Teste N.º 1 de Matemática A • 11.º Ano
Proposta de resolução

1. $D\hat{B}C = 180° - 115° = 65°$

$$\begin{cases} \text{tg } 65° = \frac{\overline{CD}}{\overline{BD}} \\ \text{tg } 32° = \frac{\overline{CD}}{\overline{AD}} \end{cases} \Leftrightarrow \begin{cases} \overline{CD} = \text{tg } 65° \times \overline{BD} \\ \overline{CD} = \text{tg } 32° \times \overline{AD} \end{cases} \Leftrightarrow \begin{cases} \overline{CD} = \text{tg } 65° \times \overline{BD} \\ \overline{CD} = \text{tg } 32° \times (\overline{AB} + \overline{BD}) \end{cases}$$

$$\Leftrightarrow \begin{cases} \overline{CD} = \text{tg } 65° \times \overline{BD} \\ \overline{CD} = \text{tg } 32° \times (18 + \overline{BD}) \end{cases} \Leftrightarrow \begin{cases} \overline{CD} = \text{tg } 65° \times \overline{BD} \\ \overline{CD} = \text{tg } 32° \times 18 + \text{tg } 32° \times \overline{BD} \end{cases}$$

$$\Leftrightarrow \begin{cases} \rule{4cm}{0.4pt} \\ \text{tg } 65° \times \overline{BD} = \text{tg } 32° \times 18 + \text{tg } 32° \times \overline{BD} \end{cases} \Leftrightarrow \begin{cases} \rule{4cm}{0.4pt} \\ \text{tg } 65° \times \overline{BD} - \text{tg } 32° \times \overline{BD} = \text{tg } 32° \times 18 \end{cases}$$

$$\Leftrightarrow \begin{cases} \rule{4cm}{0.4pt} \\ \overline{BD} \times (\text{tg } 65° - \text{tg } 32°) = \text{tg } 32° \times 18 \end{cases} \Leftrightarrow \begin{cases} \rule{4cm}{0.4pt} \\ \overline{BD} = \frac{\text{tg } 32° \times 18}{\text{tg } 65° - \text{tg } 32°} \end{cases}$$

$$\Leftrightarrow \begin{cases} \overline{CD} = \text{tg } 65° \times \frac{\text{tg } 32° \times 18}{\text{tg } 65° - \text{tg } 32°} \\ \overline{BD} = \frac{\text{tg } 32° \times 18}{\text{tg } 65° - \text{tg } 32°} \end{cases}$$

$$A_{[ABD]} = \frac{\overline{AB} \times \overline{CD}}{2} = \frac{18 \times \text{tg } 65° \times \frac{\text{tg } 32° \times 18}{\text{tg } 65° - \text{tg } 32°}}{2} \approx 143 \text{ cm}^2$$

2. Opção (D)

$-1548 = -360 \times 4 - 108$

$360 \div 10 = 36$

$108 \div 36 = 3$

$-1548° = -360° \times 4 - 36° \times 3$

A imagem do ponto *B* por meio de uma rotação de centro O e ângulo de amplitude $-1548°$ é o ponto *I*.

3. Opção (B)

O ponto A tem coordenadas $(3 \cos 150°, 3 \text{ sen } 150°)$.

$\cos 150° = \cos(180° - 30°) = -\cos 30° = -\frac{\sqrt{3}}{2}$

$\text{sen } 150° = \text{sen}(180° - 30°) = \text{sen } 30° = \frac{1}{2}$

ou seja, o ponto A tem coordenadas $\left(-\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$, pelo que, as coordenadas do transformado do ponto A por uma meia-volta de centro O são $\left(\frac{3\sqrt{3}}{2}, -\frac{3}{2}\right)$.

4. Sabemos que as coordenadas do ponto C são $(cos(90° + \beta), \text{sen}(90° + \beta))$.

Sabe-se que, para um determinado valor de β, o polígono $[OABC]$ tem área $\frac{5}{13}$, pelo que:

$2 \times \frac{1 \times \text{sen}(90°+\beta)}{2} = \frac{5}{13} \Leftrightarrow \text{sen}(90° + \beta) = \frac{5}{13} \Leftrightarrow \cos\beta = \frac{5}{13}$

$\text{sen}^2\,\beta + \cos^2\beta = 1 \Leftrightarrow \text{sen}^2\,\beta + \left(\frac{5}{13}\right)^2 = 1 \Leftrightarrow \text{sen}^2\,\beta = 1 - \frac{25}{169}$

$\Leftrightarrow \text{sen}^2\,\beta = \frac{144}{169}$

$\Leftrightarrow \text{sen}\,\beta = \pm\frac{12}{13}$

Como $\beta \in]0°, 90°[$, então $\text{sen}\,\beta = \frac{12}{13}$.

5. Opção (C)

$\text{sen}\,\alpha \times \text{tg}\,\alpha > 0 \Leftrightarrow \text{sen}\,\alpha \times \frac{\text{sen}\,\alpha}{\cos\alpha} > 0 \Leftrightarrow \frac{\text{sen}^2\alpha}{\cos\alpha} > 0$

Como $\text{sen}^2\alpha \geq 0, \forall\alpha \in]-180°, 180°[$, podemos concluir que se $\frac{\text{sen}^2\alpha}{\cos\alpha} > 0$, então $\cos\alpha > 0$, o que implica que $\alpha \in]-90°, 90°[$.

Desta forma, e uma vez que $\frac{\cos\alpha}{1+\text{tg}\,\alpha} < 0$, então $1 + \text{tg}\,\alpha < 0 \Leftrightarrow \text{tg}\,\alpha < -1$, o que nos permite concluir que $\alpha \in]-90°, -45°[$ e, portanto, que $-150° < \alpha - 60° < -105°$.

Assim, o ângulo de amplitude $\alpha - 60°$ pertence ao terceiro quadrante.

6.

6.1 $\text{sen}^2\,\alpha + \cos(\pi - \alpha) + \frac{1}{9} = 0 \Leftrightarrow \text{sen}^2\,\alpha - \cos\alpha + \frac{1}{9} = 0$

$\Leftrightarrow 1 - \cos^2\alpha - \cos\alpha + \frac{1}{9} = 0$

$\Leftrightarrow \cos^2\alpha + \cos\alpha - \frac{10}{9} = 0$

$\Leftrightarrow \cos\alpha = \frac{-1 \pm \sqrt{1 - 4 \times 1 \times \left(-\frac{10}{9}\right)}}{2 \times 1}$

$\Leftrightarrow \cos\alpha = \frac{-1 \pm \frac{7}{3}}{2}$

$\Leftrightarrow \cos\alpha = -\frac{5}{3} \;\vee\; \cos\alpha = \frac{2}{3}$

$-1 \leq \cos\alpha \leq 1, \forall x \in \mathbb{R}$, logo $\cos\alpha = \frac{2}{3}$.

$\text{sen}^2\,\alpha + \cos^2\alpha = 1 \Leftrightarrow \text{sen}^2\,\alpha + \left(\frac{2}{3}\right)^2 = 1 \Leftrightarrow \text{sen}^2\,\alpha = 1 - \frac{4}{9}$

$\Leftrightarrow \text{sen}^2\,\alpha = \frac{5}{9}$

$\alpha \in \left]\frac{3\pi}{2}, 2\pi\right[$, logo $\text{sen}\,\alpha = -\frac{\sqrt{5}}{3}$.

$$\operatorname{tg}\alpha = \frac{-\frac{\sqrt{5}}{3}}{\frac{2}{3}} \Leftrightarrow \operatorname{tg}\alpha = -\frac{\sqrt{5}}{2}$$

6.2 $3\operatorname{sen}(3\pi+\alpha) + \cos^2\left(\frac{\pi}{2}-\alpha\right) + 2\operatorname{tg}(\alpha-2\pi) = 3\operatorname{sen}(2\pi+\pi+\alpha) + (-\operatorname{sen}\alpha)^2 + 2\operatorname{tg}\alpha =$

$= 3\operatorname{sen}(\pi+\alpha) + \operatorname{sen}^2\alpha + 2\operatorname{tg}\alpha =$

$= -3\operatorname{sen}\alpha + \operatorname{sen}^2\alpha - \operatorname{tg}\alpha =$

$= -3\left(-\frac{\sqrt{5}}{3}\right) + \left(-\frac{\sqrt{5}}{3}\right)^2 + 2\left(-\frac{\sqrt{5}}{2}\right) =$

$= \sqrt{5} + \frac{5}{9} - \sqrt{5} =$

$= \frac{5}{9}$

7. Opção (B)

$D_f = \left\{x \in \mathbb{R}: 1 + \cos\left(\frac{\pi}{2} - 2x\right) \neq 0\right\} = \mathbb{R} \setminus \left\{x: x = \frac{3\pi}{4} + k\pi, k \in \mathbb{Z}\right\}$

Cálculo auxiliar

$1 + \cos\left(\frac{\pi}{2} - 2x\right) = 0 \Leftrightarrow 1 + \operatorname{sen}(2x) = 0$

$\Leftrightarrow \operatorname{sen}(2x) = -1$

$\Leftrightarrow 2x = -\frac{\pi}{2} + 2k\pi, k \in \mathbb{Z}$

$\Leftrightarrow x = -\frac{\pi}{4} + k\pi, k \in \mathbb{Z}$

$\Leftrightarrow x = \frac{3\pi}{4} + k\pi, k \in \mathbb{Z}$

8.

8.1 $g(x) = (\cos x - 1)^2 - \cos\left(\frac{\pi}{2} + \pi x\right) - 2\operatorname{sen}\left(\frac{3\pi}{2} - x\right) + 3\operatorname{sen}(\pi + \pi x) - 1 + \operatorname{sen}^2(-x) =$

$= \cos^2 x - 2\cos x + 1 - (-\operatorname{sen}(\pi x)) - 2(-\cos x) - 3\operatorname{sen}(\pi x) - 1 + \operatorname{sen}^2 x =$

$= \cos^2 x - 2\cos x + 1 + \operatorname{sen}(\pi x) + 2\cos x - 3\operatorname{sen}(\pi x) - 1 + \operatorname{sen}^2 x =$

$= \underbrace{\operatorname{sen}^2 x + \cos^2 x}_{1} - 2\cos x + 2\cos x + \operatorname{sen}(\pi x) - 3\operatorname{sen}(\pi x) + 1 - 1 =$

$= 1 + 0 - 2\operatorname{sen}(\pi x) + 0 =$

$= 1 - 2\operatorname{sen}(\pi x)$

8.2 Opção (A)

$P = \frac{2\pi}{|\pi|} = 2$

$f = \frac{1}{2}$

9. $f\left(x-\frac{\pi}{2}\right)+f\left(x+\frac{\pi}{2}\right)=3+5\cos\left(5\left(x-\frac{\pi}{2}\right)+\frac{\pi}{7}\right)+\left(3+5\cos\left(5\left(x+\frac{\pi}{2}\right)+\frac{\pi}{7}\right)\right)=$

$$=6+5\cos\left(5x-\frac{5\pi}{2}+\frac{\pi}{7}\right)+5\cos\left(5x+\frac{5\pi}{2}+\frac{\pi}{7}\right)=$$

$$=6+5\cos\left(5x+\frac{\pi}{7}-2\pi-\frac{\pi}{2}\right)+5\cos\left(5x+\frac{\pi}{7}+2\pi+\frac{\pi}{2}\right)=$$

$$=6+5\cos\left(5x+\frac{\pi}{7}-\frac{\pi}{2}\right)+5\cos\left(5x+\frac{\pi}{7}+\frac{\pi}{2}\right)=$$

$$=6+5\,\text{sen}\left(5x+\frac{\pi}{7}\right)+\left(-5\,\text{sen}\left(5x+\frac{\pi}{7}\right)\right)=$$

$$=6+5\,\text{sem}\left(5x+\frac{\pi}{7}\right)-5\,\text{sem}\left(5x+\frac{\pi}{7}\right)=$$

$$=6$$

Daqui se conclui que para todo o valor real de x, $f\left(x-\frac{\pi}{2}\right)+f\left(x+\frac{\pi}{2}\right)=6$.

10.

10.1 $A(\cos\beta,\text{sen}\,\beta)$ $\quad B(1,\text{tg}\,\beta)$ $\quad C(1,-\text{sen}\,\beta)$ $\quad D(\cos\beta,0)$

Como $\beta\in\left]-\frac{\pi}{2},0\right[$, $\cos\beta>0$ e $\text{sen}\,\beta<0$.

$\overline{AD}=-\text{sen}\,\beta$

$\overline{BC}=-\text{tg}\,\beta-\text{sen}\,\beta$

$h=1-\cos\beta$

$$A_{[ABCD]}=\frac{-\text{tg}\,\beta-\text{sen}\,\beta+(-\text{sen}\,\beta)}{2}\times(1-\cos\beta)=$$

$$=\frac{-\text{tg}\,\beta-2\,\text{sen}\,\beta}{2}\times(1-\cos\beta)=$$

$$=\frac{-\text{tg}\,\beta+\text{tg}\,\beta\times\cos\beta-2\,\text{sen}\,\beta+2\,\text{sen}\,\beta\times\cos\beta}{2}=$$

$$=\frac{-\text{tg}\,\beta+\text{sen}\,\beta-2\,\text{sen}\,\beta+2\,\text{sen}\,\beta\times\cos\beta}{2}=$$

$$=\frac{-\text{tg}\,\beta-\text{sen}\,\beta+2\,\text{sen}\,\beta\times\cos\beta}{2}=$$

$$=-\frac{\text{tg}\,\beta+\text{sen}\,\beta-2\,\text{sen}\,\beta\cos\beta}{2}$$

10.2 $\text{sen}\left(-\frac{3\pi}{2}-\beta\right)-2\cos(\pi-\beta)=\frac{6}{5}\Leftrightarrow\cos\beta+2\cos\beta=\frac{6}{5}$

$$\Leftrightarrow\cos\beta=\frac{2}{5}$$

$\text{sen}^2\,\beta+\cos^2\beta=1\Leftrightarrow\text{sen}^2\,\beta=1-\frac{4}{25}$

$$\Leftrightarrow\text{sen}^2\,\beta=\frac{21}{25}$$

$$\Leftrightarrow\text{sen}\,\beta=\pm\frac{\sqrt{21}}{5}$$

Como $\beta\in\left]-\frac{\pi}{2},0\right[$, então $\text{sen}\,\beta=-\frac{\sqrt{21}}{5}$.

$$\operatorname{tg} \beta = \frac{-\frac{\sqrt{21}}{5}}{\frac{2}{5}} \Leftrightarrow \operatorname{tg} \beta = -\frac{\sqrt{21}}{2}$$

Assim :

$$A(\beta) = -\frac{-\frac{\sqrt{21}}{2} + \left(-\frac{\sqrt{21}}{5}\right) - 2\left(-\frac{\sqrt{21}}{5} \times \frac{2}{5}\right)}{2} =$$

$$= -\frac{-\frac{\sqrt{21}}{2} - \frac{\sqrt{21}}{5} + \frac{4\sqrt{21}}{25}}{2} =$$

$$= \frac{\sqrt{21}}{4} + \frac{\sqrt{21}}{10} - \frac{4\sqrt{21}}{50} =$$

$$= \frac{25\sqrt{21}}{100} + \frac{10\sqrt{21}}{100} - \frac{8\sqrt{21}}{100} =$$

$$= \frac{27\sqrt{21}}{100}$$