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FUNCTIONAL MATHEMATICS: MORE THAN "BACK TO BASICS"

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Introduction

There is increasing recognition that current mathematics curricula do not adequately equip people to use and apply mathematics effectively in different spheres of their lives, for example, as learners, citizens and workers. For a flavour of the national debate, see Roberts, 2002; Smith, 2004; Working Group on 14-19 Reform, 2004. Whilst not neglecting the importance of individuals being mathematically literate as learners and citizens, policy makers in the UK are particularly concerned that we, as a nation, should have a well-educated workforce and are therefore well-placed to remain economically competitive and, in this regard, recognise the key role that mathematics has to play across many employment sectors and different skills levels of workers. In this country there is, therefore, increasing attention being paid to how one might be better equipped and prepared to use mathematics to make sense of situations at a time when there is increasing access to what might be termed quantitative data. The UK is not alone in focusing on this problem: similar concerns about inadequacies of current mathematics curricula have been raised in many countries; there is perhaps some convergence of mathematics curricula because of the international comparative studies that measure students' performance in mathematics and the resulting pressure on nations to improve their position in international league tables. Terms such as 'quantitative literacy', 'mathematical literacy', 'numeracy', and now in the UK, 'functional mathematics', have been used to try to capture the essence of what might form a new curriculum that ensures that people are in future better equipped to use mathematical knowledge and skills in a way that empowers them to solve problems and be able to make critical and informed choices based on quantitative information.

Current UK government white papers on education and skills (DfES, 2005) use the place-holder 'functional mathematics' for this potential new curriculum. Although these documents cannot set out the detail of what might comprise the resulting new courses, they do give some indication of early thinking, at least from policy makers, suggesting that it is the intention that young people will achieve "high standards in the basics", due to the development of "specific modules which focus on the functional and practical application of English and maths."

In this paper I explore what such 'functional mathematics' modules might encompass in terms of their curriculum whilst giving some thought to the implications for associated teaching/pedagogy. I consider it essential to take into account the likely experience of learners from the outset of the development of the crucial design features of a 'functional mathematics' curriculum, as not to do so will almost inevitably result in another unsuccessful attempt to develop a core or key skill qualification in the area of mathematics such as we have recently experienced with Application of Number (Hodgson and Spours, 2003). Unfortunately, in this relatively recent development we have evidence of how a qualification can prove unattractive to both learners and teachers, and apparently lack value to employers and higher education institutions. It is, therefore, essential that any new 'functional mathematics' curriculum is seen to have value and relevance to all concerned.

The current mathematics curriculum 14-19

Official reports of the state of mathematics post-14, particularly at Entry and Levels 1 and 2, in general make depressing reading. In the main, these are critical of teachers and their learners' experience. The latest Ofsted subject Report for Mathematics in Secondary Schools (Ofsted, 2005) highlights the findings of the Smith Inquiry (2004) that there is often a lack of engagement and motivation for many Key Stage 4 pupils in mathematics, that lower attaining pupils are particularly badly affected by this, and that too few pupils are successful in transferring their mathematical skills to different situations and using them to solve problems. It seems that, in many ways, the 14-19 curriculum is out of touch with the current needs and future aspirations of the young people it serves. It was presumably never the intention that when the National Curriculum for mathematics and current examinations for 16 year-olds (GCSE) were designed that this would be the case. The curriculum in its "using and applying" strand (Attainment Target Ma1) attempts to address many of the key ideas that we might find

in a 'functional mathematics' curriculum, such as pupils being able to tackle substantial tasks and analyse complex situations, interpret mathematical information and communicate findings. However, these are the very skills that appear to be lacking in many who have had recent experience of this curriculum: hence, the demand for a new 'functional mathematics' curriculum.

It is often argued that the study of mathematics can equip learners with a 'way of thinking'; but somehow, in general, pupils' experiences of mathematics in classrooms does not seem to result in them being well equipped to think and reason with mathematics across and within a diverse range of situations. Attempting to identify why this may be the case, influential mathematics educator and clearly successful mathematician, Alan Schoenfeld (2001), points to how his own mathematics education was impoverished in a number of ways:

- i. mainly consisting of the application of tools and techniques that he had just been shown;
- ii. being mainly 'pure' and lacking in opportunity to be involved with mathematical modelling;
- iii. not involving real data;
- iv. not being required to communicate using mathematics.

I would suggest that Schoenfeld's experience characterises the experience of pupils currently in the vast majority of our post- (and pre-) 14 mathematics classrooms.

To many of us in the mathematics education community, then, the development of a 'functional mathematics' curriculum could provide a welcome opportunity to revitalise a curriculum that appears to be increasingly out of touch with the needs of the young people it serves. However, at this stage I should perhaps sound a word of warning, as we may be in danger of optimistically taking note of the first part of the paragraph I quote below from the Skills White Paper (DfES, 2005), whilst perhaps foolishly ignoring the later part which suggests that the curriculum might be best informed by the Skills for Life qualifications.

We have set out in the 14-19 White Paper the way we intend to ensure that young people achieve high standards in the basics, including the design of specific modules which focus on the functional and practical application of English and maths. These units will be incorporated within English and mathematics GCSEs but will be assessed separately. Passing these units will be a prerequisite for

gaining a full GCSE. For those who pass the functional element without succeeding in the GCSE, separate certification will be available. The knowledge and skills that make up these units will draw on the Skills for Life standards, curricula and tests. These units will replace the current Skills for Life qualifications for use by adults seeking recognition for their progress and achievements.

A problem I detect here is that being functional with mathematics (that is, being able to apply mathematics to make sense of one's world), requires more than familiarity with the knowledge and skills defined by the Skills for Life qualifications (aimed at adults), as I shall argue in more detail in later sections of this paper. There will be those who might argue that these qualifications, whilst being focused on basic competence with mathematical knowledge and skills, also require the use of mathematics in a range of contexts. However, these contexts appear to continue to be of the type that we almost exclusively meet in the mathematics classroom and do not reflect the authenticity of situations that arise in the 'real world'. This lack of authenticity is perhaps most easily exemplified by quantities which learners are expected to work with: for example, pie-charts where sectors have angles of 90° , data collected from samples of 800 people and so on. One can easily envisage situations where real data is less compliant. However, this is but just one aspect in which quantifying real situations adds to complexity; another aspect that we need to take into account is the 'novelty' of the situations that can be explored using mathematics. Often these are markedly different to the rather clinical examples of the mathematics classroom. Introducing a variety and novelty of suitable situations will cause problems for teachers, as in many ways mathematics lessons currently focus on presenting to learners a restricted range of types of problems that can be tackled by methods and techniques that the student comes to learn as procedures that often lack meaning, and consequently prove difficult to transfer or transform into new and different situations. Adopting a new range of activities and experiences in classrooms to ensure that learners have opportunities to explore novel and realistic situations with real data will no doubt prove a major challenge to teachers and curriculum developers.

Mathematics in real situations

Often real situations can be quite complex and can be explored and made sense of using relatively straight forward mathematics. As examples of this consider just two examples from case studies of workplace practice. I have written about these in detail elsewhere (for example, see Wake and Williams, 2001; Wake and Williams, 2003; and

Wake, in press) to exemplify the type of mathematics met by workers in their daily activity, but here I will describe each in only a little detail to demonstrate how, although each measure met by a worker has only relatively simple mathematical structure, it requires relatively deep understanding and access to ways of thinking mathematically if we are to make sense of, and come to thoroughly understand its meaning.

- i. 'Debtor days' is a measure commonly used in finance offices. It is found using the formula:

$$\text{'debtor days'} = (\text{outstanding debts} / \text{annual turnover}) * 365'.$$

The worker who calculated this measure each month, although having a sense of what the measure conveys, was not able to make sense of how it related to the data involved. A researcher and office manager eventually gained an understanding by substituting the simplified values, "annual turnover = 2 million [pounds]" and "outstanding debts = 1 million [pounds]" giving "debtor days = 182.5" or half a year. This, they concluded therefore, gives an indication of how long customers are taking to clear debts. Perhaps the worker's confusion about the meaning of the measure was compounded because she actually knew exactly how long each customer took to clear each individual debt associated with their purchases. Although the worker had built the spreadsheet which calculated the measure, and in doing so had dealt with tricky arithmetical issues of sales taxes, she was unable to interpret it clearly and consequently unable to communicate the meaning of the measure to others.

- ii. Average gradient

As part of his work a railway engineer calculates the average gradient over a length of track. This requires calculating the total fall divided by the horizontal distance covered by multiple stretches of track. As part of a research project where we explored this with college engineering students we found that they were technically competent with fractions and finding averages but wanted to calculate the average gradient by adding the gradients, expressed as fractions, for the different sections of the track and finding the mean of these quantities without taking into account the different horizontal distances involved in calculating the individual gradients of the different sections of track.

These are just two examples that perhaps illustrate that being functional with mathematics requires a deep understanding of mathematical ideas and concepts and is certainly requires more than fluency with 'the basics'.

The debate about how mathematics might contribute to the education of individuals in the years to come, particularly as, at least in the western world, we are living in increasingly complex industrial societies in which in all walks of life we have access to large amounts of quantitative data is not bounded by national boundaries. Potentially informative debate has not only been taking place under headings instantly recognisable as relevant, such as those referring to 'mathematical' and/or 'quantitative literacy', but also in perhaps less immediately obvious areas of mathematics education, such as those relating to transferability of knowledge and problem solving and mathematical modelling and associated strategies.

There are, therefore, four main sources which I use to illustrate some of the thinking that has already taken place:

- the debate about the nature of 'quantitative literacy' that has been taking place in the USA but which has also been informed by the experiences of respected mathematics educators in other nations;
- the experience of the community in mathematics education that has for many years worked in the area of mathematical modelling;
- international comparative studies of performance in the application of mathematics;
- recent and current research into the use of mathematics in workplaces.

The 'quantitative literacy' debate

In recent years there has been considerable debate in the USA about what might be the nature of 'quantitative literacy': whilst this has been instigated at a national level it has been outward looking and drawn on strengths in the field of mathematics education research and development wherever that might be located. This debate has been instigated by the recognition that, much as in our own national setting, at the end of compulsory schooling, even those who appear to have been relatively successful at mathematics seem ill-equipped to apply mathematics to make sense of the world about them. The move to develop new curricula, whether they be 'quantitative literacy' or 'functional mathematics' to some appears to be a remedial action due to the fact

that current mathematics curricula are inadequate. They argue that surely being able to use and apply mathematics should be an outcome of any mathematics education. However, it is perhaps pragmatic to accept that while this may well be true, it is not currently the case and we should seriously consider further what such curricula might entail.

The work of Lyn Arthur Steen has been influential in the *quantitative literacy* debate. His editing of the publication *Mathematics and Democracy: The Case for Quantitative Literacy* has done much to inform thinking and in particular that aired at the “National Forum on Quantitative Literacy” in 2001 and some of the follow-up work coordinated by the Mathematical Association of America. The debate has been wide ranging and whilst not always arriving at consensus, particularly over a precise and detailed definition of what is meant by the term ‘quantitative literacy’, it is perhaps fair to summarise that there is broad agreement that:

- there is an urgent need for learners to have access to courses focussed on ‘quantitative literacy’ so that they are better equipped to function effectively in economic, political, cultural and personal capacities;
- currently mathematics education moves too quickly to abstract sophisticated concepts;
- a ‘quantitative literacy’ course would allow students to gain “experience in applying quantitative skills in subtle and sophisticated contexts” (Steen, 2001);
- mathematics and ‘quantitative literacy’ should be complementary aspects of the school curriculum (Steen, 2001).

This emerging consensus in the USA seems to focus on developing competencies in working with quantitative data in many and different, and often quite sophisticated, contexts using relatively simple mathematical knowledge and skills.

In an attempt to define the boundaries of what might be meant by ‘quantitative’ in this sense de Lange (2001) argues that we should be concerned about literacy across a greater playing field than suggested by ‘quantitative literacy’. He suggests that this should be considered a subset of ‘mathematical literacy’ which takes into account other aspects of mathematics. He points to the four phenomenological categories used to organise the Organisation for Economic Co-operation and Development (OECD) Programme for International Student Assessment (PISA) study (OECD, 2002): ‘quantity’; ‘space and shape’; ‘change and relationships’; and ‘uncertainty’, suggesting

that often, as conceived, 'quantitative literacy' neglects to include 'spatial literacy' which derives from the shape and space category, and which when combined with a 'quantitative literacy' based on the remaining categories might be taken to define a more complete 'mathematical literacy'. De Lange points to the relatively broad definition of the International Life Skills Survey (ILSS, 2000) as perhaps succinctly capturing the essence of what might be the ethos of a mathematical literacy:

An aggregate of skills, knowledge, beliefs, dispositions, habits of mind, communication capabilities and problem solving skills that people need in order to engage effectively in quantitative situations arising in life and work.

He goes on to suggest that in comparison with traditional school mathematics,

ML [Mathematical Literacy] is less formal and more intuitive, less abstract and more contextual, less symbolic and more concrete. ML also focuses more attention and emphasis on reasoning, thinking and interpreting as well as on other very mathematical competencies (de Lange, 2001)

It is perhaps worth here emphasising the idea of "habits of mind" referred to by de Lange and others. To achieve this as an outcome of a 'quantitative literacy' or 'functional mathematics' curriculum would effectively require a sea-change in how mathematics is perceived. Its potential to equip users with a powerful tool, or way of thinking, which allows them to quickly make sense of new situations in ways that otherwise would not be possible would need to be grasped. Students would need to learn to organise, make sense of, seek structures, identify, understand, develop and use quantitative arguments. Thus such a curriculum would indeed develop ways of thinking: *and* these would have significance and meaning to students, both immediately and in the future.

Mathematical modelling

Some, if not many, of the habits of mind suggested above, being central to using mathematics effectively, whether under the banner of 'quantitative literacy' or 'functional mathematics', can be developed by involvement in the process of mathematical modelling. This is perhaps not surprising, as central to mathematical modelling is the use of mathematics to solve problems and make sense of situations in the real world. For many years the work of the International Conference for the Teaching of Mathematics and its Applications (ICTMA) group has explored how

mathematical modelling can inform teaching and learning of mathematics at all levels. An important result of the work of members of this group is the conceptualisation of mathematical modelling and how this relates to applications of mathematics.

A detailed discussion and analysis of mathematical modelling and how aspects of this might be incorporated into a 'functional mathematics' curriculum is not possible here, (for examples of the discussion of the modelling process see the biennial proceedings of ICTMA such as Houston et al, 1997; Lamon et al, 2003). However, attention should be drawn to principles of mathematical modelling in general and how these might inform the development, in some way, of a 'functional mathematics' curriculum and pedagogy. Essential to using mathematics to model a real world situation or problem is the genesis of the activity in the real world itself. Mathematising this situation, that is simplifying and structuring it so that it can be described and analysed using mathematical ideas and constructs, leads to the mathematical model. Following analysis using mathematical knowledge, skills, techniques and understanding the outcomes and results are interpreted in terms of the original problem, being checked to determine whether or not they are valid. At this stage it may be decided that the model is adequate, or that it needs to be modified in some way, perhaps making it more sophisticated so that the results/solution to the problem are more appropriate. This can therefore be conceived of as a cyclical process with the 'modeller' translating between real world and mathematical representation. Some mathematical model types are commonly found and used to describe many different situations (for example, in the sciences models of direct proportion, exponential growth and decay and inverse square laws abound) and in some instances a recognition of this allows one to short circuit some of the process and work quickly between mathematical model and real world. In the discussion document which set out an agenda for the forthcoming International Commission on Mathematical Instruction (ICMI) study of Applications and Modelling in Mathematics Education (Blum, 2002) care was taken to distinguish between use of the term 'modelling' on the one hand, to describe the mathematisation as one moves from reality to mathematical model, and 'application' on the other, as one interprets mathematical analysis in real terms.

It seems likely that these ideas could be very useful in assisting to define a 'functional mathematics' curriculum giving us access to useful ways of describing the processes and types of mathematical activities we might expect learners to engage with.

International comparison of students using and applying mathematics

Another key area which should prove profitable in informing our debate about how we might specify a 'functional mathematics' curriculum is the OECD PISA study which attempts to measure how well young people are prepared to meet the challenges of living in the 21st century, and in particular focuses on the four domains of mathematical, reading and scientific literacies and problem solving skills (OECD, 2002). Part of this study, therefore, examines the degree to which fifteen year-olds can be considered as being mathematically literate in terms of being informed, reflective citizens and intelligent consumers.

The development of the conceptual framework which is used to organise the assessment items that probe 'mathematical literacy' has been informed by the areas of debate discussed in the previous two sections, and as a number of influential mathematics educators have been involved in developments in each of these areas each has been informed and influenced by the other. However, whilst not defining a curriculum the PISA framework is possibly as close as we get to a tangible expression of the interests expressed so far by those contributing to the debate. The study defines 'mathematical literacy' to be:

An individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to use and engage with mathematics in ways that meets the needs of that individual's life as a constructive, concerned and reflective citizen.

It elaborates that it:

is concerned with the ability of students to analyse, reason, and communicate ideas effectively as they pose, formulate, solve and interpret solutions to mathematical problems in a variety of situations.

The assessment of this particular literacy pays attention to content organised in terms of four overarching themes (quantity; space and shape; change and relationships; and uncertainty), process defined in terms of general mathematical competencies (including use of mathematical language, modelling and problem solving skills), and situations in which mathematics is used (personal, educational, occupational, public and scientific).

The framework gives further detail of the competencies upon which someone might require when mathematising a situation. Drawing on the work of Niss (1999) eight characteristic competencies have been elaborated:

- i. *Thinking and reasoning.* This involves not only asking questions that are characteristic of mathematics but when it is appropriate to ask such questions and the kinds of answers that one might expect.
- ii. *Argumentation.* This involves following and understanding mathematical arguments and having an understanding of the nature of proof.
- iii. *Communication.* This involves expressing one's own mathematics in a variety of forms and making sense of the mathematics of others.
- iv. *Modelling.* This includes involvement in all aspects of the modelling process as outlined in the previous section.
- v. *Problem solving and posing.* Posing, formulating, defining mathematical problems of different types in different domains is at the core of this competency.
- vi. *Representation.* This involves being able to work with a wide range of mathematical representations, both interpreting those of others and being able to develop appropriate representations to communicate mathematical ideas to others.
- vii. *Using symbolic, formal and technical language and operations.* This involves working with a range of mathematically technical language to develop and communicate mathematical arguments.
- viii. *Use of aids and tools.* Being able to use appropriate aids and tools, including information technology tools, is expected.

These competencies appear to be potentially useful in identifying the ways in which we might expect those involved with functional maths to be working mathematically. Such competencies might therefore allow us to come to be able to consider in general terms what might comprise a type of pedagogy that could be associated with 'functional mathematics' courses and in turn what might be appropriate assessment.

Mathematical activity in workplaces

Research into the use of mathematics by workers in workplaces has usefully informed the debate about difficulties associated with transfer or transformation of mathematical knowledge and skills into unfamiliar settings, and in particular into situations where the division of the total activity of a worker cannot always be easily subdivided into strict 'academic disciplines'. To do so can, at times, be unhelpful and even inappropriate. Mathematics with which workers are involved is not often easily recognisable as it does not look like the kind of mathematical activity that we see in classrooms. Occasionally they are involved in mathematics that is focused on solving a problem, but perhaps more often their mathematical activity is concerned with monitoring and measuring workplace routines and output.

Much recent research in the field of the use of mathematics in the workplace has been strongly influenced by ideas of situated cognition which, at its most extreme, suggests that mathematical understanding and competence cannot be separated from the socio-cultural setting in which it is constructed (for example see Lave, 1988). Researchers point to how often idiosyncratically developed artefacts mediate the actions of workers so that they, as individuals and teams, successfully achieve the outcomes required of them (see for example, Strässer, 2003). Sometimes these artefacts can be used successfully by workers without recourse to mathematical thought or understanding. Other influences that are seen to mediate the mathematical activities of workers include formal and informal 'rules' and the way in which individuals operate cooperatively and divide labour within a workplace 'community'. These aspects of workplaces are often most pertinent in shaping and forming day-to-day workplace relationships and activity and are often responsible for the mathematics looking very different from that met in mathematics classrooms.

Other analyses have led researchers to develop constructs that take account of the situated nature of the mathematics they have observed in workplaces, whilst bridging to the perhaps more familiar mathematics of education and academic communities. These include:

- the idea of 'situated abstraction' (Pozzi et al, 1998), which allows one to understand how workers may develop a generalised mathematical understanding, but within the situational context of their work, using a

discourse other than that of standard/formal mathematics but which may be mapped to this;

- 'general mathematical competences' (Williams et al, 1999; Williams and Wake, 2000), which were developed from a mathematics education standpoint to attempt to take account of common ways that workers might bring together coherent bodies of mathematical knowledge, skills and models, for example when "handling experimental data graphically";
- *techno-mathematical literacies* (Kent et al, 2004), which are currently being developed to assist understanding of how mathematics in workplaces is not only very much grounded in day-to-day workplace activity but is also often highly integrated and dependent on the use of modern technologies.

These constructs, whilst not always referring explicitly to ideas associated with modelling, do, often by implication, suggest that mathematical modelling and mathematical models are central to the activity of workers.

This research and emerging ways of conceptualising mathematical activity as a part of wider activity (in this case situated in workplaces) again emphasises how mathematics as currently experienced by students in school and college classrooms is of a particular stylised form and that adults, as citizens and workers, adapt and reformulate mathematics into different forms to suit their purposes and objectives. For example, they point to how workers may develop a generalised understanding of a mathematical concept, but within a particular situational context (which indeed may have enabled them to develop this understanding in a way that may be very different to that expected in mathematics classrooms). They also highlight how technology often plays a large role in the overall activity of workers, and how this forms and shapes mathematical processes and consequently understanding.

However, perhaps their most important observation is again how often relatively straightforward mathematical ideas and techniques are used by workers to make sense of quite complex situations and to solve problems. It is the complexity of the situation and the use of mathematics to make sense of it that poses the challenge: fluency with basic numerical skills and techniques are absolutely necessary, but other skills are required. The problems posed and situations investigated are very different to those currently experienced by students, working at all levels, in mathematics classrooms.

Pedagogy

I hope that to some extent the previous section has painted a picture of 'functional mathematics' as being something that requires the full involvement of the learner; that the emphasis has been on mathematical processes and important competencies rather than content. To develop a curriculum that has these as central and indeed in the foreground will require a considerable shift in the way that mathematics is conceived, practised and experienced in classrooms at present.

The recent Ofsted Report of Mathematics in Secondary Schools (Ofsted, 2005), in discussing current teaching and learning in classrooms, is highly critical:

the main part of too many mathematics lessons for all age groups consists of demonstration by the teacher followed by standard exercises for pupils to practise the technique. The assumption is that completing the exercise in itself will ensure effective learning. However, pupils' learning is not sufficiently well assessed so teachers' assumptions that learning has happened are not verified.

This report is particularly critical of the resulting experiences of post-14 students pointing to the fact that much of what occurs is dominated by teacher explanation at the expense of learners developing secure understanding of mathematical concepts and that tasks seem irrelevant to learners who do not have a clear view of what they are being asked to learn. It appears that current practice is the antithesis of what might be required to ensure that learners come to appreciate mathematics as a problem solving tool and so that they have the confidence to explore 'novel' situations with their mathematics.

Being functional with mathematics requires fluency with basic mathematical techniques and procedures; however, this is necessary but not sufficient. Deep understanding of mathematical concepts and ideas is necessary. In a recent research project we investigated students using their mathematics in the physical sciences (Wake and Hardy, 2005). This demonstrated how even high achieving students (in terms of their mathematics grades) have difficulty in seeing the common structure of laws and models of direct proportion such as Ohm's Law, Hooke's Law, Newton's Second Law, the definition of density, and so on. Each was seen by the students as a new situation and their lack of insight into the common mathematical structure of each meant that it had to be learnt as a separate and new entity. Our investigations concluded that at no stage of current science or mathematics teaching is it likely that deep understanding of

the underlying structure of these physical laws could emerge, and perhaps more worryingly it appears that there is currently no place where this is encouraged in either curriculum.

Teaching towards a 'functional mathematics' curriculum, therefore, will require mathematics classrooms to allow time and space for understanding of concepts to be developed and fostered; there needs to be a move away from mathematics as being a series of procedures to memorise. Fundamental to being functional with mathematics is being able to bring deep understanding of key ideas and concepts to bear to make sense of 'novel' situations. Teaching must therefore ensure that students not only have opportunities to apply mathematics in this way, but careful thought also needs to be given to how learning can be developed out of such experiences. It is perhaps fair to characterise much of current learning as being in the opposite direction with the student learning techniques or procedures in the general and often abstract case and then being shown how these might be applied to a narrow set of problems. 'Functional mathematics' might allow us to develop courses that grasp the ideas espoused by Freudentahl and others (see for example, Gravemeijer, 1994) that the mathematical understanding can be developed by involvement with real situations from the outset. There is evidence that this approach is not only effective but also that students find it motivating (see for example, Nicholson, 2005).

Important resources that we need to mobilise in classrooms are those of computer technology and associated data projectors and interactive whiteboards that are increasingly becoming commonplace in our classrooms. These have enormous potential to allow teachers and pupils to explore what it might be to use mathematics functionally in a range of different contexts that we couldn't previously consider in classrooms. Teaching and learning of 'functional mathematics' needs to be supported by high quality resources exemplifying how maths can be used powerfully, for example, in workplaces, but also to help one make sense of the world in which we live. It is equally important that, as a matter of course, students have access to computer technology which they can use as a tool to support their exploration and analysis of data and communication of their findings.

Of course, we should not underestimate the challenge that shifts in the directions suggested here pose for teachers, who have little or no experience of working in such ways, as well as there currently being a lack of resources to support such a style of teaching and learning. Perhaps most important in developing teachers' pedagogic

beliefs and practices is their own experience of learning the discipline. What is being proposed with a 'functional mathematics' curriculum of the type being suggested here is a fundamental shift in what mathematics education should encompass and perhaps more importantly what doing mathematics would entail for students. What is being proposed is probably somewhat alien to the experience of mathematics teachers themselves and if we are therefore to shift towards a curriculum that goes some way to meeting the ideas suggested here, we should not underestimate the level of curriculum development and support this will require as well as support for teachers in terms of carefully structured continuing professional development.

Conclusion

The move to develop a 'functional mathematics' curriculum in the UK is being paralleled by developments across the international mathematics education community. There are a number of factors motivating such developments, but perhaps most important is the recognition that to be well prepared to function effectively and democratically in a world with increasing access to a wide range of quantitative and other data requires that you have access to a range of important mathematical literacies. Whilst these are not clearly defined at present there is an emerging consensus that they require one to be able to make sense of often relatively complex situations with often basic mathematics. This requires that learners have experience of working to develop a far wider range of skills than is currently the case in mathematics classrooms and that they will therefore require access to richer and more diverse tasks than are currently provided. We can look to research and developmental studies across the world that, whilst often at early stages of conceptualising the field, point to exciting new horizons for mathematics education. They suggest ways in which the teaching and learning of the discipline might develop to reflect the quickly developing ways in which it is used in all walks of life, particularly capturing the way in which information and communications technology, whilst adding to the amount and complexity of data available, can be liberating in its use as a powerful tool that can be used to analyse and communicate mathematical arguments and solutions to problems.

The debates, discussions, research and frameworks I have referred to here are just some of the informed and critical thinking that is being developed to inform possible curriculum development in mathematics across the world. It is to be hoped that the development of 'functional mathematics' in the UK will be carried out to a timetable

that allows it to be fully informed by this debate. It is clear that those who have invested a considerable amount of time, energy and scholarship in grappling with this problem are thinking beyond a 'return to basics' and have a vision where mathematics potentially becomes an empowering tool available to future citizens and workers. It is important that our policy makers ensure that mathematics curricula of the future are built on the sound foundations of such work.

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