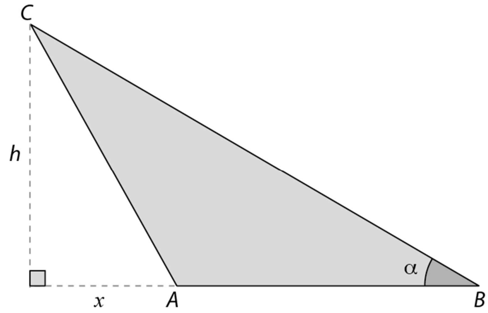
TESTE N.º 1 – Proposta de resolução

Grupo I

1. Opção (A)



$A_{[ABC]} = 16\sqrt{3} \Leftrightarrow \frac{8\times h}{2} = 16\sqrt{3} \Leftrightarrow 8h = 32\sqrt{3}$

$\Leftrightarrow h = 4\sqrt{3}$

Pelo Teorema de Pitágoras:

$8^2 = \left(4\sqrt{3}\right)^2 + x^2 \Leftrightarrow 64 = 48 + x^2 \Leftrightarrow x^2 = 16$

Logo, $x = 4$. Assim, $\tan\alpha = \frac{4\sqrt{3}}{4+8} = \frac{\sqrt{3}}{3}$.

2. Opção (D)

$f(x-\pi) = 2\sin(x-\pi)\cos(x-\pi) = -2\sin(\pi-x)\cos(\pi-x) = 2\sin x\cos x = f(x)$

$f(\pi-x) = 2\sin(\pi-x)\cos(\pi-x) = -2\sin x\ \cos x = -f(x)$

$f\left(\frac{\pi}{2}-x\right) = 2\sin\left(\frac{\pi}{2}-x\right)\cos\left(\frac{\pi}{2}-x\right) = 2\cos x\sin x = 2\sin x\cos x = f(x)$

$f\left(x-\frac{\pi}{2}\right) = 2\sin\left(x-\frac{\pi}{2}\right)\cos\left(x-\frac{\pi}{2}\right) = -2\sin\left(\frac{\pi}{2}-x\right)\cos\left(\frac{\pi}{2}-x\right) = -2\cos x\sin x =$

$= -2\sin x\cos x = -f(x)$

3. Opção (B)

Sendo α a amplitude do ângulo AOB, então $\sin\alpha = \frac{1}{2}$ e $\cos\alpha = \frac{\sqrt{3}}{2}$. Assim, as coordenadas de C são:

$\left(\cos\left(\frac{3\pi}{2}-\alpha\right), \sin\left(\frac{3\pi}{2}-\alpha\right)\right) = (-\sin\alpha, -\cos\alpha) = \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

4. Opção (C)

$\sin\alpha = \frac{2}{3}$

$\cos^2\alpha = 1 - \sin^2\alpha \Leftrightarrow \cos^2\alpha = 1 - \frac{4}{9} \Leftrightarrow \cos^2\alpha = \frac{5}{9}$

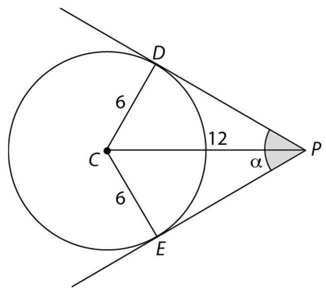
Logo, $\cos\alpha = \frac{\sqrt{5}}{3}$.

Assim, $\cos\left(\arcsin\frac{2}{3}\right) = \frac{\sqrt{5}}{3}$.

5. Opção (A)

Um período desta função pode ser $\frac{7\pi}{2} - \frac{7\pi}{10} = \frac{14\pi}{5}$.

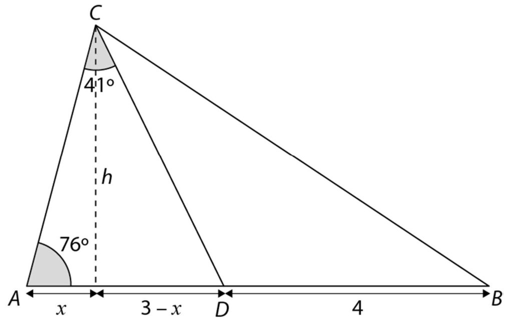
Grupo II



1. $\sin\left(\frac{\alpha}{2}\right) = \frac{6}{12} \Leftrightarrow \sin\left(\frac{\alpha}{2}\right) = \frac{1}{2}$

Logo, $\frac{\alpha}{2} = 30°$ e, portanto, $\alpha = 60°$.

2.



2.1. $A\hat{D}C = 180° - 41° - 76° = 63°$

Seja *h* a altura do triângulo $[ABC]$.

$\tan 76° = \frac{h}{x} \Leftrightarrow h = x \tan 76°$

$\tan 63° = \frac{h}{3-x} \Leftrightarrow h = (3-x)\tan 63°$

Logo:

$x \tan 76° = (3-x)\tan 63°$

$\Leftrightarrow x\tan 76° + x\tan 63° = 3\tan 63°$

$\Leftrightarrow x = \frac{3\tan 63°}{\tan 76° + \tan 63°}$

Portanto:

$h = \frac{3\tan 63°}{\tan 76° + \tan 63°} \times \tan 76°$

Então:

$A_{[ABC]} = \frac{7 \times \frac{3\tan 63°}{\tan 76° + \tan 63°} \times \tan 76°}{2} \approx 13{,}8$ u.a.

2.2. $\frac{\sin 76°}{\overline{CD}} = \frac{\sin 41°}{3} \Leftrightarrow \overline{CD} = \frac{3\sin 76°}{\sin 41°}$

$B\hat{D}C = 180° - 63° = 117°$

$\overline{BC}^2 = 4^2 + \left(\frac{3\sin 76°}{\sin 41°}\right)^2 - 2 \times 4 \times \frac{3\sin 76°}{\sin 41°}\cos 117°$

Logo, $\overline{BC} = \sqrt{4^2 + \left(\frac{3\sin 76°}{\sin 41°}\right)^2 - 2 \times 4 \times \frac{3\sin 76°}{\sin 41°}\cos 117°} \approx 7{,}2$ u.c.

3.

3.1. $D_f = \{x \in \mathbb{R}: \cos^4 x + \cos^2 x \sin^2 x \neq 0\} = \mathbb{R} \backslash \left\{x \in \mathbb{R}: x = \frac{\pi}{2} + k\pi, k \in \mathbb{Z}\right\}$

Cálculo auxiliar

$$\begin{aligned}\cos^4 x + \cos^2 x \sin^2 x = 0 &\Leftrightarrow \cos^2 x\,(\cos x^2 + \sin^2 x) = 0\\ &\Leftrightarrow \cos^2 x = 0\\ &\Leftrightarrow \cos x = 0\\ &\Leftrightarrow x = \frac{\pi}{2} + k\pi, k \in \mathbb{Z}\end{aligned}$$

3.2. $f(x) = \frac{\sin x - \sin^3 x}{\cos^4 x + \cos^2 x \sin^2 x} = \frac{\sin x(1-\sin^2 x)}{\cos^2 x(\cos x^2 + \sin^2 x)} = \frac{\sin x \cos^2 x}{\cos^2 x \times 1} = \sin x$

3.3. $D'_f =]-1, 1[$

3.4.
$$\begin{aligned}\frac{f^2(-x) f\left(\frac{5\pi}{2}-x\right) + f\left(\frac{7\pi}{2}+x\right)}{f\left(\frac{3\pi}{2}-x\right)} + f^2(x-\pi) &= \frac{\sin^2(-x)\sin\left(\frac{5\pi}{2}-x\right) + \sin\left(\frac{7\pi}{2}+x\right)}{\sin\left(\frac{3\pi}{2}-x\right)} + \sin^2(x-\pi) =\\ &= \frac{\sin^2 x \sin\left(\frac{\pi}{2}-x\right) + \sin\left(-\frac{\pi}{2}+x\right)}{\sin\left(-\frac{\pi}{2}+x\right)} + \sin^2(\pi - x) =\\ &= \frac{\sin^2 x \cos x - \sin\left(\frac{\pi}{2}-x\right)}{-\sin\left(\frac{\pi}{2}-x\right)} + \sin^2 x =\\ &= \frac{-\sin^2 x \cos x + \cos x}{\cos x} + \sin^2 x =\\ &= -\sin^2 x + 1 + \sin^2 x =\\ &= 1\end{aligned}$$

4.

4.1.
$$\begin{aligned}A_{[ABCDO]} = \frac{2\cos\alpha \times \sin\alpha}{2} \times 2 + \frac{2\sin\alpha \times \cos\alpha}{2} &= 2\sin\alpha\cos\alpha + \sin\alpha\cos\alpha =\\ &= 3\sin\alpha\cos\alpha = A(\alpha)\end{aligned}$$

4.2. $\sin\left(\frac{\pi}{2} - \alpha\right) = \frac{1}{3} \Leftrightarrow \cos\alpha = \frac{1}{3}$

$\sin^2\alpha = 1 - \cos^2\alpha \Leftrightarrow \sin^2\alpha = 1 - \frac{1}{9} \Leftrightarrow \sin^2\alpha = \frac{8}{9}$

Como $\alpha \in \left]0, \frac{\pi}{2}\right[$, então $\sin\alpha = \frac{2\sqrt{2}}{3}$.

Assim, $A(\alpha) = 3 \times \frac{2\sqrt{2}}{3} \times \frac{1}{3} = \frac{2\sqrt{2}}{3}$.

4.3. $A(\alpha) = \sin\alpha\,(\cos\alpha + 1) \Leftrightarrow 3\sin\alpha\cos\alpha = \sin\alpha\;(\cos\alpha + 1)$

$$\Leftrightarrow 3\sin\alpha\cos\alpha - \sin\alpha\;(\cos\alpha + 1) = 0$$

$$\Leftrightarrow \sin\alpha\,(3\cos\alpha - \cos\alpha - 1) = 0$$

$$\Leftrightarrow \sin\alpha\,(2\cos\alpha - 1) = 0$$

$$\Leftrightarrow \sin\alpha = 0 \vee 2\cos\alpha - 1 = 0$$

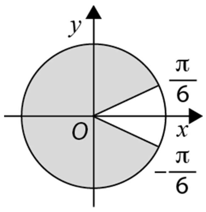
$$\Leftrightarrow \sin\alpha = 0 \vee \cos\alpha = \frac{1}{2}$$

$$\Leftrightarrow \alpha = k\pi \vee \alpha = \frac{\pi}{3} + 2k\pi \vee \alpha = -\frac{\pi}{3} + 2k\pi, k \in \mathbb{Z}$$

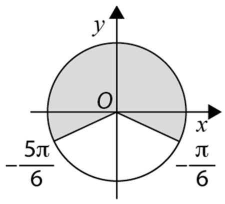
Como $\alpha \in \left]0, \frac{\pi}{2}\right[$, então $\alpha = \frac{\pi}{3}$.

5. $\cos x < \frac{\sqrt{3}}{2} \;\wedge\; \sin x > -\frac{1}{2}$

$\cos x < \frac{\sqrt{3}}{2} \Leftrightarrow \cos x < \cos\frac{\pi}{6} \Leftrightarrow x \in \left[-\pi, -\frac{\pi}{6}\right[\cup \left]\frac{\pi}{6}, \pi\right]$



$\sin x > -\frac{1}{2} \Leftrightarrow \sin x > \sin\left(-\frac{\pi}{6}\right) \Leftrightarrow x \in \left[-\pi, -\frac{5\pi}{6}\right[\cup \left]-\frac{\pi}{6}, \pi\right]$



Uma vez que $x \in [-\pi, \pi]$, então C.S. $= \left[-\pi, -\frac{5\pi}{6}\right[\cup \left]\frac{\pi}{6}, \pi\right]$.