TESTE N.º 1 – Proposta de resolução

1. Seja *C'* a projeção ortogonal de *C* sobre [*AB*].

$\cos 45° = \dfrac{\overline{C'B}}{2} \Leftrightarrow \overline{C'B} = 2 \times \dfrac{\sqrt{2}}{2} \Leftrightarrow \overline{C'B} = \sqrt{2}$

$\operatorname{sen} 45° = \dfrac{\overline{CC'}}{2} \Leftrightarrow \overline{CC'} = 2 \times \dfrac{\sqrt{2}}{2} \Leftrightarrow \overline{CC'} = \sqrt{2}$

$\operatorname{tg} 30° = \dfrac{\sqrt{2}}{\overline{AC'}} \Leftrightarrow \overline{AC'} = \dfrac{\sqrt{2}}{\frac{\sqrt{3}}{3}} \Leftrightarrow \overline{AC'} = \dfrac{3\sqrt{2}}{\sqrt{3}}$

$\Leftrightarrow \overline{AC'} = \frac{3\sqrt{6}}{3}$

$\Leftrightarrow \overline{AC'} = \sqrt{6}$

$\overline{AB} = \overline{AC'} + \overline{C'B} = \sqrt{6} + \sqrt{2}$

2. $A\hat{C}B = 90°$, pois o triângulo [*ABC*] está inscrito na circunferência de diâmetro [*AB*].

$\operatorname{sen} \alpha = \dfrac{\overline{CB}}{6} \Leftrightarrow \overline{CB} = 6\operatorname{sen} \alpha$

$\cos \alpha = \dfrac{\overline{AC}}{6} \Leftrightarrow \overline{AC} = 6\cos \alpha$

$A_{[ABC]} = \dfrac{\overline{AC} \times \overline{BC}}{2} = \dfrac{6\cos\alpha \times 6\operatorname{sen}\alpha}{2} = 18\operatorname{sen}\alpha\cos\alpha$

$A_{\text{semicircunferência}} = \dfrac{1}{2} \times \pi \times r^2 = \dfrac{1}{2} \times \pi \times 9 = \dfrac{9}{2}\pi$

Logo, a área da região a sombreado é igual a $\frac{9}{2}\pi - 18\operatorname{sen}\alpha\cos\alpha$.

3. Para $x \in \left]\frac{\pi}{2}, \pi\right[, 0 < \operatorname{sen} x < 1$

$0 < k^2 + 2k + 1 < 1$

$\Leftrightarrow k^2 + 2k + 1 < 1 \ \wedge \ k^2 + 2k + 1 > 0$

$\Leftrightarrow k^2 + 2k < 0 \ \wedge \ k^2 + 2k + 1 > 0$

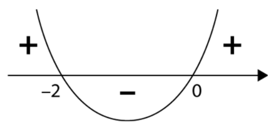
$\Leftrightarrow -2 < k < 0 \ \wedge \ (k > -1 \vee k < -1)$

$\Leftrightarrow -2 < k < -1 \ \vee \ -1 < k < 0$

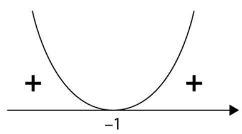
C.S. = $]-2, -1[\cup]-1, 0[$

Cálculos auxiliares

- $k^2 + 2k = 0 \Leftrightarrow k(k+2) = 0$
  $\Leftrightarrow k = 0 \ \vee \ k = -2$



- $k^2 + 2k + 1 = 0 \Leftrightarrow (k+1)^2 = 0 \Leftrightarrow k = -1$



4. Opção (D)

$\alpha \in 3.º\text{ Q}$ e $\beta \in 2.º\text{ Q}$

- $\text{sen}\,\alpha < 0$ e $\cos\beta < 0$, logo $\text{sen}\,\alpha \times \cos\beta > 0$
- $\text{tg}\,\alpha > 0$ e $\cos\beta < 0$, logo $\text{tg}\,\alpha \times \cos\beta < 0$
- $\cos\alpha < 0$ e $\text{tg}\,\beta < 0$, logo $\cos\alpha + \text{tg}\,\beta < 0$
- $\text{sen}\,\alpha < 0$ e $\text{sen}\,\beta > 0$, logo $\text{sen}\,\alpha - \text{sen}\,\beta < 0$

5. $\cos^4 x + \text{sen}^2 x \cos^2 x + \text{sen}^2 x + \text{tg}^2 x = \cos^2 x \left(\underbrace{\cos^2 x + \text{sen}^2 x}_{1}\right) + \text{sen}^2 x + \text{tg}^2 x =$

$= \cos^2 x + \text{sen}^2 x + \text{tg}^2 x =$

$= 1 + \text{tg}^2 x =$

$= \frac{1}{\cos^2 x}$ c.q.d.

6. Opção (B)

$\alpha + \beta = \frac{3\pi}{2} \Leftrightarrow \beta = \frac{3\pi}{2} - \alpha$

$\alpha + \gamma = 2022\pi \Leftrightarrow \gamma = 2022\pi - \alpha$

$-\cos\gamma - \text{sen}\,\beta + \cos\alpha = -\cos(2022\pi - \alpha) - \text{sen}\left(\frac{3\pi}{2} - \alpha\right) + \cos\alpha =$

$= -\cos\alpha + \cos\alpha + \cos\alpha =$

$= \cos\alpha$

7. $\text{sen}\,\beta\,\text{tg}\,\beta = \frac{9}{20} \Leftrightarrow \frac{\text{sen}^2\beta}{\cos\beta} = \frac{9}{20} \Leftrightarrow 1 - \cos^2\beta = \frac{9}{20}\cos\beta$

$\Leftrightarrow 20 - 20\cos^2\beta = 9\cos\beta$

$\Leftrightarrow 20\cos^2\beta + 9\cos\beta - 20 = 0$

$\Leftrightarrow \cos\beta = \frac{-9 \pm \sqrt{81 - 4 \times 20 \times (-20)}}{40}$

$\Leftrightarrow \cos\beta = \frac{-9 \pm 41}{40}$

$\Leftrightarrow \cos\beta = -\frac{5}{4} \ \vee\ \cos\beta = \frac{4}{5}$

Como $-1 \leq \cos\beta \leq 1, \forall\, \beta \in \text{IR}$, então $\cos\beta = \frac{4}{5}$.

8. Opção (C)

$A(x) = \cos^2\left(-\frac{3\pi}{2} + x\right) + \text{tg}(-2023\pi + x) \times \text{sen}\left(x + \frac{3\pi}{2}\right) + \cos^2(2023\pi + x) =$

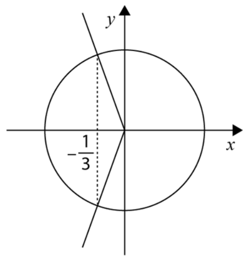
$= (-\text{sen}\,x)^2 + \text{tg}\,x \times (-\cos x) + (-\cos x)^2 =$

$= \text{sen}^2 x + \frac{\text{sen}\, x}{\cos x} \times (-\cos x) + \cos^2 x =$

$= \text{sen}^2 x + \cos^2 x - \text{sen}\, x =$

$= 1 - \text{sen}\, x$

9. Opção (A)



$3\cos x = -1 \Leftrightarrow \cos x = -\frac{1}{3}$

Em $\left]-\frac{3\pi}{2}, 0\right[$ a equação tem 2 soluções.

Em $[0, 4\pi[$ a equação tem 4 soluções.

10.

10.1. $A(\underbrace{\cos\alpha}_{-}, \underbrace{\text{sen}\,\alpha}_{+})$ $\quad B(\cos\alpha, -\text{sen}\,\alpha)$ $\quad D\left(1, \text{tg}\left(\alpha - \frac{\pi}{2}\right)\right)$

$$\text{tg}\left(\alpha - \frac{\pi}{2}\right) = \frac{\text{sen}\left(\alpha - \frac{\pi}{2}\right)}{\cos\left(\alpha - \frac{\pi}{2}\right)} = \frac{-\cos\alpha}{\text{sen}\,\alpha} = -\frac{1}{\text{tg}\,\alpha}$$

$D\left(1, -\underbrace{\frac{1}{\text{tg}\,\alpha}}_{+}\right)$ $\quad C\left(1, \frac{1}{\text{tg}\,\alpha}\right)$

$A_{[ABCD]} = \frac{\overline{DC} + \overline{BA}}{2} \times h = \frac{-\frac{2}{\text{tg}\,\alpha} + 2\text{sen}\,\alpha}{2} \times (1 - \cos\alpha) =$

$= \frac{-\frac{2\cos\alpha}{\text{sen}\,\alpha} + 2\text{sen}\,\alpha}{2} \times (1 - \cos\alpha) =$

$= \left(-\frac{\cos\alpha}{\text{sen}\,\alpha} + \text{sen}\,\alpha\right) \times (1 - \cos\alpha) =$

$= \frac{-\cos\alpha + \text{sen}^2\alpha}{\text{sen}\,\alpha} \times (1 - \cos\alpha) =$

$= \frac{-\cos\alpha + 1 - \cos^2\alpha}{\text{sen}\,\alpha} \times (1 - \cos\alpha) =$

$= \frac{-\cos^2\alpha - \cos\alpha + 1}{\text{sen}\,\alpha} \times (1 - \cos\alpha)$ c.q.d

10.2. $\cos\left(\frac{\pi}{2} + \alpha\right) = -\frac{1}{3} \Leftrightarrow -\text{sen}\,\alpha = -\frac{1}{3} \Leftrightarrow \text{sen}\,\alpha = \frac{1}{3}$

$\text{sen}^2\alpha + \cos^2\alpha = 1 \Leftrightarrow \frac{1}{9} + \cos^2\alpha = 1$

$\Leftrightarrow \cos^2\alpha = \frac{8}{9}$

$\Leftrightarrow \cos\alpha = \frac{\pm 2\sqrt{2}}{3}$

Como $\alpha \in 2.°$ Q, então $\cos\alpha = -\frac{2\sqrt{2}}{3}$.

Logo, a área do trapézio é igual a:

$$\frac{-\frac{8}{9}+\frac{2\sqrt{2}}{3}+1}{\frac{1}{3}} \times \left(1+\frac{2\sqrt{2}}{3}\right) = \left(\frac{1}{3}+2\sqrt{2}\right)\left(1+\frac{2\sqrt{2}}{3}\right) = \frac{1}{3}+\frac{2\sqrt{2}}{9}+2\sqrt{2}+\frac{8}{3} = 3+\frac{20\sqrt{2}}{9}$$

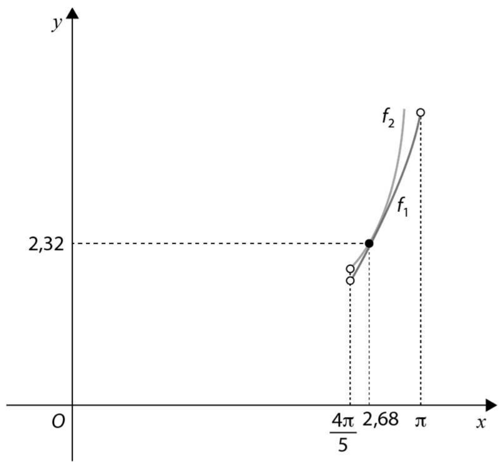
10.3. $A\left(\beta-\frac{\pi}{6}\right)=\frac{1}{2}A(\beta), \quad \frac{4\pi}{5}<\beta<\pi$

Utilizando a letra x como variável independente: $A\left(x-\frac{\pi}{6}\right)=\frac{1}{2}A(x)$

Utilizando as capacidades gráficas da calculadora:

$$f_1(x)=\frac{-\cos^2\left(x-\frac{\pi}{6}\right)-\cos\left(x-\frac{\pi}{6}\right)+1}{\operatorname{sen}\left(x-\frac{\pi}{6}\right)}\times\left(1-\cos\left(x-\frac{\pi}{6}\right)\right)$$

$$f_2(x)=\frac{-\cos^2 x-\cos x+1}{\operatorname{sen} x}(1-\cos x)\times\frac{1}{2}$$



$\beta \approx 2{,}68$

11. Opção (C)

$$\begin{cases}\alpha r=\frac{4\pi}{7}\\ \frac{\alpha r^2}{2}=\frac{8\pi}{7}\end{cases} \Leftrightarrow \begin{cases}\alpha r=\frac{4\pi}{7}\\ \alpha r^2=\frac{16\pi}{7}\end{cases} \Leftrightarrow \begin{cases}\alpha r=\frac{4\pi}{7}\\ \alpha r\times r=\frac{16\pi}{7}\end{cases}$$

$$\Leftrightarrow \begin{cases}\alpha r=\frac{4\pi}{7}\\ \frac{4\pi}{7}r=\frac{16\pi}{7}\end{cases}$$

$$\Leftrightarrow \begin{cases}\alpha r=\frac{4\pi}{7}\\ r=4\end{cases}$$