

### 3.

Do enunciado sabe-se que:

- $\lim_{x \rightarrow \pm\infty} \frac{g(x)}{x} = 6 \Rightarrow \lim_{x \rightarrow \pm\infty} \frac{x}{g(x)} = \frac{1}{6}$
- $\lim_{x \rightarrow \pm\infty} (g(x) - 6x) = -2$
- $\lim_{x \rightarrow \pm\infty} g(x) = \lim_{x \rightarrow \pm\infty} (6x - 2) = \pm\infty$

Tem-se que:

$$m = \lim_{x \rightarrow +\infty} \frac{f(x)}{x} = \lim_{x \rightarrow +\infty} \frac{\frac{xg(-x)}{g(x)}}{x} = \lim_{x \rightarrow +\infty} \frac{xg(-x)}{xg(x)} = \lim_{x \rightarrow +\infty} \left( \frac{x}{g(x)} \times \frac{g(-x)}{x} \right) = \lim_{x \rightarrow +\infty} \frac{x}{g(x)} \times \lim_{x \rightarrow +\infty} \frac{g(-x)}{x} =$$

$$= \frac{1}{6} \times \lim_{x \rightarrow +\infty} \frac{g(-x)}{x} = -\frac{1}{6} \times \lim_{\substack{y=-x \\ y \rightarrow -\infty}} \frac{g(-x)}{-x} = -\frac{1}{6} \times \lim_{y \rightarrow +\infty} \frac{g(y)}{y} = -\frac{1}{6} \times 6 = -1$$

$$b = \lim_{x \rightarrow +\infty} (f(x) - mx) = \lim_{x \rightarrow +\infty} (f(x) + x) = \lim_{x \rightarrow +\infty} \left( \frac{xg(-x)}{g(x)} + x \right) = \lim_{x \rightarrow +\infty} \frac{xg(-x) + xg(x)}{g(x)} =$$

$$= \lim_{x \rightarrow +\infty} \frac{x(g(-x) + g(x))}{g(x)} = \lim_{x \rightarrow +\infty} \frac{x}{g(x)} \times \lim_{x \rightarrow +\infty} (g(-x) + g(x)) = \frac{1}{6} \times \lim_{x \rightarrow +\infty} (g(-x) + 6x + g(x) - 6x) =$$

$$= \frac{1}{6} \times \left( \lim_{x \rightarrow +\infty} (g(-x) + 6x) + \lim_{x \rightarrow +\infty} (g(x) - 6x) \right) = \frac{1}{6} \times \left( \lim_{x \rightarrow +\infty} (g(-x) + 6x) - 2 \right) =$$

$$\stackrel{\substack{y=-x \\ y \rightarrow -\infty}}{=} \frac{1}{6} \times \left( \lim_{y \rightarrow -\infty} (g(y) - 6y) - 2 \right) = \frac{1}{6} \times (-2 - 2) = \frac{1}{6} \times (-4) = -\frac{4}{6} = -\frac{2}{3}$$

Logo, a recta de equação  $y = -x - \frac{2}{3}$  é assíntota oblíqua do gráfico de  $f$ , quando  $x \rightarrow +\infty$ .