Derivatives of elementary functions

Let $u = u(x)$ be a real function of real value and k a constant.

$(k)' = 0$	
$x' = 1$	$(x^\alpha)' = \alpha x^{\alpha-1}$
$(e^x)' = e^x$	$(e^u)' = e^u u'$
$(a^x)' = a^x \ln(a)$	$(a^u)' = a^u \ln(a) u'$
$\big(\ln(x)\big)' = \dfrac{1}{x}$	$\big(\ln(u)\big)' = \dfrac{u'}{u}$
$\big(\log_a(x)\big)' = \dfrac{1}{x\ln(a)}$	$\big(\log_a(u)\big)' = \dfrac{u'}{u\ln(a)}$
$\big(\sin(x)\big)' = \cos(x)$	$\big(\sin(u)\big)' = \cos(u)u'$
$\big(\cos(x)\big)' = -\sin(x)$	$\big(\cos(u)\big)' = -\sin(u)u'$
$\big(\mathrm{tg}(x)\big)' = \dfrac{1}{\cos^2(x)} = \sec^2(x)$	$\big(\mathrm{tg}(u)\big)' = \dfrac{u'}{\cos^2(u)} = \sec^2(u)u'$
$\big(\mathrm{cotg}(x)\big)' = -\dfrac{1}{\sin^2(x)} = -\csc^2(x)$	$\big(\mathrm{cotg}(u)\big)' = -\dfrac{u'}{\sin^2(u)} = -\csc^2(u)u'$
$\big(\sec(x)\big)' = \sec(x)\mathrm{tg}(x)$	$\big(\sec(u)\big)' = \sec(u)\mathrm{tg}(u)u'$
$\big(\csc(x)\big)' = -\csc(x)\mathrm{cotg}(x)$	$\big(\csc(u)\big)' = -\csc(u)\mathrm{cotg}(u)u'$
$\big(\arcsin(x)\big)' = \dfrac{1}{\sqrt{1-x^2}}$	$\big(\arcsin(u)\big)' = \dfrac{u'}{\sqrt{1-u^2}}$
$\big(\arccos(x)\big)' = -\dfrac{1}{\sqrt{1-x^2}}$	$\big(\arccos(u)\big)' = -\dfrac{u'}{\sqrt{1-u^2}}$
$\big(\mathrm{arctg}(x)\big)' = \dfrac{1}{1+x^2}$	$\big(\mathrm{arctg}(u)\big)' = \dfrac{u'}{1+u^2}$
$\big(\mathrm{arccotg}(x)\big)' = -\dfrac{1}{1+x^2}$	$\big(\mathrm{arccotg}(u)\big)' = -\dfrac{u'}{1+u^2}$
$\big(\mathrm{arcsec}(x)\big)' = \dfrac{1}{\lvert x\rvert\sqrt{x^2-1}}$	$\big(\mathrm{arcsec}(u)\big)' = \dfrac{u'}{\lvert u\rvert\sqrt{u^2-1}}$
$\big(\mathrm{arccsc}(x)\big)' = -\dfrac{1}{\lvert x\rvert\sqrt{x^2-1}}$	$\big(\mathrm{arccsc}(u)\big)' = -\dfrac{u'}{\lvert u\rvert\sqrt{u^2-1}}$

Derivation rules

Let $u = u(x)$ and $v = v(x)$ real functions of real value and k a constant.

$(ku)' = ku'$

$(u \pm v)' = u' \pm v'$ (derivative of the summation)

$(uv)' = u'v + uv'$ (derivative of the product)

$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ (derivative of the quotient)

$(u \circ v)'(x) = u'(v(x)) \cdot v'(x)$ (derivative of the composition)

$(u^{-1})'(y) = \frac{1}{u'\big(u^{-1}(y)\big)}$ (derivative of the inverse function)

Some trigonometric formulas

$$\mathrm{tg}(u) = \frac{\sin(u)}{\cos(u)}$$

$$\mathrm{cotg}(u) = \frac{1}{\mathrm{tg}(u)}$$

$$\sec(u) = \frac{1}{\cos(u)}$$

$$\csc(u) = \frac{1}{\sin(u)}$$

$$\sin^2(u) + \cos^2(u) = 1$$

$$\mathrm{tg}^2(u) + 1 = \frac{1}{\cos^2(u)}$$

$$\sin(u \pm v) = \sin(u)\cos(v) \pm \cos(u)\sin(v)$$

$$\cos(u \pm v) = \cos(u)\cos(v) \mp \sin(u)\sin(v)$$

$$\mathrm{tg}(u \pm v) = \frac{\mathrm{tg}(u) \pm \mathrm{tg}(v)}{1 \mp \mathrm{tg}(u)\mathrm{tg}(v)}$$

$$\mathrm{cotg}^2(u) + 1 = \frac{1}{\sin^2(u)}$$

$$\sin(2u) = 2\sin(u)\cos(u)$$

$$\cos(2u) = \cos^2(u) - \sin^2(u)$$

$$\mathrm{tg}(2u) = \frac{2\mathrm{tg}(u)}{1 - \mathrm{tg}^2(u)}$$

$$\sin^2(u) = \frac{1 - \cos(2u)}{2}$$

$$\cos^2(u) = \frac{1 + \cos(2u)}{2}$$

$$\mathrm{tg}^2(u) = \frac{1 - \cos(2u)}{1 + \cos(2u)}$$

Trigonometry - table of angles

	$-\frac{\pi}{2}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π
sin	-1	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0
cos	0	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1
tg	$-\infty$ (n.d.)	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$+\infty$ (n.d.)	0

Inverse trigonometric functions

Arc sine:

$$\arcsin = \operatorname{sen}^{-1} : [-1,1] \longrightarrow [-\tfrac{\pi}{2}, \tfrac{\pi}{2}]$$

Arc cosine:

$$\arccos = \cos^{-1} : [-1,1] \longrightarrow [0,\pi]$$

Arc tangent:

$$\operatorname{arctg} = \operatorname{tg}^{-1} : \mathbb{R} \longrightarrow]-\tfrac{\pi}{2}, \tfrac{\pi}{2}[$$

Arc cotangent:

$$\operatorname{arccotg} = \operatorname{cotg}^{-1} : \mathbb{R} \longrightarrow]0,\pi[$$

Arc secant:

$$\operatorname{arcsec} = \sec^{-1} : [-\infty,-1] \cup [1,+\infty[\longrightarrow [0,\tfrac{\pi}{2}[\cup]\tfrac{\pi}{2},\pi]$$

Arc cosecant:

$$\operatorname{arccsc} = \csc^{-1} : [-\infty,-1] \cup [1,+\infty[\longrightarrow [-\tfrac{\pi}{2},0[\cup]0,\tfrac{\pi}{2}]$$

Table of immediate primitives

Let $u = u(x)$ be a real function of real value and k a constant. Consider C varying in $\mathbb{R}$.

$\mathrm{P}1 = x + C$	$\mathrm{P}k = kx + C$
$\mathrm{P}x^\alpha = \frac{x^{\alpha+1}}{\alpha+1} + C, \quad \text{with } \alpha \neq -1$	$\mathrm{P}u^\alpha u' = \frac{u^{\alpha+1}}{\alpha+1} + C, \quad \text{with } \alpha \neq -1$
$\mathrm{P}e^x = e^x + C$	$\mathrm{P}e^u u' = e^u + C$
$\mathrm{P}a^x = \frac{a^x}{\ln(a)} + C, \quad \text{with } 1 \neq a \in \mathbb{R}^+$	$\mathrm{P}a^u u' = \frac{a^u}{\ln(a)} + C, \quad \text{with } 1 \neq a \in \mathbb{R}^+$
$\mathrm{P}\frac{1}{x} = \ln\lvert x\rvert + C$	$\mathrm{P}\frac{u'}{u} = \ln\lvert u\rvert + C$
$\mathrm{P}\frac{1}{x\ln(a)} = \log_a\lvert x\rvert + C, \quad \text{with } 1 \neq a \in \mathbb{R}^+$	$\mathrm{P}\frac{u'}{u\ln(a)} = \log_a\lvert u\rvert + C, \quad \text{with } 1 \neq a \in \mathbb{R}^+$
$\mathrm{P}\sin(x) = -\cos(x) + C$	$\mathrm{P}\sin(u)u' = -\cos(u) + C$
$\mathrm{P}\cos(x) = \sin(x) + C$	$\mathrm{P}\cos(u)u' = \sin(u) + C$
$\mathrm{P}\sec^2(x) = \mathrm{tg}(x) + C$	$\mathrm{P}\sec^2(u)u' = \mathrm{tg}(u) + C$
$\mathrm{P}\csc^2(x) = -\mathrm{cotg}(x) + C$	$\mathrm{P}\csc^2(u)u' = -\mathrm{cotg}(u) + C$
$\mathrm{P}\sec(x)\mathrm{tg}(x) = \sec(x) + C$	$\mathrm{P}\sec(u)\mathrm{tg}(u)u' = \sec(u) + C$
$\mathrm{P}\csc(x)\mathrm{cotg}(x) = -\csc(x) + C$	$\mathrm{P}\csc(u)\mathrm{cotg}(u)u' = -\csc(u) + C$
$\mathrm{P}\frac{1}{\sqrt{k^2 - x^2}} = \arcsin(\frac{x}{k}) + C = -\arccos(\frac{x}{k}) + C$	$\mathrm{P}\frac{u'}{\sqrt{k^2 - u^2}} = \arcsin(\frac{u}{k}) + C = -\arccos(\frac{u}{k}) + C$
$\mathrm{P}\frac{1}{k^2 + x^2} = \frac{1}{k}\mathrm{arctg}(\frac{x}{k}) + C = -\frac{1}{k}\mathrm{arcotg}(\frac{x}{k}) + C$	$\mathrm{P}\frac{u'}{k^2 + u^2} = \frac{1}{k}\mathrm{arctg}(\frac{u}{k}) + C = -\frac{1}{k}\mathrm{arccotg}(\frac{u}{k}) + C$
$\mathrm{P}\frac{1}{\lvert x\rvert\sqrt{x^2 - 1}} = \mathrm{arcsec}(x) + C = -\mathrm{arccsc}(x) + C$	$\mathrm{P}\frac{u'}{\lvert u\rvert\sqrt{u^2 - 1}} = \mathrm{arcsec}(u) + C = -\mathrm{arccsc}(u) + C$

General rules of primitive integration

Let $u = u(x)$ and $v = v(x)$ be real functions of real value and k a constant.

$$\mathrm{P}(u \pm v) = \mathrm{P}u \pm \mathrm{P}v$$

$$\mathrm{P}(ku) = k\mathrm{P}u$$

Integration by parts

$$\mathrm{P}(u'v) = uv - \mathrm{P}(uv')$$

Integration by substitution

$$\mathrm{P}f(x) = \mathrm{P}\big(f(\varphi(t)) \cdot \varphi'(t)\big), \text{ where } x = \varphi(t) \text{ is an adequate substitution.}$$

Function expression	**Substitution** $x = \varphi(t)$
$\sqrt{ax+b}$	$ax + b = t^2$
$\sqrt{a^2 - x^2}$	$x = a\sin(t)$ or $x = a\cos(t)$
$\sqrt{a^2 + x^2}$	$x = a\mathrm{tg}(t)$ or $x = a\mathrm{cotg}(t)$
$\sqrt{x^2 - a^2}$	$x = a\sec(t)$ or $x = a\csc(t)$
e^{ax+b}	$ax + b = \ln(t)$
$\ln(ax + b)$	$ax + b = e^t$
$f(x) = R(x^{\frac{m}{n}}, x^{\frac{p}{q}}, \ldots, x^{\frac{r}{s}})$	$x = t^\mu$, with $\mu = \text{l.c.m}(n, q, \ldots, s)$
$f(x) = R\big(x, (\frac{ax+b}{cx+d})^{\frac{m}{n}}, (\frac{ax+b}{cx+d})^{\frac{p}{q}}, \ldots, (\frac{ax+b}{cx+d})^{\frac{r}{s}}\big)$	$\frac{ax+b}{cx+d} = t^\mu$, with $\mu = \text{l.c.m}(n, q, \ldots, s)$
$f(x) = R(x, \sqrt{ax^2 + bx + c})$	$\sqrt{ax^2+bx+c} = \sqrt{a}x + t$, if $a > 0$ $\sqrt{ax^2+bx+c} = tx + c$, if $c > 0$
$f(x) = R(\sin(x), \cos(x))$	$t = \mathrm{tg}(\frac{x}{2})$ (*)
$f(x) = R(\sin(x), \cos(x))$, with R even function	$t = \mathrm{tg}(x)$ (**)

(*) In this case, $\sin(x) = \frac{2t}{1+t^2}$ and $\cos(x) = \frac{1-t^2}{1+t^2}$

(**) In this case, $\sin(x) = \frac{t}{\sqrt{1+t^2}}$ and $\cos(x) = \frac{1}{\sqrt{1+t^2}}$