Prueba Modelo 1

1

$$P(\overline{A}) = 0{,}75 \iff P(A) = 1 - 0{,}75$$
$$= 0{,}25$$

$$P(\overline{B}) = 0{,}55 \iff P(B) = 0{,}45$$

$$P(\overline{A} \cap B) = 0{,}3 \iff P(B) - P(A \cap B) = 0{,}3$$
$$P(A \cap B) = 0{,}15$$

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{0{,}15}{0{,}25}$$
$$= 0{,}6$$
$$= \frac{6}{10} = \frac{3}{5}$$

— (A) —

(2)

$$(n+2)! = k$$

$$(n+1)! = p$$

$$\boxed{n=2}$$

$k = 24$

$p = 6$

$$\frac{k-2p}{p}$$

$$\frac{24-12}{6}$$

$$\frac{12}{6} = 2$$

$$(n+2) \times (n+1)! = k$$

$$(n+2) \times p = k$$

$$n+2 = \frac{k}{p}$$

$$n = \frac{k}{p} - 2 = \frac{k-2p}{p}$$

(C)

③ f é contínua em $x=0$ se

$0 \in D_f$

$f(0) = \lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$

$f(0) = e^0(0+1) = 1$

$\lim_{x \to 0^-} \left(e^x(x+1)\right) = 1$

$\lim_{x \to 0^+} \left(x - \frac{\ln(x)}{x}\right) = 0 - \frac{\ln(0^+)}{0^+}$

$= +\infty$

Como $\lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x)$

f não é contínua em $x=0$

Toda a função não contínua, não tem $f'(0)$.

3.2.

Visto che f è continua in $\mathbb{R}\setminus\{0\}$ e $\lim_{x\to 0^+} f(x) = +\infty$

allora $x = 0$ è a unica

A.V. unilaterale.

$\boxed{x \to -\infty}$

$$m = \lim_{x\to -\infty} \frac{f(x)}{x}$$

$$= \lim_{x\to -\infty} \frac{e^x(x+1)}{x} = \frac{\infty}{\infty}$$

$$= \lim_{x\to -\infty} \frac{e^x \cancel{x}\left(1+\frac{1}{x}\right)}{\cancel{x}}$$

$$= e^{-\infty}\left(1+\frac{1}{-\infty}\right) = 0$$

$$L = \lim_{x \to -\infty} \left(f(x) - \underbrace{a}_{0}\, x \right)$$

$$= \lim_{x \to -\infty} e^{x}(x+1)$$

$$= 0 \times (-\infty) \quad \text{Ind.}$$

T.C.V.

$x \to -\infty$ | $-x = y$

$-x \to +\infty$ | $x = -y$

$y \to +\infty$

$$= \lim_{y \to +\infty} e^{-y}(-y+1)$$

$$= \lim_{y \to +\infty} \frac{\cancel{-y+1}}{e^{y}} = \frac{\infty}{\infty}$$

$$= \frac{-1+0}{+\infty} \quad \left(\text{L. Natural } \lim_{x \to +\infty} \frac{e^x}{x^p} = +\infty \right.$$

$$= 0$$

$y = 0$ A.H ; $x \to -\infty$

$\boxed{x \to +\infty}$

$$M = \lim_{x \to +\infty} \left(\frac{x}{x} - \frac{\frac{\ln(x)}{x}}{x} \right) = \frac{\infty}{\infty} \text{ Ind}$$

C.N

$$= 1 - \frac{0}{+\infty}$$

$$= 1$$

$$S = \lim_{x \to +\infty} \left(f(x) - 1x \right)$$

$$= \lim_{x \to +\infty} \left(\cancel{x} - \boxed{\frac{\ln(x)}{x}} - \cancel{1x} \right)$$

$$= \frac{\infty}{\infty}$$

L.N.

$$= -0$$

$$= 0$$

$$y = 1x + 0$$

$$y = x$$

A. obliquo

$x \to +\infty$

3.3.

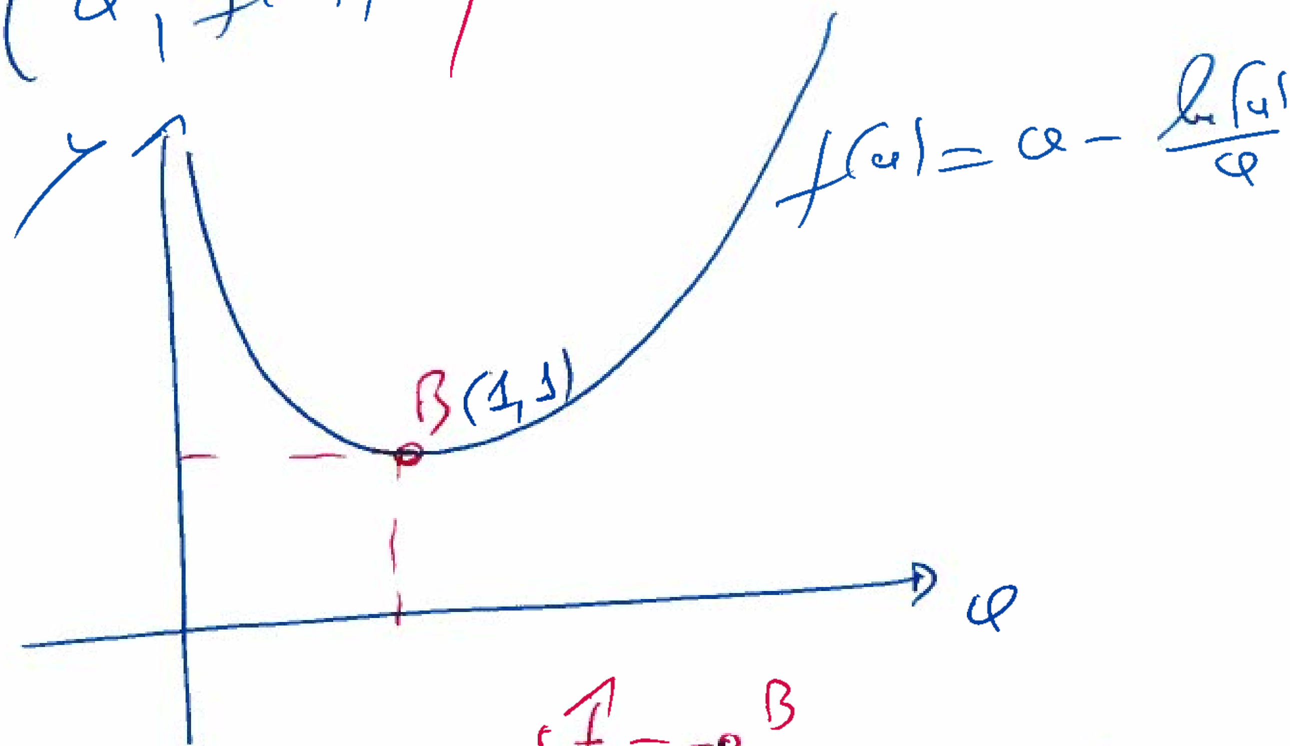
$$f(x) = 0$$

$$x \leq 0 \quad \wedge \quad e^{x}(x+1) = 0$$

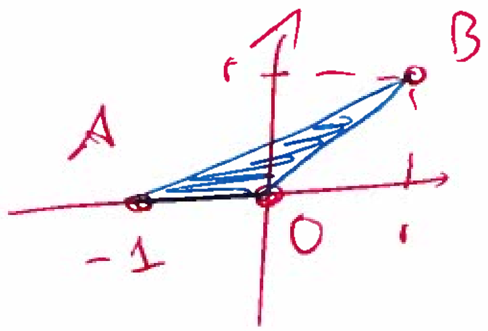
$$e^{x} = 0 \quad \vee \quad x = -1$$

$A(-1, 0)$

$B(x, f(x))$; $x > 0$



$A = ?$

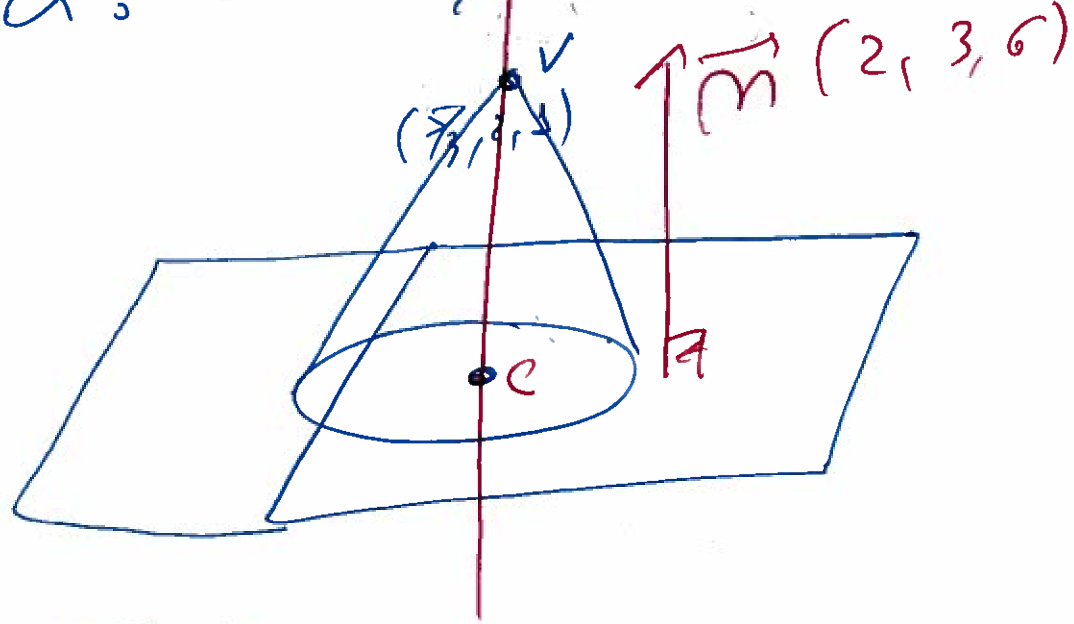


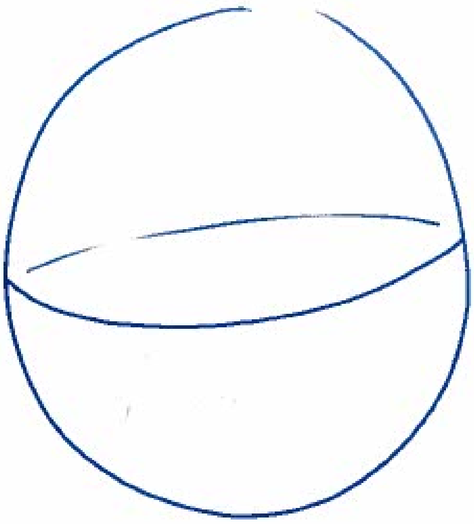
$$A = \frac{1 \times 1}{2}$$

$$A = \frac{1}{2} \text{ u.a.}$$

(4)

$\alpha: 2x + 3y + 6z + 1 = 0$





$C(1, 1, -1)$

$r = \sqrt{1} = 1$

$\vec{m} \parallel \vec{VC}$ bp

$$(x, y, z) = \underbrace{\left(\frac{5}{3}, 2, 1\right)}_{V} + k\overbrace{(2, 3, 6)}^{\vec{m}}; \quad k \in \mathbb{R}$$

4.2

$$\begin{cases} x = \boxed{\frac{5}{3} + 2k} \\ y = \boxed{2 + 3k} \\ z = \boxed{1 + 6k} \\ 2x + 3y + 6z + 1 = 0 \end{cases}$$

$$\begin{cases} \text{—} \\ \text{—} \\ \text{—} \\ \frac{10}{6} + 4k + 6 + 9k + 6 + 36k + 1 = 0 \end{cases}$$

$$\begin{cases} \text{—} \\ \text{—} \\ \text{—} \\ 49k = -13 - \frac{10}{6} \end{cases} \qquad \begin{cases} \text{—} \\ \text{—} \\ \text{—} \\ 49k = \frac{-88}{6} \end{cases}$$

$$\begin{cases} \text{—} \\ \text{—} \\ \text{—} \\ k = -\frac{88}{6 \times 49} \end{cases} \qquad \begin{cases} x = 1 \\ y = 1 \\ z = -1 \end{cases}$$

4.3.

Raio = 1

$alt = \overline{VC} = \sqrt{(x_c - x_v)^2 + (y_c - y_v)^2 + (z_c - z_v)^2}$

ou $\overrightarrow{VC} = (1, 1, -1) - (\frac{5}{3}, 2, 1)$

$= (-\frac{2}{3}, -1, -2)$

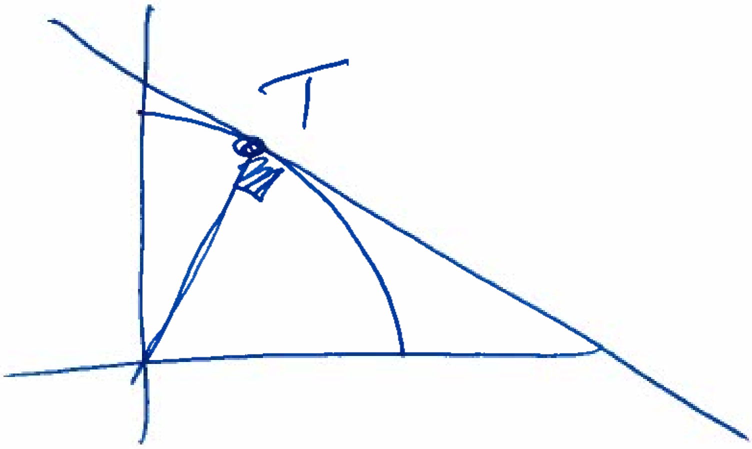
$a = \|\overrightarrow{VC}\| = \sqrt{\frac{4}{9} + 1 + 4}$

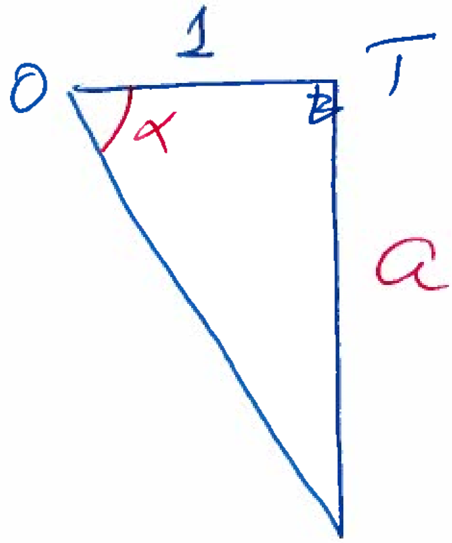
$= \sqrt{\frac{49}{9}} = \frac{7}{3}$

$V = \dfrac{\pi \times 1^2 \times \frac{7}{3}}{3} = \dfrac{7\pi}{9}$ u.v.

5

$r = 1$





$$A = \frac{1 \times a}{2}$$

$$Tg(\alpha) = a$$

$$A(\alpha) = \frac{Tg(\alpha)}{2}$$

—— (1) ——

⑥ $$\lim_{x \to 0} \frac{\frac{\cancel{Kx}\left(e^{Kx}-1\right)}{Kx}}{2\cancel{x}} = \frac{0}{0}$$

L.N.

$$= \frac{K \times 1}{2}$$

$$\lim_{x \to 0} \frac{e^{\cancel{Kx}}-1}{\cancel{Kx}} = 1$$

$Kx \to 0$

$y \to 0$

$$= \frac{K}{2} = 3$$

$K = 6$

$$\lim_{y \to 0} \frac{e^{y}-1}{y} = 1$$

$D_g = \{x \in \mathbb{R} : K - x > 0\}$

$6 - x > 0$

$-x > -6$

$x < 6$

(B)

7

$\underline{3\ 2\ 8\ 2\ 5\ 7}$

$$\boxed{\frac{6!}{2!}} \rightarrow \text{porque existem } \underline{2 \text{ alg}}$$

Ou

$$\underbrace{{}^{6 \text{ lug.}}C_2}_{\text{Alg } 2} \times \underbrace{4!}_{4 \text{ alg } \neq}$$

Ou

$$\underbrace{{}^{6 \text{ lug.}}A_4}_{\substack{4 \text{ alg} \\ \neq}} \times \underbrace{{}^{2}C_2}_{\substack{2 \text{ alg} \\ =}}$$

⑧ 7 jovens c/ 7 G.C. F

A_g $\boxed{VLA}$ ⊖ ⊖ ⊖ ou $\boxed{VLA}$ ... A_g

2 extremos

$$\boxed{A_1 \times 3! \times 4!}$$

8.2.

N° casos possíveis $= 7!$

N° casos favoráveis $= {}^3C_2 \times 2! \times 5!$

Por aplicação da Regra de Laplace

$$P = \frac{{}^3C_2 \times 2! \times 5!}{7!} = \ldots$$

9

$AV: \frac{3-x}{6} = \frac{z-4}{8} \wedge y=0$

$A \in$ eixo xx $(x>0)$

$A(x,0,0) \in$ ret. AV

$\frac{3-x}{6_{(4)}} = \frac{0-4}{8_{(3)}} \wedge \quad 0=0$

$12-4x=-12$

$-4x=-24$

$x=6$

$A(6,0,0)$

$B(0,6,0)$

$C(-6,0,0)$

$D(0,-6,0)$

$V \in$ eixo zz

$V(0,0,z)$

$\frac{3-0}{6_{(4)}} = \frac{z-4}{8_{(3)}}$

$12=3z-12$

$3z=24$

$z=8$

$V(0,0,8)$

9.2.

$z = 2$ intersecta a reta AV

$$\frac{3-x}{6}_{(4)} = \frac{2-4}{8}_{(3)} \wedge y = 0$$

$$12 - 4x = -6$$

$$-4x = -18$$

$$x = \frac{9}{2}$$

$B(0,6,0)$ $V(0,0,8)$

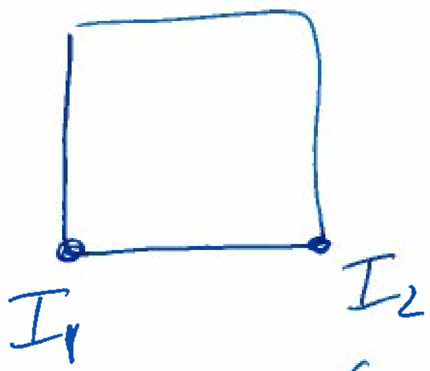
BV:

$$\overrightarrow{BV} = (0,-6,8)$$

$$(x,y,z) = (0,6,0) + k(0,-6,8)$$

$z = 2$

$$\begin{cases} x = 0 \\ y = 6 - 6k \\ z = 0 + 8k \end{cases} \qquad \begin{cases} y = 6 - \frac{6}{4} = \frac{9}{2} \\ k = \frac{1}{4} \end{cases}$$



$\left(\frac{9}{2}, 0, 2\right)$ $\left(0, \frac{9}{2}, 2\right)$

$$\overline{I_1 I_2} = l = \sqrt{\frac{81}{4} + \frac{81}{4} + 0}$$

$$= \sqrt{\frac{2 \times 81}{4}}$$

$$= \frac{\sqrt{2} \times 9}{2}$$

$$A = l^2 = \sqrt{\frac{\cancel{2} \times 81}{\cancel{2} \times 2}}$$

$$= \frac{81}{2} \quad \underline{u.a.}$$

(10) De $]-\infty, 0[$ e $]0, +\infty[$ f é contínua uma vez que está definida por operações entre f. contínuas.

em $x=0$?

$$f(0) = k+1$$

$$\lim_{x \to 0} \left(\frac{\cancel{x} \times 1}{\frac{e^{x}-1}{x} \cancel{x}} + k \right) = \frac{0}{0}$$

$$= \frac{1}{1 \text{ (L.N.)}} + k$$

$$= 1 + k$$

Como $0 \in D$ e

$$\lim_{x \to 0^+} f(x) = f(0)$$

então f é contínua em $x=0$

logo f é contínua em $\mathbb{R}$, qualquer que seja $k \in \mathbb{R}$

10.2. $\boxed{k=2}$

Não tem A.V. pois f é contínua em $\mathbb{R}$.

$\boxed{x \to -\infty}$

$$m = \lim_{x \to -\infty} \left(\frac{x}{\frac{e^x - 1}{x}} + \frac{2}{x} \right)$$

$$= \lim_{a \to -\infty} \left(\frac{\cancel{a} \times 1}{\cancel{a}(e^{a}-1)} + \frac{2}{a} \right)$$

$$= \frac{1}{0-1} + \frac{2}{-\infty} = -1$$

$$b = \lim_{a \to -\infty} \left(f(a) - a a \right)$$

$$= \lim_{a \to -\infty} \left(\frac{a}{e^{a}-1} + 2 + 1a \right)$$

$$= \lim_{a \to -\infty} \frac{\cancel{a} + 2e^{a} - 2 + ae^{a} - \cancel{a}}{e^{a}-1}$$

$a \longrightarrow -\infty$ | $-a = y$

$-a \longrightarrow +\infty$ | $a = -y$

$y \longrightarrow +\infty$

$$= \lim_{y \to +\infty} \frac{2e^{-y} - \frac{2}{1(e^y)} - y e^{-y}}{e^{-y} - 1}$$

$$= \lim_{y \to -\infty} \frac{\frac{2}{\cancel{e^y}} - \frac{2e^y}{\cancel{e^y}} - \frac{y}{\cancel{e^y}}}{\frac{1}{\cancel{e^y}} - \frac{e^y}{\cancel{e^y}}}$$

$$= \lim_{y \to -\infty} \frac{\cancel{e^y}\left(\frac{2}{e^y} - 2 - \frac{\cancel{y} \times \frac{1}{y}}{\frac{e^y}{y} \times \cancel{y}}\right)}{\cancel{e^y}\left(\frac{1}{e^y} - 1\right)}$$

$$= \frac{\frac{2}{+\infty} - 2 - \frac{1}{+\infty}}{\frac{1}{+\infty} - 1} = \frac{-2}{-1} = 2$$

$y = -1x + 2$ A. Obl. $x \to -\infty$

$\boxed{u \to +\infty}$

$$m = \lim_{u \to +\infty} \left(\frac{\frac{u}{e^u - 1}}{u} + \frac{2}{u} \right)$$

$$= \lim_{u \to +\infty} \frac{\cancel{u} \times 1}{\cancel{u}(e^u - 1)} + \frac{2}{u}$$

$$= \frac{1}{+\infty} + \frac{2}{+\infty}$$

$$= 0$$

$$b = \lim_{u \to +\infty} \left(\frac{\frac{u}{u}}{\frac{e^u}{u} - \frac{1}{u}} + 2 \cancel{- 0u} \right)$$

$$= \frac{1}{+\infty - 0} + 2 = 0 + 2 = 2$$

$y = 2$ A.H

(11)

$$P(B \cup \bar{A}) - P(\bar{B}|A) \times P(\bar{A}) = \ldots$$

$$P(B) + \boxed{P(\bar{A})} - P(B \cap \bar{A}) - \frac{P(\bar{B} \cap A)}{P(A)}(1 - P(A))$$

$$P(B) + 1 - P(A) - \left(P(B) - P(B \cap A)\right) - \frac{P(\bar{B} \cap A)}{P(A)} + \frac{P(\bar{B} \cap A)}{\cancel{P(A)}} \cancel{P(A)}$$

$$\cancel{P(B)} + \cancel{1} - \cancel{P(A)} - \cancel{P(B)} + \cancel{P(B \cap A)} - \cancel{\frac{P(A)}{P(A)}} + \frac{P(A \cap B)}{P(A)} + \cancel{P(A)} - \cancel{P(A \cap B)}$$

$$= \frac{P(A \cap B)}{P(A)} = \underline{\underline{P(B|A)}}$$

c.q.d.

(12)

$$f(a) = (k - a^2)\, e^{-\frac{a^2}{2}}$$

$$= \frac{k - a^2}{e^{\frac{a^2}{2}}}$$

$$m = \lim_{a \to +\infty} \frac{f(a)}{a}$$

$$= \lim_{a \to +\infty} \frac{k - a^2}{a\, e^{\frac{a^2}{2}}} = \frac{\infty}{\infty}$$

$$= \lim_{a \to +\infty} \frac{\cancel{a^2}\left(\frac{k}{a^2} - 1\right)}{\cancel{a^2}\, \frac{e^{\frac{a^2}{2}}}{a}}$$

$$= \frac{0 - 1}{+\infty} = 0$$

L.N.

$$b = \lim_{a\to+\infty}\left(f(a) - \cancel{0a}\right)$$

$$= \lim_{a\to+\infty} \frac{k-a^2}{e^{\frac{a^2}{2}}} = \frac{\infty}{\infty}$$

$$= \lim_{a\to+\infty} \frac{\cancel{a^2}\left(\frac{k}{a^2}-1\right)}{\cancel{a^2}\,\frac{e^{\frac{a^2}{2}}}{2\frac{a^2}{2}}}$$

$$= \frac{0-1}{+\infty} = 0$$

Para $-\infty$, el cálculo de un es so idénticos by a así, el igual

$$\cancel{y = 0a+0}$$

$$y = 0 \quad \text{A.H.}$$

$$\lim_{a\to+\infty} \frac{e^{\frac{a}{2}}}{a^{b}}$$

12.2.

$$f'(a) = (k - a^2) \times e^{-\frac{a^2}{2}}$$

$$f'(a) = \left(\frac{k - a^2}{e^{\frac{a^2}{2}}}\right)'$$

$$= \frac{-2a\, e^{\frac{a^2}{2}} - \left(\frac{a^2}{2}\right)' e^{\frac{a^2}{2}} (k - a^2)}{\left(e^{\frac{a^2}{2}}\right)^2}$$

$$= \frac{e^{\frac{a^2}{2}}\left(-2a - \frac{\cancel{2}a}{\cancel{2}}(k - a^2)\right)}{e^{\cancel{2} \times \frac{a^2}{\cancel{2}}}}$$

$$= \frac{e^{\frac{a^2}{2}} \times a\,(-2 - k + a^2)}{e^{a^2}}$$

$$f'(a)=0$$

$$e^{\frac{a^2}{2}}=0 \vee \boxed{a=0} \vee -2-k+a^2=0$$

$$a^2=2+k$$

$$a=\pm\sqrt{2+k}$$

Para que só tenha 1 único zero então

$$2+k<0$$

$$k<-2$$

$$k\in\,]-\infty,-2[$$

12.3.

Se os zeros de f são -1 e 1

$$f(a)=0$$

$$k-a^2=0 \quad \vee \quad e^{-\frac{a^2}{2}}=0$$

$$a=\pm\sqrt{k} \qquad \qquad \emptyset$$

logo $\underline{k=1}$

$$f'(a)=\frac{e^{\frac{a^2}{2}}\times a\times(-3+a^2)}{e^{a^2}}$$

$$\boxed{f'(a)=0}$$

$$e^{\frac{a^2}{2}}=0 \quad \vee \quad a=0 \quad \vee \quad a=\pm\sqrt{3}$$

$\times$

x	$-\infty$		$-\sqrt{3}$		0		$\sqrt{3}$		$+\infty$
f'		$-$	0	$+$	0	$-$	0	$+$	
f		$\searrow$	mín $f(-\sqrt{3})$	$\nearrow$	máx $f(0)$	$\searrow$	mín $f(\sqrt{3})$	$\nearrow$	

Os extremos de f têm abscissa $\{-\sqrt{3}, 0, \sqrt{3}\}$