ISCTE IUL

Group Assignment

Optimization - GAi

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Table of Contents

Introduction

In this assignment we were given a linear programming problem, and asked to solve it using the Solver, as a group. First we will define the variables of the problem and explain the problem constraints of our formulation, which will make the rest of the assignment easier for us. Afterwards, we will use the Solver, in order to answer the next question of the assignment, i.e finding the optimal solution of the problem, which without the Solver would be very hard for us to answer. We also faced different scenarios, during the problem, where we should answer questions if one of the variables of the problem was changed.

Formulate the linear programming problem described above. Define the variables and explain the problem constraints of you formulation.

MachinesConstraints

Machines	Time ofone unit produced	Available hours	Maximum number of units
A(1)	15 m/u = 0,25 h/unit	450 h	$\frac{450}{0,25} = 1800\ units$
B(2)	12 m/u = 0, 20 h/unit	400 h	$\frac{400}{0,2} = 2000\ units$
C(1)	10 m/u ≅ 0,17h/unit	500 h	$\frac{500}{0,17} \cong 2941\ units$
C(2)	8 m/u ≅ 0,13 h/unit	500 h	$\frac{500}{0,13} \cong 3846\ units$

Production Process 1:

Using A1 and C1, the maximum numbers are, respectively, 1800 and 2941 units. This gives us the maximum number of units we can produce by using the process 1, **not using raw materials.**

And since 1800 < 3000, in total we can **only produce 1800 units** which means $\frac{1800\ x\ 10}{60}$= **300 labor hours of machine C,** because we use all 450 available labor hours of A1.

So, the **total production of process 1 regarding machines is 1800 units**.

Production Process 2:

Using B2 and C2, the maximum numbers are, respectively, 2000 and 3846 units. This gives us the maximum number of units we can produce by using the process 2, **not using raw materials.** We consider 500 of C, because if we choose the process 2, the machine C was never used in process 1.

And since 2000 < 3846, in total we can **only produce 2000 units** which means $\frac{2000\ x\ 8}{60}$ **≅266, 66 labor hours of machine C,** because we use all 400 available hours of B2.

Therefore, the total **production of process 2 regarding machines is 2000 units.**

Raw Materials Constraints

Raw materials	Quant. forone unit produced	Available quantity	Maximum number of units
RM1(1)	3 rm/unit	5550 rm1	$\frac{5550}{3} = 1850\ units$
RM2 (2)	2,5 rm/unit	3500 rm2	$\frac{3500}{2,5} = 1400\ units$
RM3 (1)	4 rm/unit	13500 rm3	$\frac{13500}{4} = 3375\ units$
RM3 (2)	5 rm/unit	13500 rm3	$\frac{13500}{5} = 2700\ units$

Production Process 1:

Using RM1(1) and RM3(1), the maximum numbers are, respectively, 1850 and 3375 units. This gives us the maximum number of units we can produce using the raw material in process 1 (RM1 and RM3).

Since 1850 < 3375, in total we can **only produce 1850 units** which means (1850 X 4) = **7400 quantity of RM3 used.**

So, the **total production of process 1** regarding raw materials (1850) is:

RM1 <= 5550

RM3 <= 7400

Max units produced =1850

Production Process 2:

Using RM2 (2) and RM3 (2), the maximum numbers are, respectively, 1400 and 2700 units. We consider the total RM3 here, because if we choose the process 2, we don't use the process 1, thus the RM3 were never consumed.

As 1400 < 2700, in total we can **only produce 1400 units** which means (1400 X 5) = **7000 quantity of RM3 used.**

So, the **total production of process 2** regarding raw materials (1400) is:

RM2<= 1400

RM3 <= 7000

Costs concerning different process

Machines

Machines	Time ofone unit produced	Costs per hour	Costs per machine
A(1)	15 m/u = 0,25 h/unit	10 m.u/h	(0.25*10)=2,5 m.u
B(2)	12 m/u = 0, 20 h/unit	18 m.u/h	(0.20*18)=3,6 m.u
C(1)	10 m/u ≅ 0,17h/unit	12 m.u /h	(0.17*12)=2 m.u
C(2)	8 m/u ≅ 0,13 h/unit	12 m.u /h	(0.13*12)=1,6 m.u

Production Process 1:

Using this process, we will spend 2,5 (machine A) + 2 (machine C1) = **4,5 m.u per unit produced in machinery.**

Production Process 2:

Using this process, we will spend 3,6 (machine B) + 1,6 (machine C) **= 5,2 m.u per unit produced in machinery.**

Raw Materials

Raw materials	Quant. of one unit produced	Costs per unit	Costs per RM

RM1(1)	3	0,6 m.u	(0.6*3) =1,8 m.u
RM2 (2)	2,5	0,6 m.u	(2.5*0.6)=1,5 m.u
RM3 (1)	4	0,5 m.u	(4*0.5)=2 m.u
RM3 (2)	5	0,5 m.u	(5*0.5)=2,5 m.u

Production Process 1:

Using this process, we will spend 1,8 (RM1) + 2 (RM3) = **3,8m.u per unit produced in raw materials.**

Production Process 2:

Using this process, we will spend 1,5 (RM2) + 2,5 (RM3) = **4m.u per unit produced in raw materials**.

Total Cost

The total cost for Production Process 1 is 4**,5 + 3,8 = 8,3 m.u**

The total cost for Production Process 2 is **5,2 + 4= 9,2 m.u**

TC = 8,3 x + 9,2 y

X: number of units produced concerning PP 1 / number of times PP1 was used

Y: number of units produced concerning PP 2 / number of times PP2 was used

Constraints:

Machine hours:

X <= 1800

Y <= 2000

0,17X + 0,13Y < = 500 (available hours of machine C)

Raw materials:

X <= 1850

Y <= 1400

4X + 5Y <= 13500 (raw material 3)

Production Minimum:

X + y >= 3000

Given the problem we can ignore X <= 1800 and Y <= 2000 because we don't have enough resources to fulfill 1800 units and 2000 units.

So, the linear programming problem is:

Min 8,3X + 9,2Y

X<= 1800

Y<= 1400

0,17X + 0,13Y <= 500

4X + 5Y <=13500

X + Y >= 3000

X,Y>= 0

Using solver, compute the optimal solution of this problem. With detail, interpret the generated output including the table related with the constraints.

If we insert the linear programming problem that we found previously, on Excel, and using the Solver, we will have something like the image below:

Linear Programming						
	Coefficients of OF					
	PP1	PP2		OF		
UNIT	8,3	9,2		25980		
Decision Variables	1800	1200				
	x1					
	x1	x2		Restriction		Independent Term
r1	1	0		1800	<=	1800
r2	0	1		1200	<=	1400
r3	0,17	0,13		462	<=	500
r4	4	5		13200	<=	13500
r5	1	1		3000	>=	3000

Linear Programming Problem
Min 8,3X + 9,2Y
X<= 1800
Y<= 1400
0,17X + 0,13Y <= 500
4X + 5Y <= 3000
X + Y >= 3000
X,Y >= 0

We found the optimal solution that is 25980 m.u, considering machinery, raw materials, and the minimum production.

Therefore:

Total cost = 25980

Optimal solution = 1800 of PP1 and 1200 of PP2

How much can the cost of one hour of machine A rise without affecting the value of the optimal solution?

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B7	Decision Variables PP1	1800	0	8,3	0,9	1E+30
C7	Decision Variables PP2	1200	0	9,2	1E+30	0,9

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
E11	r1 Restriction	1800	-0,9	1800	950	200
E12	r2 Restriction	1200	0	1400	1E+30	200
E13	r3 Restriction	462	0	500	1E+30	38
E14	r4 Restriction	13200	0	13500	1E+30	300
E15	r5 Restriction	3000	9,2	3000	60	1200

Using the Solver, after finding the optimal solution, we can have three different reports, answer, sensitivity and limits, and given the question we decided to use the sensitivity report.

Since the machine A is exclusively used in Process 1, its costs will only be allocated to the same process, therefore we will consider that all other costs remain unchanged.

According to the Sensitivity Report, the objective coefficient 8.3 can rise 0.9 without affecting the value of the optimal solution.

Now, we can conclude that the cost of one hour of machine A, can rise 3.4 m.u (2.5 + 0.9) without affecting the value of the optimal solution.

If the minimum production per week changes to 3050 units, what can you say about the cost and the optimal solution of the new problem? Use the results from the previous point.

First of all, to find the new total cost, we need to change the inputs.

Our minimum production per week instead of 3000 will now be 3050.

Computing these values into Solver, we get:

Linear Programming						
	Coefficients of OF					
	PP1	PP2		OF		
UNIT	8,3	9,2		26440		
Decision Variables	1800	1250				
	x1					
	x1	x2		Restriction		Independent Term
r1	1	0		1800	<=	1800
r2	0	1		1250	<=	1400
r3	0,17	0,13		468,5	<=	500
r4	4	5		13450	<=	13500
r5	1	1		3050	>=	3050

Units produced by P1 = 1800

Units produced by P2 = 1250

And our total cost will now be 26440

We notice an increase of 460 in our total cost, since it passed form 25980 to 26440, since raising the minimum production will have higher cost, therefore our total cost will also increase.

Starting from the solution obtained in 2, without solving the new problem and justifying with detail all your conclusions, answer to the following questions:

- If, by legal motives, the company couldn't distributed more than 3500 units of its production, what would happen with the optimal solution?

If, by legal motives, the company couldn't distribute more than 3500 units of its production, nothing would happen to the optimal solution, because our optimal solution has a maximum production of 3000 units.

- If the available labor hours of the machines B and C had been reduced, at the same time, to 300 and 480 hours, respectively, what would happen to the optimal solution?

Machines	Time ofone unit produced	Available hours	Maximum number of units
A(1)	15 m/u = 0,25 h/unit	450 h	$\frac{450}{0,25} = 1800\ units$
B(2)	12 m/u = 0, 20 h/unit	300 h	$\frac{300}{0,2} = 1500\ units$
C(1)	10 m/u ≅ 0,17h/unit	480 h	$\frac{480}{0,17} \cong 2824\ units$
C(2)	8 m/u ≅ 0,13 h/unit	480 h	$\frac{480}{0,13} \cong 3692 units$

In the process 1, since we use the machine A and C(1), and 1800 < 2824 units, we don't verify any change because we will still be producing 1800.

In the process 2, we use the machine B and C(2), and the new outcomes would be 1500 < 3692, we will now be producing 1500 units, and using all available hours of machine B that is 300, and (1500*8)/60 = 200 labor hours of C, we are producing 1500 units of the product.

Assuming that the raw materials remain unchanged, our machine hours constraint related to "y" will be:

X =< 1800

Y =< 1500

X*10 + Y*8 =< 30000

We can conclude that nothing changes in the linear programming problem, so everything will remain exactly the same, therefore our optimal solution won't change.

- What would be the new optimal solution if, due to inflation, the prices rose all 10%?

Due to inflation, the prices of Raw Materials would be:

- RM1: 0,6*1,1 = 0,66 m.u
- RM2: 0,6*1,1 = 0,66 m.u

- RM3: 0,5*1,1 = 0,55 m.u

These increase in the raw materials price will also lead to an increase of the costs related to each process.

Cost Related to each process :

- *Process 1:*
  Instead of spending 3,8= (1,8 + 2) we will spend 3*0,66 + 4*0,55 = 4,18 m.u.

Assuming that the cost with machinery will remain the same, our cost with the same will be 4,5 m.u and so we would get a total cost of 4,18+4,5 = 8,68 m.u.

- *Process 2:*

Our costs here, will also change; instead of having 4 m.u in Raw Materials we would now have 2,5*0,66 + 5*0,55 = 4,4 m.u.

Also, assuming that the cost with machinery will not change, we will continue having a cost of 5,2 m.u here, so we would get a total cost of 9,6 m.u per unit produced.

Equation for total cost:

Organizing, the information we have above, we have the following equation:

TC = 8,68X + 9,6Y

Since we want the optimal point, we should substitute X by 1800 and Y by 1200, leading us to :

TC = 8,68 * 1800 + 9,6 * 1200 = 27144 m.u.

- ## What would you say for sure about the optimal cost if the company had a third production process?

If the company had a third production process, there are two possibilities for the optimal cost:

- If this new production process turns out to be inefficient, our optimal cost would remain the same, because it wouldn't make sense to use this process;
- If this new production process turns out to be efficient, our optimal cost would be either equal or lower than the current optimal cost (25980 m.u.)

Conclusion

This assignment was very useful to us, because we were able to apply what we are learning in real life problems. And we also learned to work with Solver, a real useful program, where we can easily solve this type of problems.