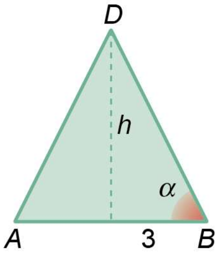
**1.** Opção **(B)**

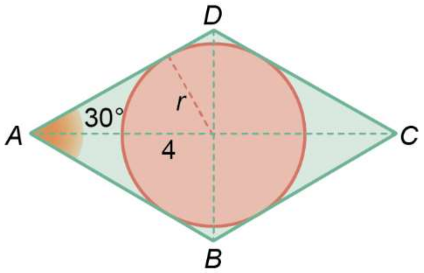
$\frac{6 \times h}{2} = 18 \Leftrightarrow h = 6$

$\overline{CB} = \sqrt{3^2 + 6^2} = \sqrt{45} = 3\sqrt{5}$

$\sin \alpha = \frac{6}{3\sqrt{5}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$




**2.** $\sin(30^\circ) = \frac{r}{4} \Leftrightarrow r = 4\sin(30^\circ) \Leftrightarrow r = 4 \times \frac{1}{2} \Leftrightarrow r = 2$




Seja $P$ comprimento da circunferência

$P = 2\pi \times r = 2\pi \times 2 = 4\pi$

**3.** Opção **(A)**

$360 : 10 = 36$. Cada setor tem de amplitude $36^\circ$.

$-2736^\circ = -7 \times 360^\circ - 216^\circ$

A imagem do setor 2 pela rotação de centro $O$ e amplitude $-216^\circ$ é o setor 8 (repara que $216 : 36 = 6$).

**4.** Opção **(B)**

Como $a \in \left]-1, 0\right[ \wedge \cos(\beta) = a$, então $\beta \in 2.^\circ\text{Q} \vee \beta \in 3.^\circ\text{Q}$

Como $\tan(\beta) < 0$, então $\beta \in 2.^\circ\text{Q}$.

**5.1.** Opção **(C)**

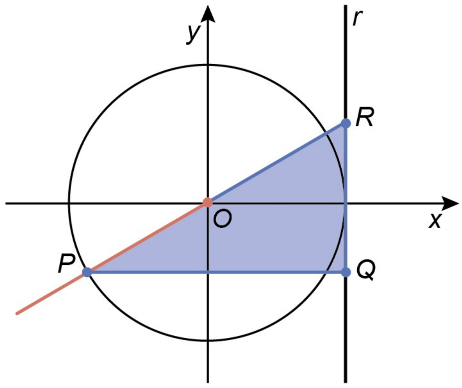
$2010^\circ = 210^\circ + 5 \times 360^\circ$

Sabe-se que $P\left(\cos(2010^\circ), \sin(2010^\circ)\right)$

$\left(\cos(2010^\circ), \sin(2010^\circ)\right) = \left(\cos(210^\circ), \sin(210^\circ)\right) = \left(\cos(180^\circ + 30^\circ), \sin\left(180^\circ + 30^\circ\right)\right) =$

$= \left(-\cos(30^\circ), -\sin(30^\circ)\right) = \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

**5.2.** $R\left(1,\ \tan\left(2010^\circ\right)\right)$, ou seja, $R\left(1,\ \tan\left(30^\circ\right)\right)$.




Daqui resulta $R\left(1,\ \frac{\sqrt{3}}{3}\right)$.

$Q\left(1,\ -\sin 30^\circ\right)=\left(1,\ -\frac{1}{2}\right)$

Então, $\overline{PQ}=1+\frac{\sqrt{3}}{2}$ e $\overline{QR}=\frac{1}{2}+\frac{\sqrt{3}}{3}$.

Seja $A$ a medida da área do triângulo $[PQR]$.

$$A=\frac{\overline{PQ}\times\overline{QR}}{2}=\frac{\left(1+\frac{\sqrt{3}}{2}\right)\times\left(\frac{1}{2}+\frac{\sqrt{3}}{3}\right)}{2}=\frac{\frac{1}{2}+\frac{\sqrt{3}}{3}+\frac{\sqrt{3}}{4}+\frac{3}{6}}{2}=\frac{1+\frac{7\sqrt{3}}{12}}{2}=\frac{1}{2}+\frac{7\sqrt{3}}{24}$$

**6.** Sabe-se que $\sin\left(\frac{3\pi}{2}-\alpha\right)=-\frac{1}{4}$ e $\alpha\in\left]\frac{3\pi}{2},\ 2\pi\right[$.

Mas, $\sin\left(\frac{3\pi}{2}-\alpha\right)=-\frac{1}{4}\Leftrightarrow -\cos\alpha=-\frac{1}{4}\Leftrightarrow \boxed{\cos\alpha=\frac{1}{4}}$.

Repara que:

$$\cos\left(-\frac{15\pi}{2}+\alpha\right)-2\sin\left(-\frac{7\pi}{6}\right)+\tan\left(-\alpha+3\pi\right)+\tan\left(\frac{9\pi}{4}\right)=$$

$$=\cos\left(\frac{\pi}{2}+\alpha\right)-2\sin\left(-\pi-\frac{\pi}{6}\right)-\tan\alpha+\tan\left(2\pi+\frac{\pi}{4}\right)=$$

$$=-\sin\alpha-2\sin\left(\frac{\pi}{6}\right)-\tan\alpha+\tan\left(\frac{\pi}{4}\right)=$$

$$=-\sin\alpha-2\times\frac{1}{2}-\tan\alpha+1=$$

$$=-\sin\alpha-\tan\alpha$$

Recorrendo à fórmula fundamental da trigonometria, tem-se:

$$\sin^2\alpha+\cos^2\alpha=1\Leftrightarrow \sin^2\alpha=1-\frac{1}{16}\Leftrightarrow \sin\alpha=\pm\frac{\sqrt{15}}{4}$$

Como $\alpha\in\left]\frac{3\pi}{2},\ 2\pi\right[$, ou seja, $\alpha\in 4.^\text{o}$ Q, então $\sin\alpha=-\frac{\sqrt{15}}{4}$

$\tan\alpha=\frac{-\frac{\sqrt{15}}{4}}{\frac{1}{4}}=-\sqrt{15}$, tendo-se: $-\sin\alpha-\tan\alpha=-\left(-\frac{\sqrt{15}}{4}\right)-\left(-\sqrt{15}\right)=\frac{5\sqrt{15}}{4}$

**7.** Opção **(B)**

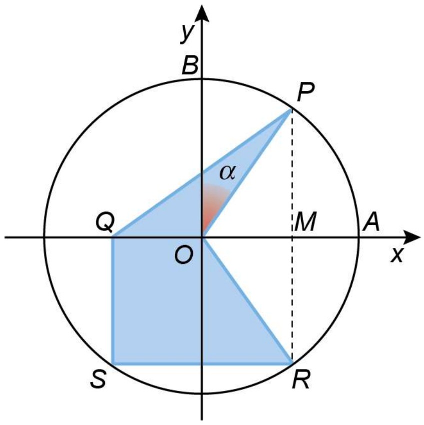
Uma vez que $\alpha$ e $\beta$ são dois ângulos de amplitudes pertencentes ao intervalo $\left]-3\pi, -\frac{5\pi}{2}\right[$ conclui-se que os lados extremidade pertencem ao 3.º quadrante.

$\alpha > \beta$ e o seno é decrescente no 3.º quadrante. Então $\sin\alpha < \sin\beta$.

**8.** $\left(1+\tan^2 x\right)\left(1+\sin x\right)+\frac{1}{1-\sin x}=\frac{1}{\cos^2 x}\left(1+\sin x\right)+\frac{1}{1-\sin x}=$

$$=\frac{\frac{1}{\cos^2 x}\left(1+\sin x\right)\left(1-\sin x\right)+1}{1-\sin x}=\frac{\frac{1}{\cos^2 x}\left(1-\sin^2 x\right)+1}{1-\sin x}=\frac{\frac{\cos^2 x}{\cos^2 x}+1}{1-\sin x}=\boxed{\frac{2}{1-\sin x}}$$

**9.1.** $A\hat{O}P=\frac{\pi}{2}-\alpha$




$P\left(2\cos\left(\frac{\pi}{2}-\alpha\right), 2\sin\left(\frac{\pi}{2}-\alpha\right)\right)$; ou seja,

$\overline{OM}=2\sin\alpha=\overline{QO}$

$\overline{MP}=2\cos\alpha$

$A_{\text{triângulo}}=\frac{\overline{QO}\times\overline{MP}}{2}=\frac{2\sin\alpha\times 2\cos\alpha}{2}=2\sin\alpha\cos\alpha$

$\overline{MR}=\overline{MP}=\overline{QS}=2\cos\alpha$

$\overline{SR}=\overline{QO}+\overline{OM}=2\times 2\sin\alpha=4\sin\alpha$

$A_{\text{trapézio}}=\frac{\overline{SR}+\overline{QO}}{2}\times\overline{QS}=\frac{4\sin\alpha+2\sin\alpha}{2}\times 2\cos\alpha=6\sin\alpha\cos\alpha$

$A_{\text{sombreada}}=A_{\text{triângulo}}+A_{\text{trapézio}}=2\sin\alpha\cos\alpha+6\sin\alpha\cos\alpha=\boxed{8\sin\alpha\cos\alpha}$

**9.2.** $\tan\left(\alpha+3\pi\right)=\frac{7}{5}\Leftrightarrow\tan\alpha=\frac{7}{5}$

$1+\tan^2\alpha=\frac{1}{\cos^2\alpha}\Leftrightarrow 1+\left(\frac{7}{5}\right)^2=\frac{1}{\cos^2\alpha}\Leftrightarrow\cos^2\alpha=\frac{25}{74}\Leftrightarrow\cos\alpha=\pm\frac{5\sqrt{74}}{74}$

Como $\alpha\in 1.^{\text{o}}$ Q, então $\cos\alpha=\frac{5\sqrt{74}}{74}$

$\sin^2\alpha+\cos^2\alpha=1\Leftrightarrow\sin^2\alpha=1-\frac{25}{74}\Leftrightarrow\sin\alpha=\pm\frac{7\sqrt{74}}{74}$

Como $\alpha\in 1.^{\text{o}}$ Q, então $\sin\alpha=\frac{7\sqrt{74}}{74}$

Assim, a área é dada por: $A=8\times\frac{5\sqrt{74}}{74}\times\frac{7\sqrt{74}}{74}=\frac{140}{37}$

**9.3.** $A_{\text{círculo}} = \pi \times r^2 = 4\pi$

$f(\alpha) = \frac{4\pi}{4} \Leftrightarrow 8\sin\alpha\cos\alpha = \pi$


$\alpha \approx 0{,}45$