Departamento de Matemática da Universidade de Aveiro

ANÁLISE MATEMÁTICA III **2007/08**

Folha 7: *soluções*

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1. $\left[\begin{array}{cccc} \frac{\delta z}{\delta x_1} & \frac{\delta z}{\delta x_2} & \frac{\delta z}{\delta x_3} & \frac{\delta z}{\delta x_4} \end{array} \right]$ onde

$\frac{\delta z}{\delta x_1} = 4x_1 - 2x_2^2 - 2x_2 + 2x_3 + 2x_4 + \cos(x_2^2 + x_1 + \sin x_4)$
$\frac{\delta z}{\delta x_2} = -2x_1 + 2x_2^2 + 2x_2 + 1 + 2x_2 \cos(x_2^2 + x_1 + \sin x_4)$
$\frac{\delta z}{\delta x_3} = 2x_1 + 2x_3 + 2x_4$
$\frac{\delta z}{\delta x_4} = 2x_1 + 2x_3 + 2x_4 + 1 + \cos(x_2^2 + x_1 + \sin x_4)\cos x_4$

2. $\left[\begin{array}{cc} \frac{\delta z_1}{\delta x_1} & \frac{\delta z_1}{\delta x_2} \\ \frac{\delta z_2}{\delta x_1} & \frac{\delta z_2}{\delta x_2} \end{array} \right]$ onde

$\frac{\delta z_1}{\delta x_1} = 2x_1 + 2x_2 - \cos(x_2^2 - x_1 + x_2) + \sec^2(x_2^2 + x_1 + \sin x_2)$
$\frac{\delta z_1}{\delta x_2} = 2x_1 + 2x_2 + (2x_2 + 1)\cos(x_2^2 - x_1 + x_2) + (2x_2 + \cos x_2)\sec^2(x_2^2 + x_1 + \sin x_2)$
$\frac{\delta z_2}{\delta x_1} = 2x_2^3 + x_2^2 + 2x_1x_2^2 - 2x_1x_2 - 3x_1^2 + 1$
$\frac{\delta z_2}{\delta x_2} = 4x_2^3 + 3x_2^3 + 6x_1x_2^2 + 2x_2 - x_1^2 + 2x_2x_1^2 + 2x_1x_2 + \cos x_2$

3. $\left[\begin{array}{cccc} \frac{\delta z_1}{\delta x_1} & \frac{\delta z_1}{\delta x_2} & \frac{\delta z_1}{\delta x_3} & \frac{\delta z_1}{\delta x_4} \\ \frac{\delta z_2}{\delta x_1} & \frac{\delta z_2}{\delta x_2} & \frac{\delta z_2}{\delta x_3} & \frac{\delta z_2}{\delta x_4} \\ \frac{\delta z_3}{\delta x_1} & \frac{\delta z_3}{\delta x_2} & \frac{\delta z_3}{\delta x_3} & \frac{\delta z_3}{\delta x_4} \end{array} \right]$ onde

$\frac{\delta z_1}{\delta x_1} = 2x_1 + 2x_4 - \cos(x_2^2 - x_1 + x_3) + \sec^2(x_3^2 + x_1 + \sin x_2)$
$\frac{\delta z_2}{\delta x_1} = 2(x_1 + x_4)(x_2^2 - x_1 + x_3) - (x_1 + x_4)^2 + 1$
$\frac{\delta z_3}{\delta x_1} = -2(x_1 + x_4)\sin(x_1 + x_4)^2 - 3(x_2^2 - x_1 + x_3)^2(x_3^2 + x_1 + \sin x_2) + (x_2^2 - x_1 + x_3)^3$
$\frac{\delta z_1}{\delta x_2} = 2x_2 \cos(x_2^2 - x_1 + x_3)^2 + \cos x_2 \sec^2(x_3^2 + x_1 + \sin x_2)$
$\frac{\delta z_2}{\delta x_2} = 2x_2(x_1 + x_4)^2 + \cos x_2$
$\frac{\delta z_3}{\delta x_2} = 2x_2^3(x_2^2 - x_1 + x_3)^2(x_3^2 + x_1 + \sin x_2) + (x_2^2 - x_1 + x_3)^3 \cos x_2$
$\frac{\delta z_1}{\delta x_3} = \cos(x_2^2 - x_1 + x_3) + 2x_3 \sec^2(x_3^2 + x_1 + \sin x_2)$
$\frac{\delta z_2}{\delta x_3} = (x_1 + x_4)^2 + 2x_3$
$\frac{\delta z_3}{\delta x_3} = 3(x_2^2 - x_1 + x_3)^2(x_3^2 + x_1 + \sin x_2) + 2x_3(x_2^2 - x_1 + x_3)^3$
$\frac{\delta z_1}{\delta x_4} = 2(x_1 + x_4)$
$\frac{\delta z_2}{\delta x_4} = 2(x_1 + x_4)(x_2^2 - x_1 + x_3)$
$\frac{\delta z_3}{\delta x_4} = -2(x_1 + x_4)\sin(x_1 + x_4)^2$

4. $\left[\begin{array}{cccc} \frac{\delta z_1}{\delta x_1} & \frac{\delta z_1}{\delta x_2} & \frac{\delta z_1}{\delta x_3} & \frac{\delta z_1}{\delta x_4} \\ \frac{\delta z_2}{\delta x_1} & \frac{\delta z_2}{\delta x_2} & \frac{\delta z_2}{\delta x_3} & \frac{\delta z_2}{\delta x_4} \\ \frac{\delta z_3}{\delta x_1} & \frac{\delta z_3}{\delta x_2} & \frac{\delta z_3}{\delta x_3} & \frac{\delta z_3}{\delta x_4} \\ \frac{\delta z_4}{\delta x_1} & \frac{\delta z_4}{\delta x_2} & \frac{\delta z_4}{\delta x_3} & \frac{\delta z_4}{\delta x_4} \end{array} \right]$ onde

$\frac{\delta z_1}{\delta x_1} = \cos(x_1 + 2x_4) - 4(x_2^2 - 2x_1 + x_3) - 2\tan(x_2^2 - 2x_1 + x_3)\sec(x_2^2 - 2x_1 + x_3)$
$\frac{\delta z_2}{\delta x_1} = 2(x_1 + 2x_4)(x_2^2 - 2x_1 + x_3) - 2(x_1 + 2x_4)^2 + 3(x_3^2 + x_1 + \cos x_1)^2(1 - \sin x_1)$
$\frac{\delta z_3}{\delta x_1} = 1 - 6(x_2^2 - 2x_1 + x_3)^2 x_2^2$
$\frac{\delta z_4}{\delta x_1} = -1$

$\frac{\delta z_1}{\delta x_2} = 4(x_2^2 - 2x_1 + x_3)x_2 + 2x_2 \tan(x_2^2 - 2x_1 + x_3)\sec(x_2^2 - 2x_1 + x_3) + 2x_2$
$\frac{\delta z_2}{\delta x_2} = 2x_2(x_1 + 2x_4)^2$
$\frac{\delta z_3}{\delta x_2} = 6x_2(x_2^2 - 2x_1 + x_3)^2 x_2^2 + 2x_2(x_2^2 - 2x_1 + x_3)^3$
$\frac{\delta z_4}{\delta x_2} = 2x_2$
$\frac{\delta z_1}{\delta x_3} = 2(x_2^2 - 2x_1 + x_3) + \tan(x_2^2 - 2x_1 + x_3)\sec(x_2^2 - 2x_1 + x_3)$
$\frac{\delta z_2}{\delta x_3} = (x_1 + 2x_4)^2 + 6x_3(x_3^2 + x_1 + \cos x_1)^2$
$\frac{\delta z_3}{\delta x_3} = 3(x_2^2 - 2x_1 + x_3)^2 x_2^2$
$\frac{\delta z_4}{\delta x_3} = 1$
$\frac{\delta z_1}{\delta x_4} = 2\cos(x_1 + 2x_4)$
$\frac{\delta z_2}{\delta x_4} = 4(x_1 + 2x_4)(x_2^2 - 2x_1 + x_3)$
$\frac{\delta z_3}{\delta x_4} = 2$
$\frac{\delta z_4}{\delta x_4} = 2$

5. $-1dm^3/s$
6. $-4dm^2/s$
7. $\frac{3}{800}K$
8. O gradiente de uma superfície de nível num ponto desta é um vector normal à superfície nesse ponto.
9. $m = n$