Itens para Avaliação intercalar – Outubro

Matemática A | 12.º Ano

Cotações e Propostas de resolução

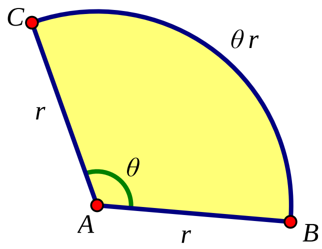
TABELA DE COTAÇÕES

Nota ao professor: Caso pretenda utilizar o conjunto de itens apresentados como uma avaliação intercalar, sugere-se a seguinte tabela de cotações:

1.	2.	3.	4.1	4.2	4.3	5.1	5.2	Total
15	15	28	28	28	28	28	30	200

PROPOSTAS DE RESOLUÇÃO

1. Na figura, está representado um setor circular BAC de raio r e amplitude θ.



O comprimento do arco BC é dado por θr, pelo que o perímetro do setor circular BAC é dado por $2r+\theta r$.

Assim, $2r+\theta r=4r \Leftrightarrow \theta \cancel{r}=2\cancel{r} \Leftrightarrow \theta=2$ rad, pelo que, $\operatorname{sen}\theta=\operatorname{sen}(2)\approx 0{,}909$ e $\cos\theta=\cos(2)\approx -0{,}416$.

Resposta: D

2. Tem-se $\beta=-\frac{125\pi}{6}=-\frac{120\pi}{6}-\frac{5\pi}{6}=-\frac{5\pi}{6}-20\pi=\frac{7\pi}{6}\underbrace{-\frac{7\pi}{6}-\frac{5\pi}{6}}_{-2\pi}-20\pi=\frac{7\pi}{6}-22\pi=\frac{7\pi}{6}-11\times 2\pi$.

Portanto, $\beta=\frac{7\pi}{6}-11\times 2\pi$, pelo que a amplitude do ângulo α pode ser $\frac{7\pi}{6}$.

Resposta: A

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3. A área do jardim é dada por $\frac{\overline{CD}\times\overline{AB}}{2}=\frac{45\times\overline{AB}}{2}=22,5\times\overline{AB}$.

Como o ângulo ACD tem mais 30° de amplitude do que o ângulo ACB, tem-se $A\hat{C}D=A\hat{C}B+30^\circ$. Assim, como $A\hat{C}B+A\hat{C}D=180^\circ$, vem que:

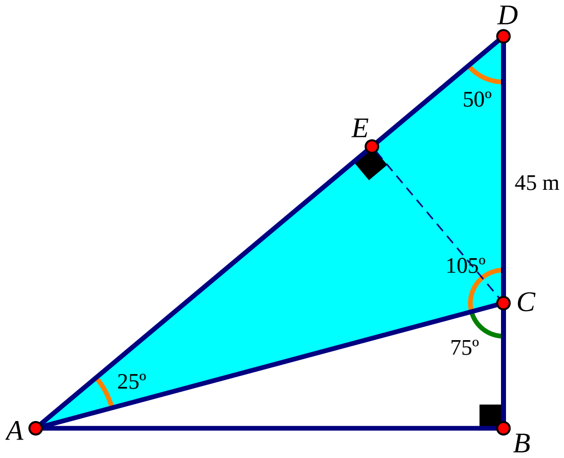
$$A\hat{C}B+A\hat{C}B+30^\circ=180^\circ \Leftrightarrow 2A\hat{C}B=180^\circ-30^\circ \Leftrightarrow 2A\hat{C}B=150^\circ \Leftrightarrow A\hat{C}B=75^\circ$$

Portanto, $A\hat{C}D=75^\circ+30^\circ=105^\circ$ e $A\hat{D}C+105^\circ+25^\circ=180^\circ \Leftrightarrow A\hat{D}C=180^\circ-130^\circ \Leftrightarrow A\hat{D}C=50^\circ$.

Consideremos a figura seguinte, em que E é a projeção ortogonal do ponto C no lado $[AD]$.

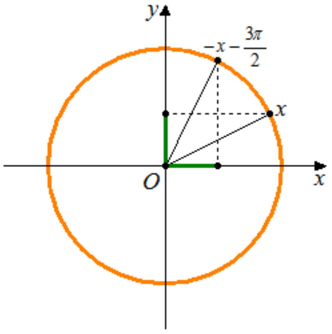
Tem-se que:

- $\operatorname{sen}(50^\circ)=\frac{\overline{CE}}{45} \Leftrightarrow \overline{CE}=45\operatorname{sen}(50^\circ)$
- $\operatorname{sen}(25^\circ)=\frac{\overline{CE}}{\overline{AC}} \Leftrightarrow \overline{AC}=\frac{45\operatorname{sen}(50^\circ)}{\operatorname{sen}(25^\circ)}$
- $\operatorname{sen}(75^\circ)=\frac{\overline{AB}}{\overline{AC}} \Leftrightarrow \overline{AB}=\frac{45\operatorname{sen}(50^\circ)\times\operatorname{sen}(75^\circ)}{\operatorname{sen}(25^\circ)}$

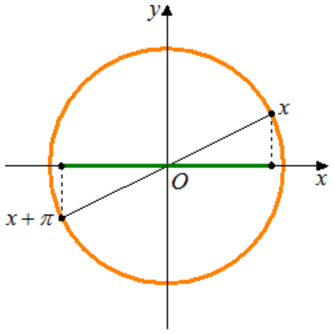


Portanto, a área do jardim é igual a $22,5\times\overline{AB}=22,5\times\frac{45\operatorname{sen}(50^\circ)\times\operatorname{sen}(75^\circ)}{\operatorname{sen}(25^\circ)}\approx 1773\text{ m}^2$.

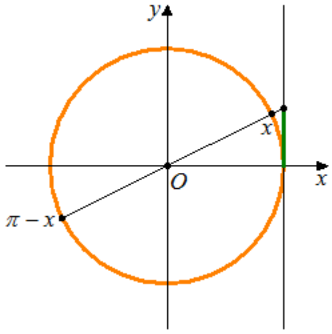
4.1



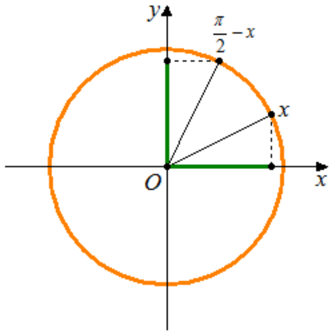
$$\cos\left(-x-\frac{3\pi}{2}\right)=\text{sen}\,x$$



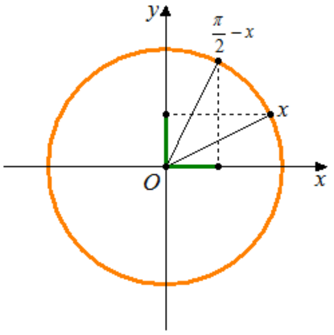
$$\cos(x+\pi)=-\cos x$$



$$\text{tg}(x-3\pi)=\text{tg}(x-\pi-2\pi)=$$
$$=\text{tg}(x-\pi)=\text{tg}\,x$$



$$\text{sen}\left(\frac{\pi}{2}-x\right)=\cos x$$



$$\cos\left(\frac{\pi}{2}-x\right)=\text{sen}\,x$$

$$\text{tg}\left(\frac{\pi}{2}-x\right)=\frac{\text{sen}\left(\frac{\pi}{2}-x\right)}{\cos\left(\frac{\pi}{2}-x\right)}=$$
$$=\frac{\cos x}{\text{sen}\,x}=\frac{1}{\text{tg}\,x}$$

Portanto, $A(x)=\dfrac{\cos\left(-x-\frac{3\pi}{2}\right)}{\text{tg}\left(\frac{\pi}{2}-x\right)}-\cos(x+\pi)-\text{tg}(x-3\pi)=\dfrac{\text{sen}\,x}{\frac{1}{\text{tg}\,x}}+\cos x-\text{tg}\,x=$

$$=\text{sen}\,x\times\text{tg}\,x+\cos x-\text{tg}\,x=\text{sen}\,x\times\frac{\text{sen}\,x}{\cos x}+\cos x-\frac{\text{sen}\,x}{\cos x}=$$

$$=\frac{\text{sen}^2 x}{\cos x}+\cos x-\frac{\text{sen}\,x}{\cos x}=\frac{\overbrace{\text{sen}^2 x+\cos^2 x}^{=1}-\text{sen}\,x}{\cos x}=\frac{1-\text{sen}\,x}{\cos x}$$

4.2 Tem-se que:

- $A\left(\frac{11\pi}{6}\right)=\dfrac{1-\text{sen}\left(\frac{11\pi}{6}\right)}{\cos\left(\frac{11\pi}{6}\right)}\underset{\frac{11\pi}{6}=\frac{12\pi}{6}-\frac{\pi}{6}=2\pi-\frac{\pi}{6}}{=}\dfrac{1-\text{sen}\left(2\pi-\frac{\pi}{6}\right)}{\cos\left(2\pi-\frac{\pi}{6}\right)}\underset{i)}{=}\dfrac{1-\text{sen}\left(-\frac{\pi}{6}\right)}{\cos\left(-\frac{\pi}{6}\right)}=\dfrac{1-\left(-\frac{1}{2}\right)}{\frac{\sqrt{3}}{2}}=\dfrac{1+\frac{1}{2}}{\frac{\sqrt{3}}{2}}=$

$$=\frac{\frac{3}{\cancel{2}}}{\frac{\sqrt{3}}{\cancel{2}}}=\frac{3}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{\cancel{3}\sqrt{3}}{\cancel{3}}=\sqrt{3}$$

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- $A\left(-\frac{4\pi}{3}\right)=\frac{1-\operatorname{sen}\left(-\frac{4\pi}{3}\right)}{\cos\left(-\frac{4\pi}{3}\right)}\underset{i)}{=}\frac{1-\operatorname{sen}\left(-\frac{4\pi}{3}+2\pi\right)}{\cos\left(-\frac{4\pi}{3}+2\pi\right)}=\frac{1-\operatorname{sen}\left(\frac{2\pi}{3}\right)}{\cos\left(\frac{2\pi}{3}\right)}=\frac{1-\frac{\sqrt{3}}{2}}{-\frac{1}{2}}=-\frac{\frac{2-\sqrt{3}}{\cancel{2}}}{\frac{1}{\cancel{2}}}=-2+\sqrt{3}$

Portanto, $A\left(\frac{11\pi}{6}\right)-A\left(-\frac{4\pi}{3}\right)=\sqrt{3}-\left(-2+\sqrt{3}\right)=\cancel{\sqrt{3}}+2-\cancel{\sqrt{3}}=2$.

$i)$ $\operatorname{sen}(\alpha+2k\pi)=\operatorname{sen}\alpha$ e $\cos(\alpha+2k\pi)=\cos\alpha$, para todo o $\alpha\in\mathbb{R}$ e $k\in\mathbb{Z}$.

4.3 Tem-se $\operatorname{tg}\alpha=-\sqrt{8}$. Assim, $\operatorname{tg}\alpha<0$ e como $\alpha\in\left]0,\pi\right[$, conclui-se que $\alpha\in\left]\frac{\pi}{2},\pi\right[$.

Como $1+\operatorname{tg}^2\alpha=\frac{1}{\cos^2\alpha}$, vem que:

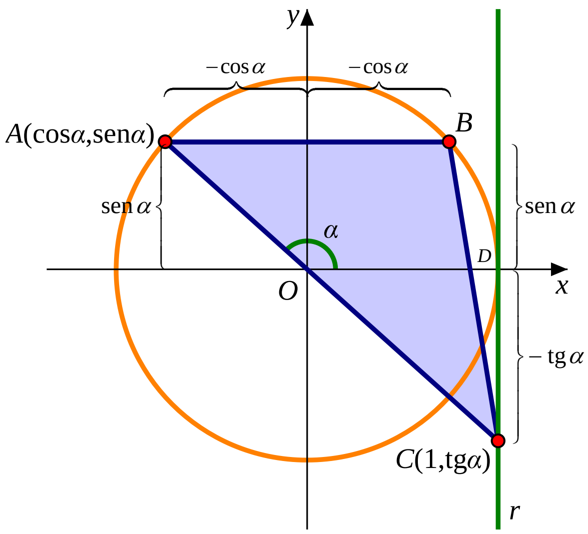
$$1+\operatorname{tg}^2\alpha=\frac{1}{\cos^2\alpha}\Leftrightarrow 1+\left(-\sqrt{8}\right)^2=\frac{1}{\cos^2\alpha}\Leftrightarrow 1+8=\frac{1}{\cos^2\alpha}\Leftrightarrow\frac{1}{\cos^2\alpha}=9\Leftrightarrow\cos^2\alpha=\frac{1}{9}\Leftrightarrow$$

$$\underset{\alpha\in\left]\frac{\pi}{2},\pi\right[\Rightarrow\cos\alpha<0}{\Leftrightarrow}\cos\alpha=-\sqrt{\frac{1}{9}}\Leftrightarrow\cos\alpha=-\frac{1}{3}$$

Como $\operatorname{tg}\alpha=\frac{\operatorname{sen}\alpha}{\cos\alpha}\Leftrightarrow\operatorname{sen}\alpha=\operatorname{tg}\alpha\times\cos\alpha$, tem-se $\operatorname{sen}\alpha=-\sqrt{8}\times\left(-\frac{1}{3}\right)=\frac{2\sqrt{2}}{3}$.

Logo, $A(\alpha)=\frac{1-\operatorname{sen}\alpha}{\cos\alpha}=\frac{1-\frac{2\sqrt{2}}{3}}{-\frac{1}{3}}=-\frac{\frac{3-2\sqrt{2}}{\cancel{3}}}{\frac{1}{\cancel{3}}}=-\left(3-2\sqrt{2}\right)=2\sqrt{2}-3$.

5.1 As coordenadas dos pontos A e C, em função de α, são, respetivamente $(\cos\alpha,\operatorname{sen}\alpha)$ e $(1,\operatorname{tg}\alpha)$, em que $\cos\alpha<0$, $\operatorname{sen}\alpha>0$ e $\operatorname{tg}\alpha<0$.



Tem-se que $\overline{AB} = -2\cos\alpha$ e a medida da altura do triângulo é dada por $\operatorname{sen}\alpha - \operatorname{tg}\alpha$, pelo que a área do triângulo $[ABC]$ é dada por:

$$\frac{-\cancel{2}\cos\alpha \times (\operatorname{sen}\alpha - \operatorname{tg}\alpha)}{\cancel{2}} = -\cos\alpha \times \operatorname{sen}\alpha + \cos\alpha \times \operatorname{tg}\alpha = -\cos\alpha \times \operatorname{sen}\alpha + \cancel{\cos\alpha} \times \frac{\operatorname{sen}\alpha}{\cancel{\cos\alpha}} =$$

$$= -\cos\alpha \times \operatorname{sen}\alpha + \operatorname{sen}\alpha = \operatorname{sen}\alpha(1-\cos\alpha)$$

5.2 Tem-se que $(\operatorname{sen}\alpha - \cos\alpha)^2 = 1 + \operatorname{sen}\alpha \Leftrightarrow \underbrace{\operatorname{sen}^2\alpha + \cos^2\alpha}_{=1} - 2\operatorname{sen}\alpha \times \cos\alpha = 1 + \operatorname{sen}\alpha \Leftrightarrow$

$$\Leftrightarrow \cancel{1} - 2\operatorname{sen}\alpha \times \cos\alpha = \cancel{1} + \operatorname{sen}\alpha \Leftrightarrow -2\cancel{\operatorname{sen}\alpha} \times \cos\alpha = \cancel{\operatorname{sen}\alpha}$$

$$\underset{\alpha \in \left]\frac{\pi}{2},\pi\right[\Rightarrow \operatorname{sen}\alpha \neq 0}{\Leftrightarrow} -2\cos\alpha = 1 \Leftrightarrow \cos\alpha = -\frac{1}{2} \underset{\alpha \in \left]\frac{\pi}{2},\pi\right[}{\Leftrightarrow} \alpha = \frac{2\pi}{3}$$

Portanto, $\alpha = \frac{2\pi}{3}$, pelo que a área do triângulo $[ABC]$ é igual a:

$$\operatorname{sen}\left(\frac{2\pi}{3}\right) \times \left(1 - \cos\left(\frac{2\pi}{3}\right)\right) = \frac{\sqrt{3}}{2} \times \left(1 + \frac{1}{2}\right) = \frac{\sqrt{3}}{2} \times \frac{3}{2} = \frac{3\sqrt{3}}{4}$$