ISCTE
BUSINESS SCHOOL

Management

Operations Research

Exam 3rd year / 1st Semester 2009 / 2010

**Data:** Tuesday, 19 of January of 2010 **Duration:** 2h30 m + 30m.

**Name:** _____

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**Instructions:**
1 – Answer all questions justifying your answers;
2 – The maximum grade (mark) of each question is indicated between brackets.

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**1.** Consider the following list of activities and their predecessors:

| Activity | A | B | C | D | E | F | G | H | I | J | K | L |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Predecessors | -- | -- | A | B | C | D | C,D | E | E | F,I | J,H | F,I |

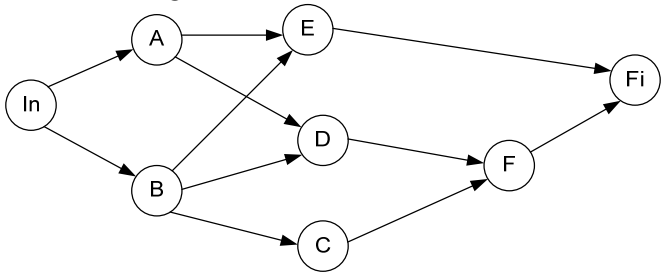
Draw the AON network.

*[1.25 in 20.0]*

**2.** Consider the following project

| Activity | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| Optimistic duration (days) | 2 | 1 | 1 | 2 | 2 | 1 |
| Most likely duration (days) | 3 | 3 | 2 | 4 | 4 | 1 |
| Pessimistic duration (days) | 4 | 5 | 9 | 6 | 12 | 1 |

represented in the following AON network:

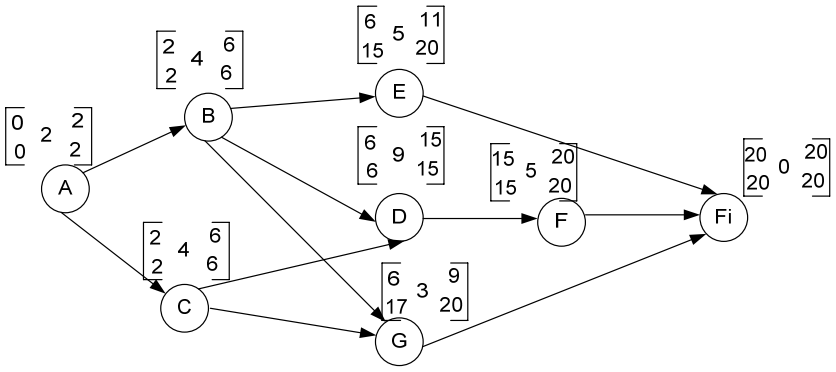



***a)*** Find the mean duration of the project. Justify. *[1.25 in 20.0]*

**b)** Find the critical activities and identify the critical path(s). *[1.25 in 20.0]*

**c)** Let T be the random variable that represents the duration of the project. It is known that T follows a normal distribution with mean $\mu_T$ and standard deviation $\sigma_T$. Calculate $\sigma_T$. *[1.25 in 20.0]*

**3.** Consider the following AON network of a project. and the results obtained by CPM (activity durations are measured in days)

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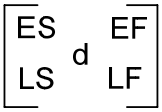
Legend:




The amount of resources needed, per day, to complete each activity are:

| Activity | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| Resources ( per day) | 5 | 3 | 6 | 4 | 3 | 8 | 4 |

**a)** What will be the consequences on the minimum duration of the project, on the critical path and on the critical activities if the activities D and G must start two days after activity B is finished? *[1.0 in 20.0]*

**b)** Find the independent float of activity G and explain what it means. *[1.0 in 20.0]*

**c)** Suppose there are available 8 units of resource, per day. Apply the SGS- Parallel heuristic to find a resource-feasible and predecessor-feasible schedule. Use as

priority rule the maximum amount of resources needed, and to break ties the maximum duration. Do not forget to write down the schedule and the new completion time for the project. *[2.0 in 20.0]*

**4.** Consider the following output from EVIEWS. Calculate the forecasted values for time periods 28 and 29. *[1.0 in 20.0]*

Sample: 1 24
Included observations: 24
Method: Holt-Winters Multiplicative Seasonal
Original Series: VENDAS
Forecast Series: VENDASSM

| | | | |
|---|---|---|---|
| Parameters: | Alpha | | 0.6299 |
| | Beta | | 0.0000 |
| | Gamma | | 0.0000 |
| Sum of Squared Residuals | | | 9912.767 |
| Root Mean Squared Error | | | 20.32319 |
| End of Period Levels: | Mean | | 756.5764 |
| | Trend | | 16.83750 |
| | Seasonals: | 21 | 0.965960 |
| | | 22 | 1.022075 |
| | | 23 | 1.139743 |
| | | 24 | 0.872222 |

**Holt-Winters' Method for multiplicative seasonality**

$$S_t = \alpha\ \left(y_t / I_{t-L}\right) + (1-\alpha)\ \left(S_{t-1} + T_{t-1}\right),\ 0 < \alpha < 1;$$

$$T_t = \beta\ \left(S_t - S_{t-1}\right) + (1-\beta)\ T_{t-1},\ 0 < \beta < 1;$$

$$I_t = \gamma\ \left(y_t / S_t\right) + (1-\gamma)\ I_{t-L},\ 0 < \gamma < 1;$$

$$\hat{y}_{t+h} = \left(S_t + T_t\ h\right)\ I_{t+h-kL},\ h=1,2,\ldots,\ t=1,2,\ldots,T,\ k = \begin{cases} 1, \text{if } 0 < h \le L \\ 2, \text{if } L < h \le 2L \\ \ldots \end{cases}$$

**5.** An investor has 60 000 Euros and intends to do an investment plan for the next three years. He can invest in three money-making activities, I1, I2, and I3. Each Euro invested in I1 returns a profit of 7 cents two years later; each Euro invested in I2 returns a profit of 9 cents three years later; each Euro invested in I3 at the beginning returns a profit of 4 cents one year later.
Consider that the investor wants to guarantee the following conditions:
- the amount invested in activity I1 cannot be less than the amount invested in I2;
- at least 10 000 Euros must be invested in activity I3.
Consider also that the profits may be reinvested.

**a)** Which investment plan maximizes the capital that can be accumulated at the end of the third year? Formulate the problem in Linear Programming. *[2.0 in 20.0]*

**b)** Consider the following additional conditions:
- At the beginning of the first year, an investment in activity I1 unables an investment in I3;
- an investment in I2 obliges to an extra payment of 100 euros.

Reformulate the problem. Justify. *[1.5 in 20.0]*

**6.** The following Linear Programming model concerns a weekly production planning problem. The decision variables $x_i$ represent the level of production, in thousands of litres, of oil of type i, i=1 (oil of peanuts), i=2 (sunflower-seed oil), i=3 (corn oil). The first constraint is concerned with the maximum demand of oils whereas the second one is related with the weekly machine-hours available for production. The last constraint concerns contracts signed up with clients. The objective is to maximize the weekly profit.

$$\text{Max } Z = 100\,x_1 + 50\,x_2 + 60\,x_3$$
$$\text{s.to} \quad x_1 + x_2 + x_3 \leq 500$$
$$0.3\,x_1 + 0.2\,x_2 + 0.1\,x_3 \leq 100$$
$$x_2 \geq 100$$
$$x_1, x_2, x_3 \geq 0$$

The following results were obtained using Solver of EXCEL:

| Name | Original Value | Final Value |
|---|---|---|
| Obj.F. | 0 | 37000 |

Adjustable Cells

| Name | Final Value | Reduced Cost | Objective Coefficient | Allowable Increase | Allowable Decrease |
|---|---|---|---|---|---|
| X1 | 200 | 0 | 100 | 80 | 40 |
| X2 | 100 | 0 | 50 | 30 | 1E+30 |
| X3 | 200 | 0 | 60 | 40 | 26,66666667 |

Constraints

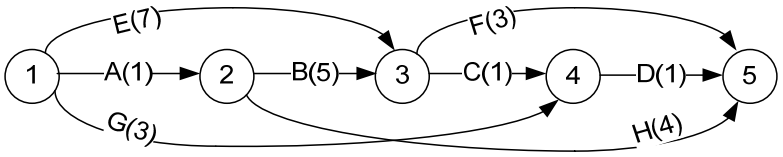
| Name | Final Value | Shadow Price | Constraint R.H. Side | Allowable Increase | Allowable Decrease |
|---|---|---|---|---|---|
| Const .1 | 500 | 40 | 500 | 400 | 133,3333333 |
| Const. 2 | 100 | 200 | 100 | 40 | 40 |
| Const. 3 | 100 | -30 | 100 | 400 | 100 |

**a)** Write a report describing, in detail, the solution obtained, the total profit, and analyzing the constraints. Justify your answer *[1.0 in 20.0]*

**b)** Suppose the unit profit of the corn oil will increase by 10 c.u.. What will be the new optimal plan and by how much the weekly profit will change? *[1.0 in 20.0]*

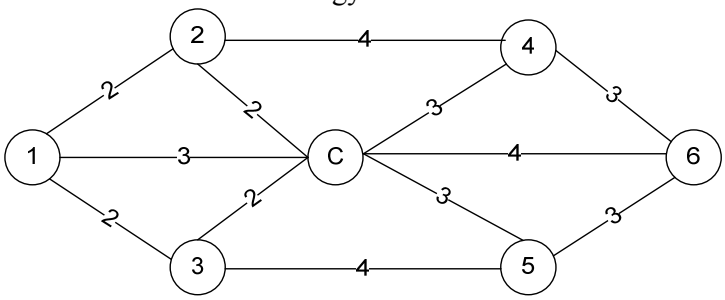
**c)** Suppose the company was asked to review the contracts signed up with the clients. More precisely, the clients want to decrease their weekly purchase of sunflower-seed oil to 80 thousand of litres. Do you think the company should accept it? Justify. *[1.0 in 20.0]*

**7.** The following network shows the possible schemes to produce a certain product P. Node 1 represents the beginning of the production whereas node 5 represents the end of the production. Each path between nodes 1 and 5 represents a way to produce one unit of P (for instance, the sequence of activities E, C and D gives a possible way to produce one unit of P). Between brackets, next to each activity, is the corresponding unit cost.




Find the sequence that minimizes the total production cost of one unit of product P. Don't forget to identify the problem and to mention the name of the algorithm used to solve it. *[1.5 in 20.0]*

**8.** An electricity distribution company needs to project an electricity distribution network for 6 cities. In the following network, node C represents the power station whereas the other nodes represent the cities. The values indicated close to the links represent the cost of installing of one unit of capacity on the link. Suppose each city has to receive 10 units of electric energy.




In order to minimize the total cost of installation describe how the problem can be solved (you don't have to solve it). *[2.0 in 20.0]*