ISCTE - IUL

Optimization

Management, Marketing Management, Finance and Accounting

28th March 2015

School year 2014/2015, 2nd semester

Kind of test: Middle test   Total test time: 1h30

Name:
Student number:
Class:
Professor:

Important observations:

1. Computers and course materials can not be used.
2. It is not allowed to write on the test with a pencil or red pen.
3. All the answers must be properly justified.
4. The staple keeping the test paper sheets can not be taken out.
5. It is not allowed to use extra paper sheets beside the ones given by the professor. The last given paper sheet can be used for auxiliary computations and, exceptionally, to answer a question of the test.
6. All the electronic devices must be shut down during the whole test time.
7. No questions about the subjects of the course will be answered.

Question points:

(2.5 pts) 1. Find the optima for the following function $f(x,y) = (x-1)^2 + (y-2)^2 + xy$.

2. Consider the set of points of $\mathbb{R}^2$ defined by $x + y = 3$.

(1.5 pts) (a) For $x \geq 0$ and $y \geq 0$, trace the above constraint function and the level curve $f(x, y) = x^2y = 1$. Consider the point $(1, 1)$ which belongs to the drawn level curve. Starting from $(1, 1)$, indicate the direction of greatest growth of f and the graphical region where it is suppose to be the maximum point of the constrained problem.

(1.5 pts) (b) Write the Lagrangean function associated to the problem $f(x, y) = x^2y$ subject to the initial constraint.

(1.5 pts) (c) Check that the only critical points are $(x, y) = (0, 3)$ and $(x, y) = (2, 1)$.

(1.5 pts) (d) Use the bordered Hessian matrix applied to the critical points of (c) to find the maximum points for the given constrained problem.

3. A company plans to have a daily optimal production. Nowadays, the company produces two products, product A and product B, and has limitations for two resources, machine time and labour-force time. The machine available time is 12 hours/per day and the labour-force available time is 8 hours/per day. For each unit production of A are needed 2 machine hours and 2 labour-force hours. For each unit production of B are needed 3 machine hours and 1 labour-force hour. One knows that a unit of product A generates a profit of 4 monetary units and a unit of product B generates a profit of 1 monetary unit. The linear programming problem to find the production which maximizes the daily profit has the following formulation:

$$\begin{aligned} \text{Max Z} = & 4x_1 + x_2 \\ \text{s.t.} \quad & 2x_1 + x_2 \leq 8 \\ & 2x_1 + 3x_2 \leq 12 \\ & x_1, x_2 \geq 0 \end{aligned}$$

(0.75 pts) (a) Which quantity does x_1 represent, the production from A or from B ? And x_2 ? Justify.

(0.75 pts) (b) Justifying, identify the respective constraints associated to the machine time and the labour-force time.

(2.5 pts) (c) Write the first simplex tableaux and construct the next one. Is the obtained solution optimal? Justify.

4. Study the nature of the following series:

(1.5 pts) (a) $\sum_{n=5}^{+\infty} \frac{n}{(n^2+1)(n+5)}$

(1.5 pts) (b) $\sum_{n=1}^{+\infty} \frac{(-1)^n}{3n^3+5n}$

(1.5 pts) (c) $\sum_{n=2}^{+\infty} \frac{1}{n3^n}$

(1.5 pts) 5. Compute the convergence interval of the power series $\sum_{n=3}^{+\infty} \left(\frac{-x+1}{4}\right)^n$

(1.5 pts) 6. If a function $f(x, y) : \mathbb{R}^2 \to \mathbb{R}$ has the absolute maximum at (x^*, y^*), what can you say about the optima of the function $g(x) : \mathbb{R} \to \mathbb{R}$ with $g(x) = f(x, y^*)$? Carefully, justify all your statements.

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