating mathematicians' technology-related teaching practices and materials could be beneficial not only for research but also for other mathematicians, academics in STEM departments, and could contribute to enhancing school-level practices and further our understanding of technology use at all levels.

3.2.4 Purposes of CAS use

Those responders who reported CAS use in teaching indicated that they primarily use CAS to visualise mathematical concepts (88%) and mostly in a lecture-like setting (84%). Engaging students in experimentation activities (89%) and solving 'real world' problems with CAS were other frequently reported types of CAS applications. The latter activities usually took place in computer lab settings (72%) or in regular classrooms using hand-held devices. Although the majority of CAS users indicated a preference for computer-based CAS packages (90%), some hand-held CAS software use was also reported (30%). The application of CAS for out-of-class activities was less frequently indicated in participants written responses, particularly by Hungarian- and UK-based participants. Finally, 82% of mathematicians reported that they use CAS to prepare and assign homework for students. But there were also some participants who commented in their written notes that they only use CAS 'behind the scenes' to check solutions and prepare exercises.

Several reasons could be found to explain the findings outlined above. Firstly, the use of CAS for visualisation in lecture settings might simply be the result of mathematicians' teaching schedules. Even if they would prefer students to use CAS in their classes, it is not possible because they are scheduled to teach in regular classrooms where computers are not available. However, most modern classrooms are equipped with projection equipment, which makes visualising images for the entire classroom possible, enabling mathematicians to use CAS in such environments. However, the availability of computer labs could frequently prevent mathematicians to use CAS in this setting. In addition, Allen et al. (1999) implied another constraint to CAS use in labs for US mathematicians. In the US, many courses are offered in a lecture-lab setting and in this setup mathematicians are only required to give lectures, while lab sessions are taught by graduate assistants. This kind of course setup prevents mathematicians to assign CAS activities directly to students, and this constraint was supported by several participants.

Secondly, Ruthven (2007a) described an even more important obstacle of technology use at the school level. Ruthven (2007a) explained that facilitating classes in computer labs not only breaks usual classroom routines but also (as Laborde (2001) suggested) poses higher demands on teachers to control the classroom activities. Monaghan (2004) additionally finds that if teachers schedule lessons in computer labs they feel obliged to use computers during the entire classroom period, whereas they might only need to use technology for a fraction of the lesson. Mathematicians can also encounter such dilemmas while determining their CAS use in their teaching. The study of Laborde (2001) suggested that initially teachers use technology in ways that enable them to retain full control of classroom activities, but as they gain experience they become more courageous and allow more technology use by students. These findings may also help to explain why mathematicians reported preferences of presentation use of CAS. However, participants and the literature (Hoyles and Lagrange 2005) suggested that technology will become increasingly mobile and accessible in the near future. It is likely that in few years students will own a mobile computer with CAS applications. This access to mobile computers and CAS could further ease the tension between the use of technology in lecture and computer lab settings. Nevertheless, the most important issue remains that mathematicians should become open to use CAS in their teaching and engage in practices that encourage the use of CAS by students both in classrooms and home.

3.2.5 The role of CAS in mathematical literacy and curriculum

The analysis of the data revealed that the majority of mathematicians believed that CAS is becoming an important element of mathematical literacy and will become integral part mathematics teaching and learning in the future.[6] In addition, participants indicated that CAS knowledge is beneficial for students' future courses and career. However, mathematicians had differing views about the applicability of CAS for different student groups. Mathematicians emphasised the importance of CAS knowledge for students studying science and engineering courses while they were less convinced by its value for students majoring in mathematics.

Mathematicians were slightly concerned about the impact of CAS on mathematical knowledge. The least-supported questionnaire item with the highest standard deviation enquired about the knowledge students acquire while learning in CAS-equipped environments. This result suggests that the changing characteristics of mathematical knowledge are a polarising issue for mathematicians. Hoyles et al. (2004) suggested that although the use of computers is widely accepted in sciences and by the society they still have a dubious role in education. It is in contrast with the widely accepted view by mathematicians that CAS is a part of mathematical literacy. In other words, mathematicians accept that CAS is part of the literacy, but at the same time they are reluctant to accept that CAS shapes mathematical knowledge. This disparity is possibly derived from the mismatch between mathematicians' CAS-related and mathematical beliefs. In the first phase of the study, it was suggested that mathematicians' teaching and technology conceptions derive from mathematicians' conceptions of mathematics (Lavicza 2007), but the mismatch outlined above implies a more delicate relationship. Hence, a closer examination of this relationship between these conceptions would be beneficial. It would be important that the mathematics community engages in a wide-ranging debate about the impact of technology on the mathematical knowledge required to learn in universities. This is crucial, especially from the perspective of assessment, since the knowledge acquired in a technology-enhanced environment may differ from traditional methods and thus the assessment should reflect this difference. Several school-level studies deal with this issue (e.g. Brown 2001; Kieran and Drijvers 2006); however, such debate is very complex and can last for decades, yet research in this area can contribute to its progress.

Responses indicate that mathematicians view the role of technology, particularly CAS, positively in mathematical literacy and in university curricula. They agree that proficiency in CAS use is beneficial for students' future studies

[6] Results are derived from two sources: (1) five-point Likert-scale items (the complete questionnaire can be found in Appendix and statements in questions 10, 11, and 12 are summarised in these results); (2) written comments of mathematicians. Means and standard deviations of Likert-scale items (strongly disagree to strongly agree) were calculated and used to reflect opinions of responders. In addition, comments were categorised and their frequencies were reported as important issues in the study. Detailed analysis can be found in Lavicza (2008a, b) and will appear in other publications.

and career, and they suggest that CAS will eventually become an integral part of the undergraduate mathematics curricula. The comparison of results between countries indicated little difference in mathematicians' perspectives on the role of CAS in mathematical literacy and curricula. However, mathematicians who use CAS in teaching value the role of CAS in mathematics teaching considerably higher than their colleagues who do not use CAS in teaching. Due to this result, it can be suggested that enthusiasts are more likely to employ CAS in teaching or the use of CAS causes them to think more positively about the usefulness of CAS in education.

The results of this study indicate that mathematicians use technology for teaching as much as or even more extensively than school teachers. Numerous mathematicians have accumulated extensive knowledge about mathematical software packages through their own research. Coupling this knowledge with their expertise in mathematics as well as with the freedom of developing their own curriculum materials provides a rich opportunity for innovations in technology-assisted teaching. In addition, mathematicians view positively the role of technology in mathematical literacy and curricula. Therefore, it is likely that there are already remarkable innovations and successful teaching practices existing at the university level. Consequently, it would be advisable to pay closer attention to mathematicians' technology-assisted teaching such as documenting and researching these practices and innovations. This could contribute significantly to the advancement in research and practice not only at universities, but also at the school level.

3.3 Further extension of the study

To broaden the results of the previously outlined study, we are carrying out a larger project called CAS in University Instruction: implementing an international research study on CAS usage and sustainability together with Canadian colleagues Chantal Buteau and Daniel Jarvis, with funding from the Social Sciences and Humanities Research Council of Canada (SSHRC). This research programme features three main components: (1) a comprehensive literature review of technology-related conference papers/journal articles (aiming for 1,500 publications) from the past decade by adapting parts of the framework developed by Lagrange et al. (2003); (2) a nation-wide, on-line, bilingual (English and French) survey of Canadian mathematicians regarding beliefs and practices relating to CAS use in university-level mathematics teaching; and (3) two case studies (one in Canada; one in the UK) in which a mathematics department has sustained technology-related instructional/curricular change over time (Jarvis et al. 2008). Preliminary results of the literature review can be found in Buteau et al. (2009) and results of the survey and departmental review will appear in forthcoming publications.

4 Summary

The reviewed ICMI and other literature highlighted two historical trends of the changing foci of research projects. Firstly, compared to early technology-related studies, which were almost entirely conducted in university classrooms, the majority of recent studies concentrate on school-level environments. Secondly, as early studies focused primarily on students' learning, more recent investigations highlighted the importance of understanding teachers' views on and conceptions of technology use.

Recent changes in tertiary education, such as the shortage of students in STEM subjects, the weak mathematical preparedness of students entering universities, and the emergence of new technologies, raised researchers attention to university-level issues. In addition, recent studies at the school level advocated a holistic review of technology integration (Monaghan, 2004), which must include tertiary education. Hence, to highlight some issues in technology integration in university-level teaching, I reported on results of a study that I conducted to examine mathematicians' views on and practices with technology. In this study, I particularly focused on CAS as it is the most prevalent software employed in universities.

Results of the outlined study suggested that a large proportion of mathematicians use CAS and other technologies for both their research and teaching. They believed that CAS is becoming an integral part of contemporary mathematics knowledge or literacy (numeracy), but they called for debates to better understand the changing nature of this knowledge. It was apparent throughout the study that mathematicians are becoming increasingly interested in discussing pedagogical issues. Therefore, this could create an invaluable opportunity for collaborative work of mathematicians and mathematics educators, which I hope could result in valuable integration of technology into mathematics teaching and learning at all levels.

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Appendix

Study questionnaire

Page 1 of 7

The Integration of Computer Algebra Systems into University-level Mathematics Education Study - K1

UNIVERSITY OF CAMBRIDGE

INTRODUCTION

Project aims: CAS is increasingly used in the teaching and learning of university level mathematics. The primary aim of this project is to investigate mathematicians' views on both current and future use of such software in university-level mathematics education.

Time required for the survey: Approximately **8 minutes** for non-users of CAS (18 questions) and **12 minutes** for CAS users (29 questions). The survey contains 7 pages. Progression is indicated in the heading of the page.

Definitions:
CAS – Any software packages, run on either computers or on handheld devices, that incorporate computational, symbolic, and visualization features. (For example, Derive, Maple, Matlab, MathCAD, Mathematica, MuPad, TI-89, TI-92, TI Voyage 200, and others)
Teaching with CAS – any use (computational, symbolic, or visual) of any CAS software in any teaching-learning settings.

Prize draw – To thank you for your help with this study I would like to invite you to enter a draw for one of the following prizes:

1) A copy of the **17th International Commission of Mathematics Instruction Study Book "Digital technologies and mathematics teaching and learning: Rethinking the terrain"**

2) A copy of **"Research on Technology in the Teaching and Learning of Mathematics"** (Editor: Kathleen, M. Heid, Pennsylvania State University)

3) A free registration for the **5th Computer Algebra in Mathematics Education Symposium and First Central- and Eastern European Conference on Computer Algebra- and Dynamic Geometry Systems in Mathematics Education Conference** that will be held University of Pécs, Pollack Mihály Faculty of Engineering, Hungary, 21-23 June, 2007. For more information about the conference: http://matserv.pmmf.hu/cadgme/

Confidentiality: your identity will not be revealed in any circumstances during or after the study (see confidentiality statement at the end of the questionnaire)

Questions/comments: If you have any questions please contact: Zsolt Lavicza – University of Cambridge (zl221@cam.ac.uk or +44 7962 488 222)

Further information: http://www.cus.cam.ac.uk/~zl221/CAS.htm

Thank you for your help in this study!
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1) How many years have you been teaching university-level mathematics?
- 1-3
- 4-7
- 8-12
- 13-20
- 20+

2) What is your age?
- 35 or less
- 36-45
- 46-55
- 56-65
- 66 or more

3) What is your gender?
- Male
- Female

4) Where have you pursued your education and career?

	In the United Kingdom	Abroad (Outside the UK)	Both (Inside and outside the UK)
Pre-university eduacation	○	○	○
Undergraduate education	○	○	○
Graduate education	○	○	○
Career (academic/non-academic)	○	○	○

5) Which of the following most closely matches your primary research area?
- ○ Pure mathematics
- ○ Applied mathematics
- ○ Statistics
- ○ Mathematics education
- ○ Engineering
- ○ Science - Computer
- ○ Science - Physical (e.g. physics, chemistry...)
- ○ Science - Life (e.g. biology, ecology...)
- ○ Science - Social (e.g. economics, sociology...)
- ○ Not pursued any research
- ○ Other, please specify: [ ]

6) In an average working month how frequently do you use CAS in your research?

Never	Less than once a month	Once a month	Once a week	2-3 a week	Daily
○	○	○	○	○	○

7) What kind of student groups do you usually teach? **Check all that apply to you.**
- ☐ Mathematics majors
- ☐ Education majors (prospective teachers)
- ☐ Engineering majors
- ☐ Science majors - Computer
- ☐ Science majors - Physical (e.g. physics, chemistry...)
- ☐ Science majors - Life (e.g. biology, ecology...)
- ☐ Science majors - Social (e.g. economics, sociology...)
- ☐ General mathematics courses for non-maths intensive majors

Other, please specify: [ ]

8) What level of mathematics students do you usually teach? (Ever taught?) **Check all that apply to you.**
- ☐ Undergraduate
- ☐ Master's
- ☐ PhD

Other, please Specify: [ ]

9) Do you teach full- or part-time?
- ○ Full-time
- ○ Part-time

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Your views on the role of CAS in Mathematical Literacy.

10) **Please choose the response that most closely matches your views on each statement.**

	Strongly disagree	Disagree	Neutral	Agree	Strongly agree
1. Knowing how to use CAS is an essential skill for mathematics graduates	○	○	○	○	○
2. Knowing how to use CAS is beneficial for students on science and engineering courses	○	○	○	○	○
3. CAS enables mathematicians to work on problems more efficiently	○	○	○	○	○
4. CAS use does not affect the mathematics that has to be learned by students in universities	○	○	○	○	○
5. CAS is changing the way in which mathematics research is done	○	○	○	○	○
6. CAS offers the possibility of introducing new topics into undergraduate mathematics	○	○	○	○	○
7. Knowing how to use CAS enhances students' future employment prospects	○	○	○	○	○
8. Science and engineering graduates should have a working knowledge of CAS	○	○	○	○	○

Your views on CAS-assisted teaching and learning.

11) **Please choose the response that most closely matches your views on each statement.**

	Strongly disagree	Disagree	Neutral	Agree	Strongly agree
1. CAS use encourages students to examine carefully the meaning of their solutions	○	○	○	○	○
2. CAS use has positive effects on students' enthusiasm for mathematics	○	○	○	○	○
3. CAS enables teachers to deliver more engaging lessons	○	○	○	○	○
4. CAS use does not make classes more interesting for students	○	○	○	○	○
5. CAS use helps students develop better understanding of mathematical concepts	○	○	○	○	○
6. CAS use can initiate in-class communication between students	○	○	○	○	○
7. CAS-generated images spark valuable discussions in class	○	○	○	○	○
8. CAS use does not help students to understand mathematical concepts	○	○	○	○	○
9. Images generated by CAS improve students' attention in class	○	○	○	○	○
10. CAS use distracts students from understanding mathematical concepts	○	○	○	○	○

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Factors that may hinder the integration of CAS into teaching and learning.

12) **Please choose the response that most closely matches your views on each statement.** (If you feel that you don't have adequate knowledge to rate these statements then please use the N/A option or skip this page.)

	Strongly disagree	Disagree	Neutral	Agree	Strongly agree	N/A
1. CAS syntax is too complex to deal with in classes	○	○	○	○	○	○
2. Most entry level classes are too large for me to incorporate CAS	○	○	○	○	○	○
3. Only a few of my colleagues are enthusiastic about using CAS in mathematics classes	○	○	○	○	○	○
4. It takes too long to develop CAS-related teaching material.	○	○	○	○	○	○
5. The poor mathematical skills of students in introductory courses mean there is little time for CAS in classes	○	○	○	○	○	○
6. CAS is too expensive for wide integration in mathematics teaching and learning mathematics	○	○	○	○	○	○
7. It is not worth using CAS in classes, because it cannot be used in tests	○	○	○	○	○	○
8. CAS is insufficiently user-friendly to be used in classes	○	○	○	○	○	○
9. My department does not encourage the use of CAS in math classes	○	○	○	○	○	○
10. It is difficult to assess what students know if they can use CAS in tests	○	○	○	○	○	○
11. The tight course schedule does not allow the involvement of technology	○	○	○	○	○	○
12. CAS is not readily available in my department to use	○	○	○	○	○	○

13) **If there are other important factors hindering CAS integration please list them briefly:**

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14) **Access at the University - Please choose the one response that best indicates your answer in each scale.**

	Strongly disagree	Disagree	Neutral	Agree	Strongly agree
CAS and projection systems are available for use in most lecture rooms	○	○	○	○	○
The number of computer labs is sufficient for CAS-assisted teaching in my department	○	○	○	○	○
It is not difficult to schedule a mathematics class in a computer lab	○	○	○	○	○
CAS is adequately accessible for everyday use	○	○	○	○	○
CAS support is available for those lecturers who need it	○	○	○	○	○

15) Have you ever participated in any CAS training provided by your department or offered in a conference?
- ○ No
- ○ Yes at the department
- ○ Yes at a conference
- ○ Other, please specify:

16) **Training** - Please rate the following statements.

	Strongly disagree	Disagree	Neutral	Agree	Strongly agree
I would be happy to attend CAS teacher training workshops	○	○	○	○	○
I would be happy to collaborate with my colleagues to develop courses that involve CAS use	○	○	○	○	○

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17) In a typical academic term approximately, in what percentage of your lessons do you use CAS?
- ○ Never
- ○ 25% or less
- ○ 26-49%
- ○ 50-74%
- ○ 75% or more

If you answered NEVER please answer the next question (18) and then go to the next page.

OTHERWISE please skip the next question and continue to question 19.

18) **Please briefly explain what are the most important reasons that hold you back from using CAS in your teaching?**

19) In what settings does your CAS-related teaching generally take place?

	Never	Occasionally	Frequently	Always
Lecture room	○	○	○	○
Computer lab	○	○	○	○
Homework/projects	○	○	○	○
Use computer-based CAS	○	○	○	○
Use hand-held CAS	○	○	○	○

20) In what way do you use CAS in your teaching? I use CAS to

	Never	Occasionally	Frequently	Always
project images to illustrate concepts in lectures	○	○	○	○
encourage students to experiment with CAS	○	○	○	○
encourage students to work in teams/groups in lectures	○	○	○	○
assign project/homework for students to work at home	○	○	○	○
develop worksheets for students to work with	○	○	○	○
develop on-line tutorials for students	○	○	○	○
develop course materials that encourage students to work with CAS	○	○	○	○

21) **If you would like to add to this list please briefly describe your way of teaching.**

22) With what kinds of student groups do you use CAS in your teaching? Courses designed for

	Never	Occasionally	Frequently	Always
Mathematics majors	○	○	○	○
Education majors (prospective teachers)	○	○	○	○
Engineering majors	○	○	○	○
Science majors - Computer	○	○	○	○
Science majors - Physical (e.g. physics, chemistry...)	○	○	○	○
Science majors - Life (e.g. biology, ecology...)	○	○	○	○
Science majors - Social (e.g. economics, sociology...)	○	○	○	○
General mathematics courses for non-maths intensive majors	○	○	○	○

23) With what level of courses do you usually use CAS?

	Never	Occasionally	Frequently	Always
Entry level undergraduate (1-2 years)	○	○	○	○
Higher level undergraduate (2-4 years)	○	○	○	○
Masters level	○	○	○	○
PhD level	○	○	○	○

24) **Departmental assignment** - Please choose the response that most closely matches your views on each statement.

	Strongly disagree	Disagree	Neutral	Agree	Strongly agree
I can freely choose whether or not I use CAS in my teaching	○	○	○	○	○
I often choose courses in which I can use CAS for teaching	○	○	○	○	○

25) How many years have you been using CAS in your teaching?

- ○ 1 or less
- ○ 1-3
- ○ 4-7
- ○ 8-12
- ○ 13-20
- ○ 20+

26) Has your use of CAS in your teaching increased over the years?

- ○ Less frequently
- ○ Almost the same
- ○ More frequently
- ○ Other, please specify:

27) Do you permit CAS to be used during assessments?

	Never	Occasionally	Frequently	Always
In-class tests	○	○	○	○
Final exams	○	○	○	○
Homework/projects	○	○	○	○
Other assessments	○	○	○	○

28) What principles guide your CAS-related teaching?

29) Why did you start using CAS in your teaching?

30) Is there anything that has significantly influenced your use of CAS in teaching?

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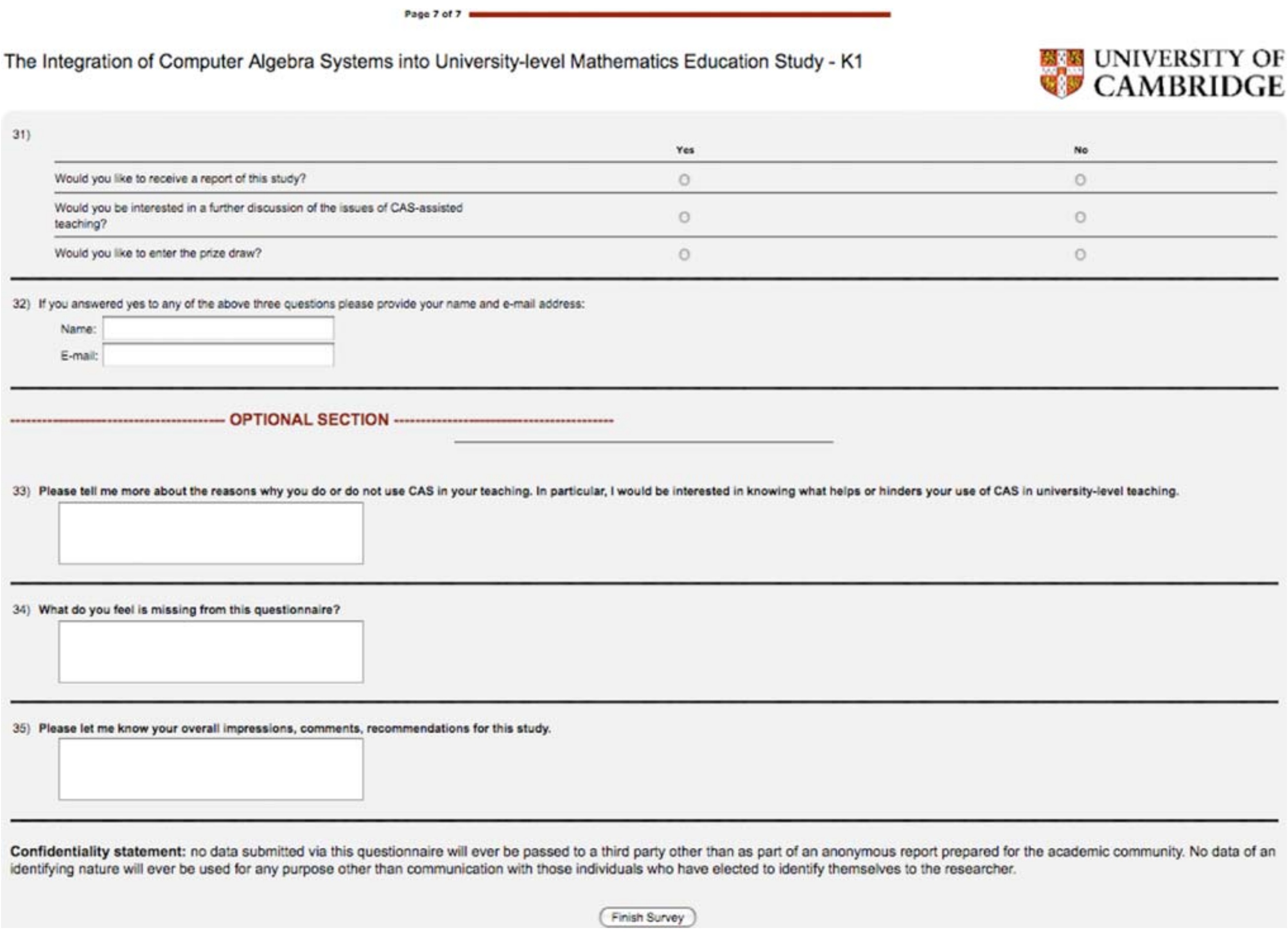

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31)

	Yes	No
Would you like to receive a report of this study?	○	○
Would you be interested in a further discussion of the issues of CAS-assisted teaching?	○	○
Would you like to enter the prize draw?	○	○

32) If you answered yes to any of the above three questions please provide your name and e-mail address:

Name:

E-mail:

OPTIONAL SECTION

33) **Please tell me more about the reasons why you do or do not use CAS in your teaching. In particular, I would be interested in knowing what helps or hinders your use of CAS in university-level teaching.**

34) **What do you feel is missing from this questionnaire?**

35) **Please let me know your overall impressions, comments, recommendations for this study.**

Confidentiality statement: no data submitted via this questionnaire will ever be passed to a third party other than as part of an anonymous report prepared for the academic community. No data of an identifying nature will ever be used for any purpose other than communication with those individuals who have elected to identify themselves to the researcher.

Finish Survey

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