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Cooperative learning and Mathematics Education: A happy marriage?

Introduction

The central questions in this paper are:

- Should knowledge be provided or generated in mathematics education?
- What do we know about the feasibility of cooperative learning in mathematics education?
- What are the underlying mechanisms of cooperative learning in mathematics education?
- What kind of effects may we expect from learning in cooperative groups in mathematics?
- What are the criteria for curriculum materials and assignments for learning in cooperative groups in mathematics education?

Purpose and Aims of cooperative learning

Cooperative learning has been championed by many advocates. It was designed and implemented in order to develop social strategies and social attitudes in students, and to improve social relations within and between groups. In addition, there is a large cluster of cooperative learning models aimed at cognitive development e.g. in mathematics. Sometimes cooperative learning is directed at both the social and the cognitive side of human development (Gillies, Ashman & Terwel, 2008).

Content of cooperative learning

The purposes and aims of cooperative learning need to be elaborated within certain domains of study. Cooperative learning is not a technique for its own sake but needs *content* in order to be useful. The specific content or subject matter is not a result of arbitrary choice, without any consequences for the design of a curriculum in which cooperative learning takes place. Content has its own characteristics, which can be utilized in the designing process and in the classroom in order to facilitate the development of thinking as a human activity. Mathematics education, for example, offers specific opportunities for cooperative learning with this purpose in view. To put it differently and to make the general idea more specific, the content of mathematics allows for specific models of cooperative learning in order to accommodate individual differences between students. Mathematical problems can be situated in real-life contexts and designed in such a way that solutions can be reached along different routes and at different levels. This makes cooperative learning in mathematics different from cooperative learning in other domains, such as languages and world orientation. Each domain has their own opportunities for teaching and learning with regard to individual differences among students.

The organization of cooperative learning

Purpose, *Organization* and *content* may be summed up in the following composite question: Should all students pursue the same purposes and content or should different programs be offered to different categories of students? My own position has always been that a common curriculum should be offered to all. I am inspired by Hans Freudenthal (1991) who proposed 'mathematics for all' in the context of comprehensive education for students between the ages of 12-16 (see also Gravemeijer & Terwel, 2000). However we should recognize large differences between students, especially in domains like mathematics and languages. Therefore, a common curriculum should always been accompanied by opportunities for enrichment, remediation and choice. The question is: how to implement this innovation in the classroom? Could cooperative learning offer a solution?

Instructional approaches: providing versus generating?

There are many instructional approaches. Most of them can be categorized in the dichotomy 'providing versus generating'. Important question is: Should knowledge be provided or generated in mathematics education? In our research, several research projects are about representations in mathematics. By representations one can think of drawings, graphs, verbal descriptions, concepts, symbols, algebraic formula's, designs and proto-types. Point of departure was one of the major questions in learning theory and curriculum design: Are representations to be provided or generated?

We have tried to overcome this dichotomy by searching for a third way. First we designed an instructional model for cooperative learning and adaptive instruction for students between the ages of 12-16 (the so called AGO-model). Second by designing an instructional approach for primary mathematics called 'guided co-construction'.

Cooperative Learning and Adaptive Instruction: AGO-model
The AGO-model is a whole-class model for cooperative learning that allows for student diversity through situational remediation and enrichment within small groups. The AGO-model consists of the following stages:

1. Whole-class introduction of a mathematics topic in real-life contexts;
2. Small-group cooperation in heterogeneous groups of four students;
3. Teacher assessments: diagnostic test and observations;
4. Alternative learning paths depending on assessments consisting of two different modes of activity:
   a) Individual work at individual pace and level (enrichment), in heterogeneous groups with the possibility of consulting other students, or;
   b) Opportunity to work in a remedial group (scaffolding) under direct guidance and supervision of the teacher;
5. Individual work at own level in heterogeneous groups with possibilities for students to help each other;
6. Whole-class reflection and evaluation of the topic;
7. Final test.

The model provides for diagnostic procedures and special instruction and guidance by the teacher in a small remedial group for low-achieving students. This cycle is extended through a series of lessons (units) over for example three to five weeks, preferably in extended units of uninterrupted instructional time.

In a pretest-posttest control-group experiment, the AGO-approach was put to the test (Terwel, Herfs, Mertens, & Perrenet, 1994). Students in the experimental (AGO) condition outperformed their counterparts in the control group (N=582). In this project an effect size of .68 was found. In addition a significant effect of class composition was found. Students in classes with a higher mean ability outperformed their counterparts in classes with a lower mean, after controlling for initial individual differences in mathematical ability. Indications were found that low-achieving students profited less from learning in small groups than high-achieving students.

Guided co-construction of mathematics

In our curriculum design and research projects we are inspired by many authors (Dewey, 1902, Brown & Palincsar, 1989; Freudenthal, 1991; Mercer, 1995; Hardman, 2008). Point of departure was Dewey's famous adage about how knowledge construction should progress:

"It is continuous reconstruction, moving from the child's present experience out into that represented by the organized bodies of truth that we call studies" (Dewey, 1902).

Freudenthal's main concept is *guided reinvention of mathematics*. Freudenthal refers to the guidance of the teacher in reinventing mathematics as a human activity. Cooperative learning in small groups of four is a central part of Freudenthal's instructional approach. Our description of the instructional approach *Guided co-construction of mathematics* entails the following three core elements.

1. 'Guided' refers to the explicit role of the teacher for whole-class instruction and the scaffolding of students either in groups or individually.
2. 'Co-' refers to cooperative learning as an essential component of mathematics as a social, human activity and a cultural tool. In contrast to mathematics as a closed system to be transmitted to students.
3. 'Construction' refers to the recognition and construction of concepts, models, symbols by students on the basis of their prior knowledge and experiences.

Taken together, these elements imply that teachers facilitate the understanding of mathematics by presenting concepts, models, symbols etc. but also elicit and scaffold contributions and constructions from students within a meaningful (real life) context. In this interactive process, the differences between students are actually called upon and mathematics is not only prescribed ahead of time but also created by the students and teacher as they interact and move along. And such a process is also called co-elaboration, co-construction or the guided reinvention of mathematics (Brown & Palincsar, 1989; Dewey, 1943; Freudenthal, 1991).

In one of our research projects on 'guided co-construction' we found promising results (Terwel, Van Oers, Van Dijk & Van den Eeden (2009). Our research question was: With regard to transfer, is it better to provide pupils with ready-made representations or is it more effective to scaffold pupils' thinking in the process of generating their own representations with the help of peers and under the guidance of a teacher in a process of guided co-construction? The sample comprises 10 classes and 239 Grade 5 primary school students, age 10–11 years. A pretest-posttest control group research design was used. In the experimental condition, pupils were taught to construct representations collaboratively as a tool in the learning of percentages and graphs. Children in the experimental condition outperformed control children on the posttest and transfer test. Both high- and low-achieving pupils profited from the intervention. This study shows that children who learn to design are in a better position to understand representations like pictures, graphs, and models. They are more successful in solving new, complex mathematical problems.

The dynamics of cooperative learning: six explaining factors

In proposing a certain instructional approach one always needs to ask: “Why one thinks it will work. What are the driving forces behind this approach? What kinds of processes cause the growth of knowledge?”

1. Students in small groups are confronted by their fellow students in the group with different solutions and points of view. This may lead to socio-cognitive conflicts that are accompanied by feelings of uncertainty. This may cause a willingness in students to reconsider their own solutions from a different perspective. The resulting processes stimulate higher cognitive skills. In principle, students can also conquer the uncertainty caused by different points of view with the help of other members of the group, particularly where difficult or complicated assignments are concerned.
2. Small groups offer group members the opportunity to profit from the knowledge that is available in the group as a whole. This may take the form of knowledge, skills and experiences that not every member of the group possesses. Students use each other as `resources' under those circumstances (resource sharing).
3. Collaboration in small groups also means that students are given the opportunity to verbalize their thoughts. Such verbalizations facilitate understanding through cognitive reorganization on the principle that 'Those who teach learn the most'. Offering and receiving explanations enhances the learning process. Group members not only profit from the knowledge and insights transmitted through `peer tutoring', but they can also internalize effective problem-solving strategies by participating in the collective solution procedures.
4. Positive effects of group work can also be expected on the basis of motivation theory. Cooperation intensifies the learning process. Students in the 12-to-16 age group are strongly oriented towards the peer group and very interested in interaction with their fellow students.
5. From the point of view of teaching methods in mathematics, positive effects may be expected from the kinds of assignments that are used in groups. Special designed assignments, which appeal to different levels of cognition and experiences, offer students the possibility of applying their strengths in the search for solutions. See, for example, Freudenthal's theory of levels in the learning process.
6. Instructional approaches like cooperative learning in which students collaboratively inquire, generate and design parts of the curriculum materials, elicit active processing and may result in knowledge that is applicable in new situations.

Assignments in mathematics

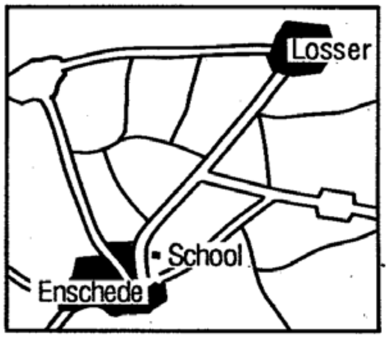
Successful mathematics education stays or falls with well designed curriculum materials.

First of all, mathematics should be conceived as a human activity rather than a ready made system to be transmitted. From this philosophy of mathematics education, the following criteria may arise:

- Point of departure should be the child's present experience and prior knowledge.
- Proceed from this starting point toward the concepts, structures and methods in mathematics.
- Utilize rich contexts from real life situations and move forward to mathematical contexts.
- Design 'multi-ability assignments' that allow students from different levels to participate.
- Assignments should be special designed for learning in cooperative groups.

Samples of assignments and student's productions

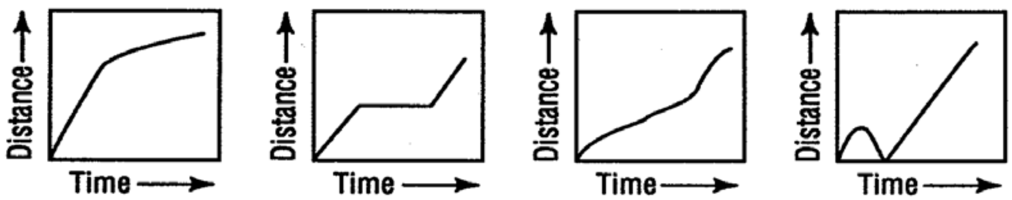
Figure 8-1 *Cycling*



Many children from Losser go to school in Enschede. They usually go by bike.

Questions:
Below you can see four graphs and four stories. Which story goes with which graph?

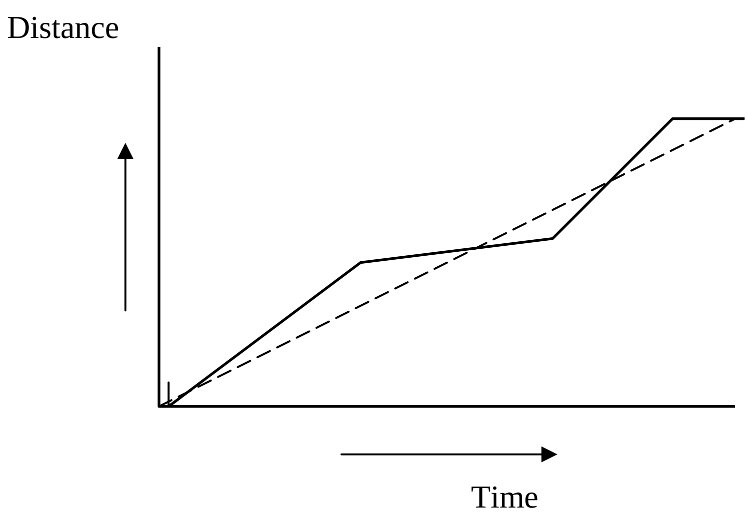
Think about what Marijke might have said.



Lucy and Evelyn running

When students in mathematics have to interpret a distance-time graph, their personal experiences sometimes play a "disruptive" role. The graph is then seen as a reflection of the hills and valleys on the road or a road with curves. An example may clarify this.

Lucy and Evelyn are going to trim. They walk a few times a week a distance of 6 km. They start together, walking the same route and finish mostly at the same time. But they don’t run in the same way. Lucy has quite a different style of running than Evelyn. The graphs show the evolution of the fitness run by Lucy and Evelyn.



Could you tell something about the difference in style of walking between Lucy and Evelyn? Describe how the speed of Lucy and Evelyn was progressing during this fitness run.

Rick says: "Lucy goes on and on without rest and Evelyn runs
irregular, she starts fast, then she slows down, and so on.”0, yes?" said José,
"why does she bend over that, stupid?”

This difference of opinion remains to exist and there is a "socio-cognitive conflict” which was escalating to an argument in which Rick says: "You don’t understand, dummy", and José slaps back: "That makes no sense, fatty."

José sees the Evelyn’s graph as a road with curves. He functions on another level than Rick.

Assignment: The global water cycle

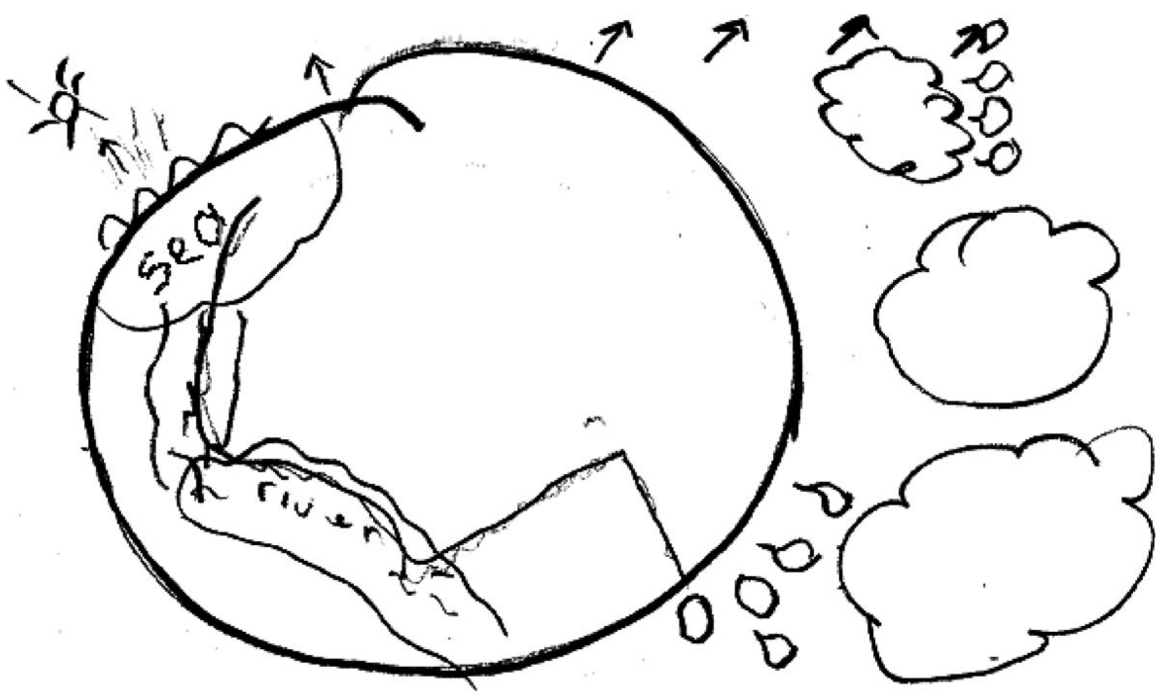
One of the assignments was about the circulation of water. After a short verbal introduction by the teacher about the global water cycle, including elements such as the oceans, the sun, evaporation, condensation, clouds, rain, mountains, rivers, sea, et cetera, students were asked:

‘‘Your friend in Groningen has never heard of this cycle in nature. Write a letter to your friend and make a model (drawing) to explain to your friend how the water cycle works?’’ Below we present two alternative solutions to this problem, produced by respectively Nienke and Daniel.

In the next part of the assignment, students were asked the following question:
‘‘A few days later you received a letter from your friend. She wrote, “Thanks for your letter. Hopefully I understood you correctly. Could I say that not a single drop of water is lost?”

The assignment continues with the following question:
‘‘Did your friend understand your explanation? How do you know this?’’ Daniel’s answer is: ‘‘Yes, not a single drop is lost because the water comes from the sea and goes back to the sea.’’.

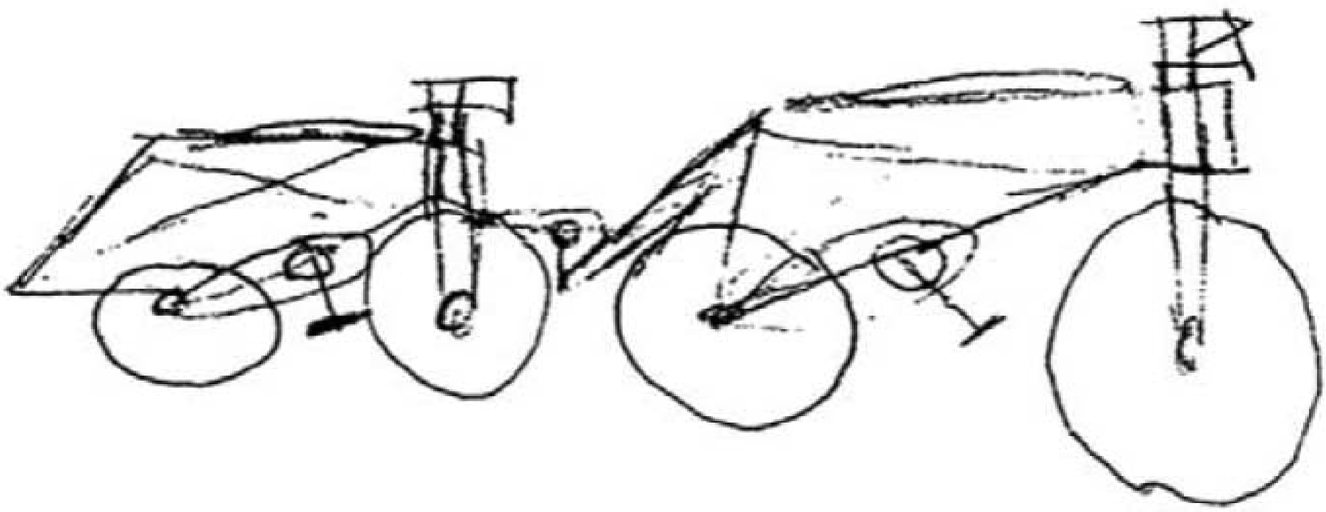
Student's productions: The global water cycle



Nienke's Model of the global water cycle

Daniel's model of the global water cycle

First design drawing by a cooperative group



Students were asked to design and construct a prototype of a tandem tricycle. Teachers assisted the students in solving problems of design or production that might occur. The students were stimulated to use or develop models to solve the problems they were faced with while working on this ‘real-life’ assignment. The student assignment was formulated as follows: ‘Design and build a prototype of a tandem tricycle for children aged 4–7 in such a way that the children have to cooperate in the process of cycling’. The students subsequently started designing during the first week, moving on to construction during the weeks following.

Conclusions and discussion

In this paper we raised the following questions:

- Should knowledge be provided or generated in mathematics education?
- What do we know about the feasibility of cooperative learning in mathematics education?
- What are the underlying mechanisms of cooperative learning in mathematics?
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- What are the criteria for curriculum materials and assignments for learning in cooperative groups in mathematics education?

From our research we may conclude that cooperative learning and guided co-construction are feasible and effective instructional approaches in mathematics education. There are reasons to believe in a happy marriage between cooperative learning and mathematics education. However these approaches are no cure for all. And cooperative learning should always be accompanied by other instructional strategies like whole class introductions and reflections which should be led by the teacher. Without special attention to low-achieving students, they may profit less from cooperative learning than their more able counterparts. The curriculum materials should be special designed for guided co-construction. Last but not least, mathematics should be taught in a meaningful, but mathematically honest way (see also Bruner, 1960). Otherwise 'mathematics for all' may result in 'no mathematics at all'. In mathematics education we need to create a knowledge-rich learning environment which is directed to the central concepts, procedures and structures of mathematics. It goes without saying that success or failure of guided co-construction depends on the teacher, the students and the curriculum materials.

The teacher and writer Daniel Pennac *(Chagrin d'école, 2009)* also stressed the importance of teaching in a meaningful, but honest way. His plea is to fight ignorance and to guide students in their efforts to appropriate the basic concepts and structures e.g. in the domains of languages and mathematics. Finally this should result in abstract, universal knowledge which can be applied by the students to solve problems in new situations. That is what transfer is about and what should be the ultimate goal of education: Non scholae, sed vitae discimus.

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