1. $2\overline{z} - i(2+3i) = z \Leftrightarrow 2\overline{x+yi} - i(2+3i) = x+yi \Leftrightarrow 2(x-yi) - 2i - 3i^2 = x+yi \Leftrightarrow$
$\Leftrightarrow 2x - 2yi - 2i + 3 = x + yi \Leftrightarrow 2x + 3 + (-2y-2)i = x + yi \Leftrightarrow 2x+3 = x \wedge -2y-2 = y \Leftrightarrow$
$\Leftrightarrow x = -3 \wedge -2y - y = 2 \Leftrightarrow x = -3 \wedge -3y = 2 \Leftrightarrow x = -3 \wedge y = -\frac{2}{3}$

Logo, $z = -3 - \frac{2}{3}i$

Resposta:
Versão 1:A
Versão 2:D

2. $\frac{(\overline{2-i})(1+i) + i^{96} - 4e^{i\frac{\pi}{2}}}{-2+i} = \frac{(2+i)(1+i) + i^0 - 4i}{-2+i} = \frac{2+2i+i+i^2+1-4i}{-2+i} = \frac{2-i}{-2+i} =$
$= \frac{2-i}{-(2-i)} = -1$

Trata-se de um número real, e o seu afixo é $A(-1;0)$

Representação no plano complexo

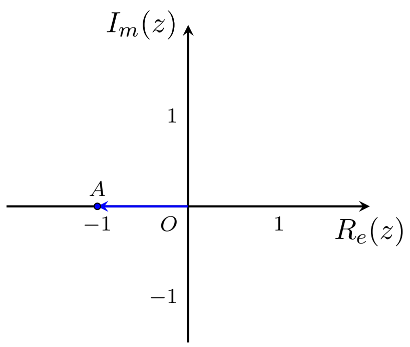

Figura 1

3. .

3.1. $w_1 = -2 + 2i$, logo, $\overline{w_1} = -2 - 2i$
$|\overline{w_1}| = \sqrt{(-2)^2 + (-2)^2} = \sqrt{8} = 2\sqrt{2}$
Seja $\theta = Arg(\overline{w_1})$
$\tan(\theta) = \frac{-2}{-2}$, com $\theta \in 3^{\text{o}}Q$
$\tan(\theta) = 1$, com $\theta \in 3^{\text{o}}Q$
Logo, $\theta = -\frac{3\pi}{4}$
Assim, $\overline{w_1} = 2\sqrt{2}e^{i\left(-\frac{3\pi}{4}\right)}$
$w_2 = \sqrt{2}e^{i\frac{\pi}{4}}$, logo, $-w_2 = \sqrt{2}e^{i\left(\frac{\pi}{4}+\pi\right)} = \sqrt{2}e^{i\frac{5\pi}{4}}$
Portanto, $(-w_2)^2 = \left[\sqrt{2}e^{i\frac{5\pi}{4}}\right]^2 = 2e^{i\left(\frac{5\pi}{4}\times 2\right)} = 2e^{i\frac{5\pi}{2}} = 2e^{i\frac{\pi}{2}}$

Assim,

$$w=\frac{\overline{w_1}}{(-w_2)^2}=\frac{2\sqrt{2}e^{i\left(-\frac{3\pi}{4}\right)}}{2e^{i\frac{\pi}{2}}}=\sqrt{2}e^{i\left(-\frac{3\pi}{4}-\frac{\pi}{2}\right)}=\sqrt{2}e^{i\left(-\frac{5\pi}{4}\right)}=\sqrt{2}e^{i\frac{3\pi}{4}}=$$
$$=\sqrt{2}\times\left(\cos\left(\frac{3\pi}{4}\right)+i\sin\left(\frac{3\pi}{4}\right)\right)=\sqrt{2}\times\left(-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\right)=-1+i$$

3.2. .

3.2.1. $z^3-3z^2+5z+w_1=13+2i \Leftrightarrow z^3-3z^2+5z-2+2i=13+2i \Leftrightarrow$
$\Leftrightarrow z^3-3z^2+5z-2+2i-13-2i=0 \Leftrightarrow z^3-3z^2+5z-15=0 \Leftrightarrow$
$\Leftrightarrow (z-3)\times Q(z)=0 \Leftrightarrow (z-3)(z^2+5)=0 \Leftrightarrow z-3=0 \vee z^2+5=0 \Leftrightarrow$
$\Leftrightarrow z=3 \vee z^2=-5 \Leftrightarrow z=3 \vee z=\pm\sqrt{-5} \Leftrightarrow z=3 \vee z=\pm\sqrt{5}i$

$C.S.=\{-\sqrt{5}i;\sqrt{5}i;3\}$
Cálculos auxiliares

Como 3 é zero de $p(z)=z^3-3z^2+5z-15$, então $p(z)$ é divisível por $z-3$, ou seja, $p(z)(z-3)\times Q(z)$
Determinemos $Q(z)$, recorrendo à regra de Ruffini

	1	−3	5	−15
3		3	0	15
	1	0	5	0

Logo, $Q(z)=z^2+5$, e portanto, $p(z)=z^3-3z^2+5z-15=(z-3)(z^2+5)$

3.2.2. $z^4-\overline{w_2}=0 \Leftrightarrow z^4=\overline{w_2} \Leftrightarrow z=\sqrt[4]{\overline{w_2}} \Leftrightarrow z=\sqrt[4]{\sqrt{2}e^{i\left(-\frac{\pi}{4}\right)}} \Leftrightarrow$
$\Leftrightarrow z=\sqrt[4]{\sqrt{2}}e^{i\left(\frac{-\frac{\pi}{4}+2k\pi}{4}\right)}, k\in\{0;1;2;3\} \Leftrightarrow$
$\Leftrightarrow z=\sqrt[8]{2}e^{i\left(\frac{-\frac{\pi}{4}}{4}+\frac{2k\pi}{4}\right)}, k\in\{0;1;2;3\} \Leftrightarrow$
$\Leftrightarrow z=\sqrt[8]{2}e^{i\left(-\frac{\pi}{16}+\frac{k\pi}{2}\right)}, k\in\{0;1;2;3\}$
Atribuindo valores a k, vem

$k=0 \rightarrow z_0=\sqrt[8]{2}e^{i\left(-\frac{\pi}{16}+0\right)}=\sqrt[8]{2}e^{i\left(-\frac{\pi}{16}\right)}$
$k=1 \rightarrow z_1=\sqrt[8]{2}e^{i\left(-\frac{\pi}{16}+\frac{\pi}{2}\right)}=\sqrt[8]{2}e^{i\left(-\frac{\pi}{16}+\frac{8\pi}{16}\right)}=\sqrt[8]{2}e^{i\frac{7\pi}{16}}$
$k=2 \rightarrow z_2=\sqrt[8]{2}e^{i\left(-\frac{\pi}{16}+\frac{2\pi}{2}\right)}=\sqrt[8]{2}e^{i\left(-\frac{\pi}{16}+\frac{16\pi}{16}\right)}=\sqrt[8]{2}e^{i\frac{15\pi}{16}}$
$k=3 \rightarrow z_3=\sqrt[8]{2}e^{i\left(-\frac{\pi}{16}+\frac{3\pi}{2}\right)}=\sqrt[8]{2}e^{i\left(-\frac{\pi}{16}+\frac{24\pi}{16}\right)}=\sqrt[8]{2}e^{i\frac{23\pi}{16}}=\sqrt[8]{2}e^{i\left(-\frac{9\pi}{16}\right)}$

Logo, $C.S.=\left\{\sqrt[8]{2}e^{i\left(-\frac{\pi}{16}\right)};\sqrt[8]{2}e^{i\frac{7\pi}{16}};\sqrt[8]{2}e^{i\frac{15\pi}{16}};\sqrt[8]{2}e^{i\left(-\frac{9\pi}{16}\right)}\right\}$

4. Seja $z=|z|e^{i\theta}$, com $\theta\in\mathbb{R}$

Assim,

$w=-2iz=-2i|z|e^{i\theta}=2e^{i\left(-\frac{\pi}{2}\right)}|z|e^{i\theta}=2|z|e^{i\left(\theta-\frac{\pi}{2}\right)}$
Portanto, o afixo do complexo w obtém-se do afixo do complexo z por uma rotação de centro na origem e ângulo de amplitude $-\frac{\pi}{2}$, seguida de uma homotetia de razão 2
Conclui-se assim que o afixo de w só poderá ser B

Resposta:
Versão 1:B
Versão 2:D

5. $z_1 = \left(e^{i\frac{\pi}{3}}\right)^4 = e^{i\frac{4\pi}{3}}$

$$z_2 = \frac{\left(e^{i\frac{\pi}{15}}\right)^7}{e^{i\left(-\frac{7\pi}{15}\right)}} = \frac{e^{i\frac{7\pi}{15}}}{e^{i\left(-\frac{7\pi}{15}\right)}} = e^{i\left(\frac{7\pi}{15}+\frac{7\pi}{15}\right)} = e^{i\frac{14\pi}{15}}$$

os afixos (imagens geométricas) de z_1 e de z_2 são vértices consecutivos de um polígono regular de n lados, com centro na origem do referencial,
Então, vem,

$$\frac{4\pi}{3} - \frac{14\pi}{15} = \frac{2\pi}{n} \Leftrightarrow \frac{6\pi}{15} = \frac{2\pi}{n} \Leftrightarrow 6n = 30 \Leftrightarrow n = 5$$

Resposta:
Versão 1:C
Versão 2:B

6. **Primeiro processo:**

$$\left(\frac{z_1}{z_2}\right)^n = \left(\frac{\sin(\theta)+i\cos(\theta)}{\sin(\theta)-i\cos(\theta)}\right)^n = \left(\frac{\cos\left(\frac{\pi}{2}-\theta\right)+i\sin\left(\frac{\pi}{2}-\theta\right)}{\cos\left(\frac{\pi}{2}-\theta\right)-i\sin\left(\frac{\pi}{2}-\theta\right)}\right)^n = \left(\frac{e^{i\left(\frac{\pi}{2}-\theta\right)}}{\overline{e^{i\left(\frac{\pi}{2}-\theta\right)}}}\right)^n =$$
$$= \left(\frac{e^{i\left(\frac{\pi}{2}-\theta\right)}}{e^{i\left(-\frac{\pi}{2}+\theta\right)}}\right)^n = \left(e^{i(\pi-\theta-\theta)}\right)^n = \left(e^{i(\pi-2\theta)}\right)^n = e^{i[n(\pi-2\theta)]}, \forall n \in \mathbb{N}$$

Segundo processo:

$$\left(\frac{z_1}{z_2}\right)^n = \left(\frac{\sin(\theta)+i\cos(\theta)}{\sin(\theta)-i\cos(\theta)}\right)^n = \left(\frac{i\left(\cos(\theta)-i\sin(\theta)\right)}{-i\left(\cos(\theta)+i\sin(\theta)\right)}\right)^n = \left(-\frac{\overline{e^{i(\theta)}}}{e^{i(\theta)}}\right)^n = \left(-\frac{e^{i(-\theta)}}{e^{i(\theta)}}\right)^n =$$
$$= \left(-e^{i(-\theta-\theta)}\right)^n = \left(e^{i(\pi-2\theta)}\right)^n = e^{i[n(\pi-2\theta)]}, \forall n \in \mathbb{N}$$

Terceiro processo:

$$\left(\frac{z_1}{z_2}\right)^n = \left(\frac{\sin(\theta)+i\cos(\theta)}{\sin(\theta)-i\cos(\theta)}\right)^n = \left[\frac{(\sin(\theta)+i\cos(\theta))^2}{(\sin(\theta)-i\cos(\theta))(\sin(\theta)+i\cos(\theta))}\right]^n =$$
$$= \left[\frac{\sin^2(\theta)+2i\sin(\theta)\cos(\theta)-\cos^2(\theta)}{\sin^2(\theta)+\cos^2(\theta)}\right]^n = \left[\frac{-(\cos^2(\theta)-\sin^2(\theta))+2i\sin(\theta)\cos(\theta)}{1}\right]^n =$$
$$= (-\cos(2\theta)+i\sin(2\theta))^n = (\cos(\pi-2\theta)+i\sin(\pi-2\theta))^n = \left(e^{i(\pi-2\theta)}\right)^n =$$
$$= e^{i[n(\pi-2\theta)]}, \forall n \in \mathbb{N}$$