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Introduction to Finance

Fundamentals of Finance with Solved Exercises and Follow-up Exercises

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The author Helena Soares dedicates this work to Ana Maria, Margarida and Maria Luisa.

The author Luis Laureano dedicates this work to Nídia, Diana and Daniela.

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Introduction

Let us start this introduction with a question, to which you can answer by choosing one of the replies presented below.

Question: «In the renewal of a financial investment, like a term deposit for example, of 25 thousand euros, your bank will credit the account in 500 euros of interest, at the end of a year. Will you accept the renewal of the investment under these conditions?»

Replies:

a) I do not accept because I have the same amount invested in an Equity Fund that last year got a profit of 1,100 euros;

b) I accept because, in times of crisis, 500 euros is an amount that one should not reject;

c) I accept because this deposit generated an interest of 990 euros last year and, making the average with the 500 euros, I get, per year, an interest that is greater than the competitor bank is offering;

d) I accept because I have all my savings in this Bank and our relationship is based on mutual trust;

e) I do not accept because the Bank must offer at least the same interest as last year's.

If you chose any of the presented replies to answer to the question, we suggest that you continue reading the next chapters of this book.

In fact, all those answers are wrong. None of them serves to answer the question posed at the beginning and, more, none serves to answer correctly in a decision--making situation of any investment. It is not just the numbers not being the adequate or the reply being inappropriate for the type of investment in case. No, all responses contradict at least one basic principle of the financial theory and, as such, the correct financial decisions cannot be taken based in those types of approach.

Let's see then what are these basic principles of financial theory that can be found in the theoretical topics and in many practical cases presented over the following 4 chapters in which the book unfolds.

Risk versus Return

If we undertake a higher risk in a certain investment, we will require a greater expected return to compensate for that additional risk. Similarly, if we choose a safer investment, we will not be able to require a level of return that corresponds to riskier investments. Concerning this principle we will work with the risk premiums in Chapter 4, for investment project analysis.

Risk Profiles

According to the level of financial wellness and the personal and professional context, each investor has a particular risk profile. Normally we recommend riskier investments, of which shares are the most traditional example, to the investors that are able to save significant amounts of money for extended periods of time (long term: more than five years). So, to decide about a particular investment, the investor should only do comparisons within the range of possibilities that relate to his risk aversion. The first criterion of choice has to be the risk and not the return, contrary of what happens in reality very frequently. The greed, so typical of human being, unduly outshines one's rationality in the decision-making process. In Chapter 1, of Time Value of Money, we will work with opportunity costs for each type of investment that we analyse.

Opportunity Costs

All the investment has an opportunity cost. Whether it's a savings deposit, securities of the capital market (bonds and shares), assets invested in companies (Capital and Operating) or an investment in a specific project, they all need to generate a profit that will be at least the same as the one which can be generated by an alternative investment (of equal risk and maturity). In the business context we are often led to forget this principle. Or because we get satisfied in that the company simply is profitable or because, just looking for the maximization of profit, we forget that this can have severe implications at the level of the financial stability of the company, putting at risk its own solvency. It is more this way of analysing the risk *vs.* return that we will try to address in Chapter 3 of the study of profitability and solvency of the firm.

We must forget the past

What distinguishes the financial theory from the accounting theory lies in the objective of each one. While the latter serves to report and conclude on the past activity, financial theory aims to help in investment decision making and, therefore, influence the future. No past event, being income or expense, must be considered in deciding on a particular investment. Only the income and expenses arising from the investment thought to be carried out, are important for assessing the relevance of undertaking the capital expenditure. It is on this view that we will only consider the incremental and ignore the so-called «sunk costs» in the analysis of investment projects on Chapter 4.

Diversification

Although we do not dedicate any chapter of this book to portfolio management, where this issue is very important, we want to point out that diversification appears as one of the most useful tools for the correct financial decision making because, as it is almost always very simple to implement, it has as a consequence that we can easily obtain the same expected return, assuming a lower level of risk.

There are two main types of risk: credit and price (market). So, whenever there is prove of not being too expensive, investors should allocate their investments by different issuers and different types of assets. Besides resulting in lower volatility of the asset values, it allows the availability of flows in different moments in time, which helps in a good financial management of the liquidity. We will describe the range of assets that we can find in the financial markets in Chapter 2.

The future is not a mirror of the past

Despite of being the principle that has triggered further studies in the theory of finance, it was, in practice, the most ignored throughout the decade of flowering of the financial engineering that culminated in the great crisis of 2008 onwards. To think that the facts of the past will repeat in the future, is to ignore completely all the randomness of events to come and the contextual changes and circumstances of our existence. There are so many examples of «nothing is as before». Take the case of September 11: a terrorist attack that changed completely the perception of the global level of risk; and the bankruptcy cases (banks and countries) in the subprime crisis: that questioned the concept that we have of the risk free rate. Basing investment decisions on past behaviours can become more risky than doing it on the basis of future perspectives. Nor in real life, nor in Finance, the past determines the future.

Chapter 1

TIME VALUE OF MONEY

Theory

Returning to the question at the beginning of the chapter of the introduction, with a slight amendment, shall we renew the 25,000 euros investment in the bank in order to have 26,000 euros in two years?

Compounding and Future Value

Before answering the question, there is a fact about which there is no doubt: the value to receive in the future will have to be always higher than the present value that we reject to spend immediately. Otherwise, what sense would it be in giving the money to an institution that, itself, will invest that money to get a return of it? Offer this opportunity for profit implies a reward in return: the interest.

So, when we talk of a particular capital, we know that it will take different values as time is passing. More time will imply more interest and, consequently, a higher future value. This is the basic concept of the financial calculation: compounding, which means to add an interest to a capital (principal) as time goes by.

Answering the question, we would never accept to receive the 25,000 euros after investing this same amount in a term deposit for two years. The value that to us is equivalent to these 25,000 euros, but two years later, will be a different one because it will have to be sufficiently increased to include interest. How enough is the increase? Which is the appropriate interest amount?

The answer to these last questions will eventually be the answer to the former question.

Opportunity Cost

In order to add interest to a capital, we first need to identify the level of interest rate suitable for the operation in question. On the question posed at the beginning of this chapter, the interest rate to consider shall meet the following requirements: be at least the one offered by the competitor banks that have the same level of credit risk of the bank where is being renewed the investment, for the same term (in this case 2 years).

For other type of investment, or financing, we shall follow the same reasoning in identifying the appropriate interest rate: we seek, respectively, the highest, or the lowest, profitability, or cost, of the alternatives of equal risk and maturity.

This rate to be found is our compounding rate or, as we shall see further, it can also work as a discount rate. It is what we could call the «fair rate» for the operation that we are evaluating.

The interest rate that effectively is used by the financial institution in a specific operation can differ from the one that we have identified as the opportunity cost, as described above. By comparing both rates, and the future values that they generate, we can conclude whether we shall accept or not the financial transaction in question.

After identifying the interest rate for the transaction, we will have to calculate the interest. There are two ways, or methodologies, to determine the interest in a financial transaction: Simple Interest and Compound Interest.

Simple Interest

The main features of this methodology are the following:

- Mostly used in short-term transactions (up to one year);
- The interest, for each period of time, is always calculated on the basis of the capital at the beginning (Principal);
- The interest, determined in a period, is not subject to interest in the following periods;
- The interest is always proportional to the elapsed time (initial capital × interest rate for a given time period × number of units of this period of time);

- With C being the initial capital, r the annual interest rate[1] and n the number of years, the future value, or accumulated value, M, can be determined by: $M = C + C \times n \times r$ or, simplifying, $M = C \times (1 + n \times r)$.

Imagine that on the question of the beginning of the chapter we conclude that the interest rate corresponding to the opportunity cost is 2.5% per year. In Simple Interest, the value that the Bank should return at the end of two years is 26,250 euros ($M = 25,000 + 25,000 \times 2 \times 2.5\%$).

Decision Making

At this point we can already answer the question that started this chapter. Comparing the «fair value» with the value promised by the Bank, we came to the conclusion that the latter is insufficient. Thus, we must reject the offer of this Bank and renew the investment in the alternative Bank that offers us the 2.5% per year.

From this conclusion, that the interest rate offered by the bank in case is less than the opportunity cost, two questions may arise:

- What is the level of the interest rate that is effectively being offered by the Bank?
- What is the maximum amount that we will be willing to invest if the Bank only offers 26,000 euros at the end of two years?

Effective Rate

The answer to the first of these questions leads us to the concept of effective rate and more specifically in that case, effective annual rate.

If we started a period of time with a capital of A and, as a result of a certain financial transaction, we ended this period with a compound value of B, the effective rate or the rate at which effectively the capital was invested or, furthermore, the «true» rate of the operation in that period of time, is easily calculated using the compound equation described above. In this case will be $B = A + A \times 1 \times r$, A and B being the values of initial and compound capital, respectively, and r our unknown variable: the effective rate on a particular length of time that, in this case, was exactly one (year).

Let us now answer the same but to the specific question of the beginning of the chapter: the Bank offers 26,000 euros, at the end of two years, to renew the invest-

(1) The reference period in financial terms is the annual period. We will see later how to deal with rates that relate to a different period.

ment of 25,000 euros. What is the effective annual rate offered? Reply: $26,000 = 25,000 \times (1 + 2 \times r) \Leftrightarrow r = 2\%$. Each year, the annual rate offered is of 2%. Note that if we consider that the period of the rate is two years, the effective rate would be 4% (n = 1) and we confirm the proportionality of the interest in relation to time, a main feature of the Simple Interest.

Discounting and Present Value

The second of the above questions leads us to a calculation that is opposite to compounding. In fact, now we do not want to find the future value that is equivalent to a certain present value. What we want to know now is what is the value that we are willing to consider in the present, for the renewal of the investment, with the perspective of receiving a given value in a future moment. We want to find the present value and to do so we will need to discount that future value.

We have to use the same compounding equation previously set, since only this considers the inclusion of the «fair» interest over time. Yet, our unknown now is the present capital and not the future value.

Considering the same data that we have used on the question of the beginning of the chapter, the value that we are willing to renew in the investment is 24,761.90 euros in face of 26,000 euros that are being offered for after two years. The result was obtained from $26,000 = C + C \times 2 \times 2.5\%$ or $C = \frac{26,000}{1 + 2 \times 2.5\%}$. By investing, at most, the 24,761.90 euros, we can get, at least, the 2.5% annual return that consist, in this example, in our cost of opportunity. If we invest the 25,000 euros, we already know that we will only get 2% per year.

Compound Interest

Although the name of this methodology, compared to the previous one, suggests a greater complexity or difficulty of calculation, this is not true and, for certain operations, such as with many financial flows, this is the system that turns out to be most simple of using. In addition, and as we will see by its main features, this is the system that includes in its calculation the interest compounding for all streams generated in an operation, without exception. For these reasons, it is no wonder that there is a clear preference for the use of this interest system by the participants in the financial markets and investments evaluation in general.

Despite the differences between them, the two regimes are bound to follow the same basic methodology, they have to respect the principle of the opportunity cost.

The main features of the Compound Interest are the following:

- Mostly used in medium and long term (more than one year and five years, respectively);
- The interest, for each period of time, is always calculated on the basis of the capital at the time of the beginning of the period in question (with accrued interest of prior periods);
- The interest, determined in a period, earns interest in following periods;
- The interest is not proportional to the elapsed time;
- Being C the initial capital, r the annual[1] interest rate and n the number of years, the future value, or compound value, M, is given by: $M = C \times (1 + r)^n$ that results from simplifying $M = C \times \underbrace{(1 + r) \times (1 + r) \times .... \times (1 + r)}_{n \text{ times}}$.

Also in this system we will consider, to answer the specific question of the beginning of the chapter, that the interest rate corresponding to the opportunity cost is 2.5% per year. So, in Compound Interest, the value that the Bank should return at the end of two years is 26,265.63 euros, because $M = 25,000 \times (1 + 2.5\%)^2$.

Note that the future value at the end of two periods of time in which the rate is defined, in this case two periods of one year, is greater in Compound Interest than in Simple Interest (26,265.63 euros against 26,250 euros), caused by the fact that in Compound Interest we have interest on interest from the second year compounding.

Decision Making

We can already answer the question that started this chapter in this other interest system. Comparing the «fair» value with the one promised by the Bank, also here we come to the conclusion that the latter is insufficient. So, we reject the offer of our current Bank and we must renew the investment in the alternative Bank that offers 2.5% per year.

The same additional issues may also be placed in this case:

- What is the level of the interest rate that is actually being offered by the Bank?
- What is the maximum amount that we will be willing to invest if the Bank only offers 26,000 euros at the end of two years?

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(1) The reference period in financial terms is the annual period. We will see later how to deal with rates that relate to a different period.

Effective Rate

Also here, the answer to the first of these questions leads us to the concept of effective annual rate, calculated now in Compound Interest.

In the same way as we did for the Simple Interest, in this we will use the equation of its specific compounding, being the unknown the effective rate that we want to find.

In the case of the question raised earlier: the Bank offers 26,000 euros, at the end of two years, to renew the investment of 25,000 euros. What is the effective annual rate that is offering? Answer: $26,000 = 25,000 \times (1 + r)^2 \Leftrightarrow r = 1.9804\%$. Each year, the annual rate offered on Compound Interest is 1.9804%. Note, that if we consider that the period of the rate is two years, the offered interest rate would be 4% ($n = 1$), and we confirm the non-proportionality of interest in relation to time, the unmistakable feature of the compound interest.

Finally we call the attention for the fact that the annual rate is, in this case, lower than the one found in Simple Interest (1.9804% against 2%). This is due to the fact that we are facing a two-year operation in which, in the case of compound interest, we managed to get an increase in interest in the second year (by compounding the interest on the interest of the first year), so the rate does not need to be as high to get to same future value at the end.

Discounting and Present Value

Once again the second of those questions leads us to do the opposite calculation of compounding: discounting. Also in Compound Interest we want to know what will be the value that we are willing to consider in the present for the renewal of the financial transaction, in face of a concrete value that is being offered to a future moment.

In Compound Interest, using the respective equation of compounding, the value with which we are willing to renew the investment of the example we have been following is 24,747.17 euros, in face of 26,000 euros that are being offered for two years from now. The result was obtained from $26,000 = C \times (1 + 2.5\%)^2$ or $C = \dfrac{26,000}{(1 + 2.5\%)^2}$. By investing a maximum of 24, 747.17 euros, we will be able to get at least the 2.5% annual return that is, in this example, our opportunity cost. If we invest 25,000 euros, we already know that we will only get 1.9804% per year.

Stated Rate *versus* Effective Rate

Let's think now in a little different question from what we have treated since the beginning of the chapter. Let us assume that we renew the financial investment by 25,000 euros, for two years, but now the Bank will calculate the interest every month.

If the annual interest rate declared by the Bank for this operation is 2%, to calculate the interest, each month, we will have to divide that by 12, obtaining a monthly rate of 0.1667%. Note, that if you do the calculation of interest month by month, 12 months in the year, it results in a different cumulative interest, at the end of the year, from the one we can get by doing a single annual calculation. And this is because, regardless of the interest being actually paid or not, by calculating interest on a monthly basis we are enabling one of the two following possibilities:

- That this interest, if paid, could be put to earn interest in a new investment;
- That this interest, if not paid and kept in the investment itself, can earn interest in the following months.

Thus, if we calculate the annual interest at once with the declared rate, we would get a smaller compound value than the one we get when we calculate the interest more than once a year, in this case 12 times. This leads us to an apparent inconsistency between the annual rate of 2%, presented by the Bank, and the annual effective rate that results from the effective interest that we can get at the end of a year by calculating it more than once during that period. That rate declared by the financial institution consists of a stated rate (facial, apparent) by counterpoint to the effective rate that reflects the true interest accumulated in the same period. This effective rate is to be calculated by including the effect of interest on interest.

Let us proceed to the calculation of the effective annual rate, for our example, corresponding to a stated annual rate (SAR) of 2%, when the calculation of interest is done on a monthly basis. So, that annual rate is an effective rate of 2.0184%, resulting from $1 + r = \left(1 + \frac{2\%}{12}\right)^{12}$ thus $r = (1 + 0.1667\%)^{12} - 1$. Note that, in reality, what we are doing is to repeat a monthly interest compounding by 12 months, on the system that calculates interest on interest, the Compound Interest.

To clarify the differences between the various types of rates and their designations, we will make the following synthesis:

- $r_{(12)}$: stated annual rate with monthly compounding or, generally, $r_{(m)}$: stated annual rate with a frequency of m compoundings per year. This is the type of rate declared by the financial institutions, unless they indicate the effective rate. This stated rate cannot be used as it is, in the case of more than one com-

pounding per year. In this case it has to be divided by the frequency of interest calculation in the annual period;

- r_{12}: effective monthly rate, that is, the result of dividing the stated annual rate by the number of compoundings in the year, in this case 12, or, generically, r_m: effective rate for the sub-period repeating m times in the year. This is the rate that can be used in the calculation of the sub-period interest: $r_m = \frac{r_{(m)}}{m}$;
- r: effective annual rate (EAR) that results from $1 + r = \left(1 + \frac{r_{(m)}}{m}\right)^m$ or $1 + r = (1 + r_m)^m$. If we want to use this rate to calculate future values, we can do it only if we use it once in the annual period.

Let's see if we are already at ease in the use of the several possible rates. Imagine that, in our example, we need to know what is the future value of our investment after six months. The same can be obtained from the following ways:

- $M = 25,000 \times \left(1 + \frac{2\%}{12}\right)^6 = 25,251.04$, using the stated annual rate, forcibly transformed into effective monthly;
- $M = 25,000 \times 1.001667^6 = 25,251.04$, using directly the effective monthly interest rate;
- $M = 25,000 \times (1 + 2.0184\%)^{0.5} = 25,251.04$, using the effective annual rate, but to compound only for half a year.

And what will be then the effective rate for the six-month period? Following the above calculations, that rate can be obtained by the following ways:

- $1 + r_2 = \left(1 + \frac{r_{(12)}}{12}\right)^6$ or $1 + r_2 = \left(1 + \frac{2\%}{12}\right)^6 \Leftrightarrow r_2 = 1.0042\%$;
- $1 + r_2 = (1 + r_{12})^6$ or $1 + r_2 = 1.001667^6 \Leftrightarrow r_2 = 1.0042\%$;
- $1 + r_2 = (1 + r)^{0.5}$ or $1 + r_2 = (1 + 2.0184\%)^{0.5} \Leftrightarrow r_2 = 1.0042\%$.

Thus, $M = 25,000 \times (1 + 1.0042\%) = 25,000 \times 1.010042 = 25,251.04$ euros, as it could not be otherwise. Note that the effective semi-annual rate is being used only once in the semester, while the effective monthly rate is used six times in the semester and the effective annual rate is used a «half» of its time in the semester. The stated annual rate was never used directly.

Annuity

Let's consider the following example, a variant of the initial case: we do the investment of 25,000 euros, at the annual rate of 2%, with the perspective of receiving from the Bank not only a future value but rather a set of future cash flows, in this case a constant annual amount during 20 years. It is the case in which an amount invested today will give rise to multiple future regular payments. At first glance it seems that this financial operation does not fit in the compounding or discounting equations that we looked before. But in fact, if we analyse better, we can conclude that, at the present moment we are not doing a sole investment but twenty. The shortest will last one year and the longest will have a maturity of 20 years. The condition that must be met is that the future annual cash flows must be all the same. This way, the 20 amounts to be invested in the present will be all different, since each one will earn different amounts of interest, caused by their different maturities. Thus, we can establish a single equation that results from the sum of the several, in this case 20, compounding/discounting equations that we identify in the operation in case. Without this single equation we would have more unknowns than there are equations. Let's see:

With A_n representing the investment, made today, that will mature in n and R the annual constant cash flow, we have,

- $A_1 \times (1 + 2\%) = R$;
- $A_2 \times (1 + 2\%)^2 = R$;
- ...;
- $A_{20} \times (1 + 2\%)^{20} = R$.

or, using the discounting equations,

- $A_1 = \dfrac{R}{(1 + 2\%)}$;
- $A_2 = \dfrac{R}{(1 + 2\%)^2}$;
- ...;
- $A_{20} = \dfrac{R}{(1 + 2\%)^{20}}$.

As we know that $A_1 + A_2 + ... + A_{20} = 25,000$, then,

$$25,000 = \frac{R}{(1+2\%)} + \frac{R}{(1+2\%)^2} + \frac{R}{(1+2\%)^3} + ...$$
$$... + \frac{R}{(1+2\%)^{19}} + \frac{R}{(1+2\%)^{20}}$$

We end up then with a sole equation with one unknown for which we can find the solution.

To this type of equation, more comprehensive than the simple compounding/discounting one analysed formerly, we give the name of equation of equivalence of capital. This represents the same concept of the other but just for more than a cash flow of each nature: in a financial transaction, the value of the set of cash flows of one nature (outflows) has to be equal to the value of the set of cash flows of different nature (inflows), when all are represented in the same moment in time. Note that all streams in that concrete equation are represented at the present moment (the future flows are discounted for today). If they were not represented in the same moment in time, the equality between the opposite nature flows could not be established, because of the effect of the interest that accrues with the passage of time.

We know that the equation represented above has a solution but getting it is time consuming and tedious. We have to add all the present values of the set of future cash flows, of the several *R*. However, when looking at the mathematical succession on the right side, we conclude that we can use the calculation formula of geometric progressions to replace the sum of that multiplicity of values by a much simpler calculation.

Thus, that same equation can be represented as follows:

$$25,000 = R \times \frac{1-(1+2\%)^{-20}}{2\%} \quad \Leftrightarrow \quad R = 1,528.92$$

Finding the solution to *R* reveals to be a much faster procedure.

To the set of *R* that mature in a regular pace in time, we give the name of Annuity, in this case of 20 annual flows. Because they are a constant value, it allows us to use that very simple calculation formula.

Generally, if we have a stream of regular payments that lasts for a certain number of periods, each payment assuming a constant value and occurring at the end of each period to which they relate, we say that we are in the presence of a constant ordinary annuity. The present value of this annuity, i.e., the sum of all the present value of the future payments, is represented as follows

$$R \times A_{\overline{n}|r} = R \times \frac{1-(1+r)^{-n}}{r}$$

Effective Global Annual Rate (EGAR)

We will use the financial operation of the previous example assuming now that the bank charges the customer a set of fees, which is not surprising in the context of the current banking practice.

Thus, we invest the 25,000 euros today to receive in a year's time and then every year for 19 times the constant value of 1,528.92 euros. We know that the interest rate used by the Bank is of 2% per year. However, the bank will charge the following fees: 0.5% of the principal at the beginning and 10 euros with each annual payment. Does the 2% annual rate reflect the true return on this financial transaction? Certainly not. The future payments were calculated above based on 2% interest rate but now, adding the fees, we are subject to an increase in the initial outflow and a reduction of the future receivables. Of course, we are led to ask this question: what is then the effective rate of return of our investment? What is the rate that already includes all the costs that we have to bear in order to obtain that financial transaction? In other words, what is the Effective Global Annual Rate (EGAR) of this operation?

Note that with the interest rate offered by the Bank we were able to determine the cash flows to be received in the future. Then, with these known streams and subsequently adjusted to all the extra fees and taxes, we can compute the overall effective rate, that is, the true rate of the transaction as a whole.

Let's calculate the EGAR for the example we have been following. We will establish a capital equivalence equation, as we did previously, but the streams are now all known and include all fees, and the rate to compound or discount on the equation cannot be a simple interest rate but rather the global effective rate that we want to determine.

Thus, being t the EGAR,

$$25,000 \times 1.005 = (1,528.92 - 10) \times \frac{1 - (1+t)^{-20}}{t} \quad \Leftrightarrow \quad t = 1.8806\%,$$

solved using, for example, the IRR function of the Microsoft Excel.

The EGAR is our unknown in a capital equivalence equation that includes the real streams that will happen in the financial transaction, and not merely those that result from the establishment of the interest. It is then an internal or intrinsic rate for the transaction in question, and can be determined for any type of operation, whether it is an investment, as in this case, or a financing, the most common case of operations with several fees.

Formulas

■ SIMPLE INTEREST

- Compound or Future Value = Present Value ×
  $$\times(1 + \text{number of periods} \times \text{interest rate})$$

- Present Value = $$\frac{\text{Future Value}}{1 + \text{number of periods} \times \text{interest rate}}$$

- Effective Annual Interest Rate = Rate of the sub-period ×
  $$\times \text{number of sub-periods in the year}$$

■ COMPOUND INTEREST

- Compound or Future Value = Present Value ×
  $$\times (1 + \text{interest rate})^{\text{number of periods}}$$

- Present Value = $$\frac{\text{Future Valule}}{(1 + \text{interest rate})^{\text{number of periods}}} =$$
  $$= FV \times (1 + \text{interest rate})^{-\text{number of periods}}$$

- 1 + effective annual interest rate =
  $$= (1 + \text{rate of the sub-period})^{\text{number of sub-periods in the year}}$$

- rate of the sub-period = $$\frac{\text{stated annual interest rate}}{\text{number of sub-periods in the year}}$$

- $1 +$ effective annual interest rate $=$

$$= \left(1 + \frac{\text{stated annual interest rate}}{\text{number of sub-periods in the year}}\right)^{\text{number of sub-periods in the year}}$$

- Present Value of the Annuity with n payments equal to $T = T \times A_{\overline{n}|r} =$

$$= \text{Value of the constant payment} \times$$

$$\times \frac{1 - (1 + \text{interest rate of the annuity period})^{-\text{number of payments of the annuity}}}{\text{interest rate of the annuity period}}$$

- Present Value of the Perpetuity $=$

$$= \frac{\text{Perpetual constant payment}}{\text{interest rate of the annuity period}}$$

- Present Value of the Perpetuity with growth $=$

$$= \frac{\text{first payment}}{\text{interest rate of the annuity period} - \text{growth rate of payments}}$$

Solved Exercises

EXERCISE 1.01

Mr Alfa lent 1,000 euros to a friend for an undefined period of time. Both agreed on a 5% annual interest rate. After three years the friend had already paid three annual values of 50 euros each, and by then wanted to reimburse his debt.

a) How much does the friend still owes to *Mr Alfa*?

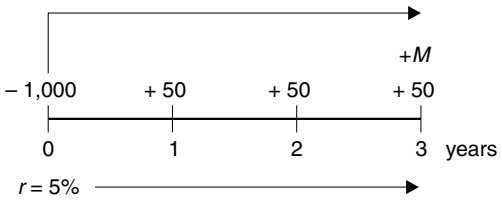
b) What is your opinion regarding the interest counting method used?

Solution

a) Annual interest:

$$J = C \times r \times n \quad \Leftrightarrow \quad J = 1,000 \times 0.05 \times 1 = 50$$

The friend paid the interest amount at the end of each year, owing the initial capital until the end of the three years.



b) At first glance it looks like *Mr Alfa* lent the money under Simple Interest (SI). However, he was given the opportunity to invest the interest earned, making it a Compound Interest operation (CI). The only instance where one cannot reinvest the interest is when its calculation is not associated with its immediate payment.

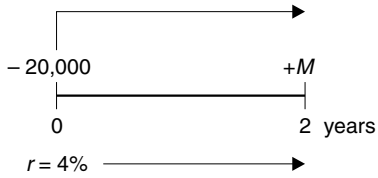
EXERCISE 1.02

Mr Sorte won a prize of 20,000 euros in a TV contest. He is given the choice between receiving that amount immediately or receiving 22,000 euros after two years. Before making a decision *Mr Sorte* called his bank account manager who offered 4% per year for a deposit for two years in CI. In your opinion, should *Mr Sorte* accept the immediate payment of the 20,000 euros or should he wait two years to receive 22,000 euros?

Solution

The question we need to answer is if *Mr Sorte* should receive the 20,000 euros immediately and make a deposit or wait two years to receive 22,000 euros. To answer this question we can follow two distinct paths:

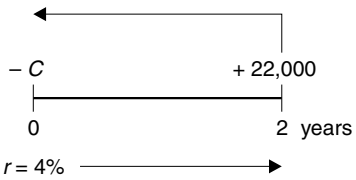
1. Determine how much are 20,000 euros worth after two years (compound) and compare the result with the 22,000 euros:



$$M = C \times (1 + r)^n \quad \Leftrightarrow \quad M = 20,000 \times (1 + 0.04)^2 = 21,632 < 22,000$$

Mr Sorte should receive the 22,000 euros after two years because if he makes a 20,000 euros deposit today he will only have 21,632 euros at the end of those two years.

2. Determine how much should *Mr Sorte* receive today (discount) for that amount to be equivalent to the 22,000 euros in two years' time:



$$C = M \times (1 + r)^{-n} \quad \Leftrightarrow$$

$$\Leftrightarrow \quad C = 22,000 \times (1 + 0.04)^{-2} = 20,340.24 > 20,000$$

For both alternatives to be equivalent, *Mr Sorte* should receive today 20,340.24 euros. Since he is only given the choice to receive 20,000 euros, he is better off waiting two years and receive the 22,000 euros.

EXERCISE 1.03

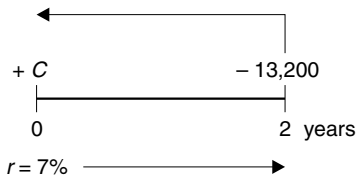
Firm *Sempo* will be receiving from the IRS a tax reimbursement of 13,200 euros two years from now. Knowing that the bank charges 7% per year for a two year loan:

a) How much should the firm ask for a loan today, assuming SI, if it wants to liquidate the loan with the amount received from the IRS?

b) Without performing additional calculations, if the bank had used CI instead of SI, the amount of the loan could have been higher or lower than the amount determined in *a)*?

Solution

a) The goal is to determine the amount that firm *Sempo* can ask today for a loan in order to reimburse the accumulated value (initial capital plus interest) in two years' time using the 13,200 euros that it will be receiving from the IRS. We can discount the 13,200 euros to the present day:



$$C = \frac{M}{1 + r \times n} \quad \Leftrightarrow \quad C = \frac{13,200}{1 + 0.07 \times 2} = 11,578.95$$

Today the firm can ask for a loan of 11,578.95 euros.

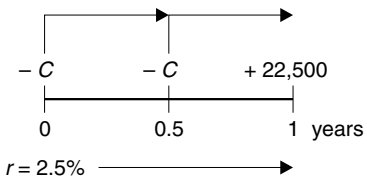
b) Since the duration of the loan (2 years) is higher than the period of the interest rate given (1 year), the CI returns a higher future value resulting from the fact that the interest in the second year will be determined over the initial capital but also over the interest due at the end of the first year. Thus, to obtain the same future value (13,200 euros) we need an amount that is less than the 11,578.95 euros determined using SI.

EXERCISE 1.04

Mr Investe wants to invest today and 6 months from now two equal values that must be sufficient to buy a new car one year from now. The car will cost 22,500 euros, at the time of the purchase. How much should *Mr Investe* invest if the bank offers an interest rate of 2.5% per year, considering CI?

Solution

The goal is to know how much Mr Investe should invest today and 6 months from now in order to have accumulated (capital plus interest) one year from now 22,500 euros, the amount necessary to buy the car. One way of solving is to equal the sum of the accumulated amounts from both investments to the 22,500 euros, one year from now:



$$22,500 = C \times (1 + 0.025)^{1} + C \times (1 + 0.025)^{0.5} \quad \Leftrightarrow \quad C = 11,043.36$$

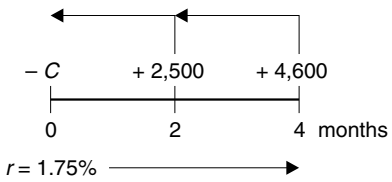
Mr Investe should make a deposit today and 6 months from now in the individual amount of 11,043.36 euros.

EXERCISE 1.05

The company *Poeira* is remodelling its headquarters. The contractor has warned that, in accordance to the budget, the company will have to pay 2,500 euros in two months and 4,600 euros 4 months from now. How much would the company deposit today, at the rate of 1.75% per year, if it wants to ensure the two cash flows in those dates?

Solution

The goal is to deposit today an amount that enables the company to withdraw 2,500 euros in two months while the remaining value stays invested for another two months yielding an accumulated value of 4,600 euros. The way of solving is to discount both values to the present day:



$$C = \frac{2,500}{(1 + 0.0175)^{\frac{2}{12}}} + \frac{4,600}{(1 + 0.0175)^{\frac{4}{12}}} = 7,066.26$$

Company *Poeira* should deposit today the amount of 7,066.26 euros.

EXERCISE 1.06

Mr Fair has two daughters which birthdays are today, one is 12 years old and the other is 10 years old. He wants to make a special deposit today, 10,000 euros in CI,

in a way that each daughter may receive the same value in their 18th birthday. The interest rate for the deposit is 4.5%.

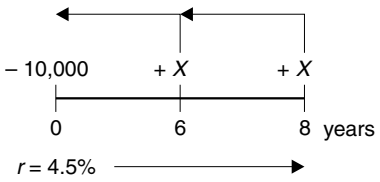
a) How much will each daughter receive for their 18th birthday?

b) In your opinion is *Mr Fair* really fair? Why?

c) Determine the amount that each daughter should receive for her 18th birthday in order for the father to be financially fair.

Solution

a) The goal is to invest today 10,000 euros, withdraw in 6 years (when the older daughter becomes 18 years old) a certain amount, while the remaining value stays invested for 2 additional years (until the younger daughter celebrates her 18th birthday) yielding an accumulated value equal to the amount received two years before by the older daughter. The way of solving is to equal the 10,000 euros invested today to the sum of the discounted amounts received by each daughter:



$$10,000 = \frac{X}{(1+0.045)^6} + \frac{X}{(1+0.045)^8} \quad \Leftrightarrow \quad X = 6,797.72$$

Each daughter will receive 6,797.72 euros for her 18th birthday.

b) No. Both daughters will receive the same amount with a two years' time difference. The amounts are financially not equivalent when we take into consideration the time value of money. The older daughter receives first the 6,797.72 euros and can invest this money for two years until the younger daughter becomes 18 years old, getting at that time an accumulated amount greater than her sister's 6,797.72 euros.

c) From the previous statement, the younger daughter should receive an amount greater than the older daughter, corresponding to the amount the latter receives, compounded for two years. We have:

$$10,000 = \frac{X}{(1+0.045)^6} + \frac{X \times (1+0.045)^2}{(1+0.045)^8} \quad \Leftrightarrow \quad X = 6,511.3$$

$$X \times (1+0.045)^2 = 6,511.3 \times (1+0.045)^2 = 7,110.5$$

The older daughter will receive 6,511.3 euros while the younger one will receive 7,110.5 euros. Intuitively, for the father to be fair to both daughters is enough to give them half the money at a specific point in time. For example, he could make a deposit today of 5,000 euros for each daughter. We can easily see that that is what is happening in the previous calculations where the 10,000 euros are worth 13,022.6 euros after 6 years. At that time this amount is split in two equal parts: 6,511.3 euros are given to the older daughter while the remaining 6,511.3 euros stay invested for the younger daughter.

EXERCISE 1.07

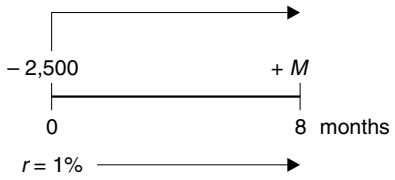
Mr Viroprego contracted with his Bank a savings deposit of 2,500 euros for three years at an interest rate of 2% in Compound Interest. After eight months he asked for its cancellation due to a sudden event. The Bank paid the interest but at a lower interest rate: 1% per year.

a) How much did *Mr Viroprego* receive?

b) If the interest had been determined at 1% but in Simple Interest, *Mr Viroprego* would have received, after the eight months, an amount higher or lower than the one determined in the previous question? Justify without performing any calculations.

Solution

a) The goal is to determine the accumulated value of the 2,500 euros after eight months at a 1% annual interest rate:



$$M = 2,500 \times (1 + 0.01)^{\frac{8}{12}} = 2,516.64$$

Mr Viroprego received 2,516.64 euros after the eight months.

b) He would have received a higher amount. This happens because the investment period is smaller than the compounding period: annual compounding (the rate offered by the Bank is annual) for an investment period of eight months. Compound Interest delivers an accumulated value higher than Simple Interest when the period of the operation is greater than the interest frequency.

EXERCISE 1.08

Identify the most common financial operations and classify them according to the type of interest more suitable for each one.

Solution

In Simple Interest the most common financial operations are typically short term (less than a year), namely deposits and treasury bills. In Compound Interest the most common financial operations are medium and long term bank loans, mortgage loans and leasing operations. Conversely to Simples Interest, Compound Interest is typically used in medium/long term financial instruments.

EXERCISE 1.09

Bank *Nostro* has launched an investment product with the following characteristics:

- Stated annual interest rate of 2.05%, guaranteed for 6 months;
- For each semester, in addition to the first, the rate increases by 0.15%, stabilizing at the end of 2 years;
- The interest is computed semi-annually (twice a year), compounding automatically.

With the purpose of creating a product flyer, complete the following table considering an initial investment of 500 euros:

	Sem 1	Sem 2	Sem 3	Sem 4
Capital invested in the beginning of the period				
SAR for the period				
Effective rate for the period				
Interest for the period				
Compounded value at the end of the period				
Effective Annual Rate				
Average Effective Annual Rate				

Solution

	Sem 1	Sem 2	Sem 3	Sem 4
Capital invested in the beginning of the period	500	505.13	510.68	516.68
SAR for the period	2.05%	2.2%	2.35%	2.5%
Effective rate for the period	1.025%(1)	1.1%	1.175%	1.25%
Interest for the period	5.13	5.56	6	6.46
Compounded value at the end of the period	505.13	510.68	516.68	523.14
Effective Annual Rate	2.14%(2)		2.44%	
Average Effective Annual Rate	2.29%(3)			

(1) To determine the effective rate for the semester (r_2), using the SAR ($r_{(2)}$), we just need to use the concept of proportionality:

$$r_2 = \frac{r_{(2)}}{2} = \frac{0.0205}{2} = 0.01025$$

(2) The effective annual rate can be obtained from the effective rates of the corresponding semesters or using the amount from the beginning and end of the year:

$$(1 + r) = (1 + 0.01025) \times (1 + 0.011) \quad \Leftrightarrow \quad r = 0.0214$$

or

$$500 \times (1 + r) = 510.68 \quad \Leftrightarrow \quad r = 0.0214$$

(3) Like in the previous calculation, it is possible to obtain the effective annual rate in two ways:

$$(1 + r)^2 = (1 + 0.0214) \times (1 + 0.0244) \quad \Leftrightarrow \quad r = 0.0229$$

or

$$500 \times (1 + r)^2 = 523.14 \quad \Leftrightarrow \quad r = 0.0229$$

EXERCISE 1.10

Mr Incha has invested the amount of 7,000 euros in his bank for a term of two years at Compound Interest, with fixed interest rate. Knowing that the compound value at the end was 7,688.13 euros and that the inflation rate in the first year was 4% and in the second 3.6%, compute:

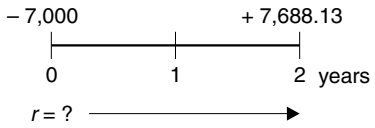
a) The real interest rate in each year;

b) The average real interest rate;

c) If *Mr Incha* wants to increase his purchasing power by 10% at the end of the two years, how much must the investment fixed interest rate be?

Solution

a) To determine the real interest rate, we must first determine the annual interest rate, in nominal terms. Since this rate is fixed (it is the same for both years), we get:



$$7,000 \times (1 + r)^2 = 7,688.13 \quad \Leftrightarrow \quad r = 0.048$$

Now it is possible to compute the real interest rate for each year since we already have the nominal and inflation rates:

$$\text{Year 1: } r_{\text{real 1}} = \frac{1 + 0.048}{1 + 0.04} - 1 \quad \Leftrightarrow \quad r_{\text{real 1}} = 0.0077$$

$$\text{Year 2: } r_{\text{real 2}} = \frac{1 + 0.048}{1 + 0.036} - 1 \quad \Leftrightarrow \quad r_{\text{real 2}} = 0.0116$$

b) The average real interest rate can be determined through the geometric mean (to be coherent with Compound Interest):

$$(1 + r_{\text{real average}})^2 = (1 + 0.0077) \times (1 + 0.0116) \quad \Leftrightarrow \quad r_{\text{real average}} = 0.0096$$

c) The increase in purchasing power is measured in real terms. In order for *Mr Incha* to increase his purchasing power by 10% at the end of the two years, the return on the investment in nominal terms must be 10% higher than the evolution in prices.

$$(1+r)^2 = (1+0.04)\times(1+0.036)\times(1+0.1) \quad \Leftrightarrow \quad r = 0.0887$$

The fixed annual interest rate should be 8.87%. We can confirm this result by determining the real annual interest rates and the real interest rate for two years:

$$\text{Year 1: } r_{\text{real 1}} = \frac{1+0.0887}{1+0.04} - 1 \quad \Leftrightarrow \quad r_{\text{real 1}} = 0.0468$$

$$\text{Year 2: } r_{\text{real 2}} = \frac{1+0.0887}{1+0.036} - 1 \quad \Leftrightarrow \quad r_{\text{real 2}} = 0.0509$$

$$\text{For two years: } r_{\text{real biannual}} = (1+0.0468)\times(1+0.0509) - 1 = 0.1$$

In real terms we obtain a return of 10% as we wanted.

EXERCISE 1.11

Bank *Rating* offered a company a 10,000 euros loan for one year with a stated annual interest rate of 6% with interest compounding every two months. The company accepted this offer while rejecting the offer presented by its former bank which had similar characteristics with the exception of having interest compounded every three months. What must have been the minimum stated annual rate offered by the former bank in order for the company to have chosen bank's *Rating* offer? (Perform the calculations on a monthly basis)

Solution

To establish preferences relations between the two alternatives one must use effective interest rates. Let us begin by determining the effective annual rate offered by bank *Rating*:

$$r = \left(1+\frac{r(6)}{6}\right)^6 - 1 = \left(1+\frac{0.06}{6}\right)^6 - 1 = 0.0615$$

Next we can determine the stated annual interest rate offered by the former bank that led to equal preferences:

$$0.0615 = \left(1 + \frac{r_{(4)}}{4}\right)^4 - 1 \quad \Leftrightarrow \quad r_{(4)} = 0.0601$$

Assuming that the firm chose the least costly alternative, which was offered by bank *Rating*, we can conclude that the stated annual interest rate offered by the former bank must have been higher than 6.01%.

EXERCISE 1.12

A company has the following alternatives for a one year funding:

- Pay interest quarterly with a stated annual interest rate of 8%;
- Pay interest monthly with an effective annual interest rate of 8.2%.

What alternative should the company choose?

Solution

Since we already know the effective annual interest rate for the second alternative, we only need to determine that same rate for the first alternative:

$$r = \left(1 + \frac{r_{(4)}}{4}\right)^4 - 1 \quad \Leftrightarrow \quad r = \left(1 + \frac{0.08}{4}\right)^4 - 1 = 0.0824$$

Since we are dealing with a financing operation, the company should chose the least costly alternative, which in this case is the second one with an effective annual interest rate of 8.2%.

EXERCISE 1.13

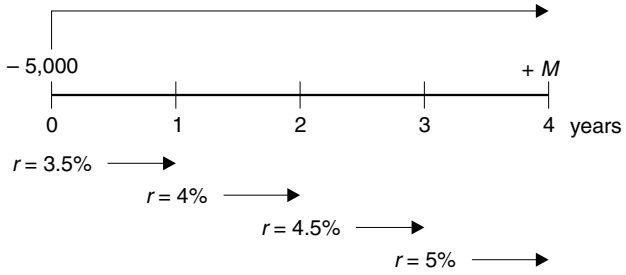
Bank *Genesis* recently created a special four years savings account with an initial stated annual interest rate of 3.5%, with compound annual interest, and with that rate increasing 0.5 percentage points every year.

a) If you invest today 5,000 euros, how much will you have at the end of the four years?

b) Compute the average annual interest rate obtained in the investment.

Solution

a) The goal is to determine the accumulated value of the 5,000 euros at the end of the four years, using an increasing stated annual interest rate. Since the interest is compounded yearly, the effective annual interest rate has the same value of the stated annual interest rate. From that it follows:



$$M = 5,000 \times (1 + 0.035) \times (1 + 0.04) \times (1 + 0.045) \times (1 + 0.05) \quad \Leftrightarrow$$

$$\Leftrightarrow \quad M = 5,905.4$$

At the end of the four years the account will have an accumulated value of 5,905.4 euros.

b) Since we already know the initial and final amounts, we can easily determine the average annual interest rate on the investment:

$$5,000 \times (1 + r)^4 = 5,905.4 \quad \Leftrightarrow \quad r = 0.0425$$

We can also calculate the same rate using the geometric mean of the four rates used previously in *a)*:

$$r = \sqrt[4]{(1 + 0.035) \times (1 + 0.04) \times (1 + 0.045) \times (1 + 0.05)} - 1 = 0.0425$$

The average annual interest rate is 4.25%.

EXERCISE 1.14

A three month bank loan was undertaken at a stated annual interest rate of 5%. What would have been the stated annual interest rate equivalent to this if the funding was made for three consecutive periods of one month with automatic compounding?

Solution

To solve this question the easiest way is to equal the effective quarterly interest rates of the two alternatives:

$$\frac{0.05}{4} = \left(1 + \frac{r_{(12)}}{12}\right)^3 - 1 \quad \Leftrightarrow \quad r_{(12)} = 0.0498$$

The equivalent stated annual interest rate is 4.98%.

EXERCISE 1.15

The effective annual interest rate of a 3 month deposit is 5%. Another deposit of 1.5 months has the same stated annual interest rate as the previous one. Compute the effective annual interest rate of the second deposit.

Solution

The link between the two deposits is made through the stated annual interest rate, which is the same in both cases. Thus, the first step is to determine the stated annual interest rate, with quarterly compounding, for the first deposit:

$$0.05 = \left(1 + \frac{r_{(4)}}{4}\right)^4 - 1 \quad \Leftrightarrow \quad r_{(4)} = 0.0491$$

The next step is to compute the effective annual interest rate of the second deposit equivalent to the stated annual interest rate of 4.91%, with interest compounding every 1.5 months:

$$r = \left(1 + \frac{r_{(8)}}{8}\right)^8 - 1 = \left(1 + \frac{0.0491}{8}\right)^8 - 1 = 0.0502$$

The effective annual interest rate is 5.02%.

EXERCISE 1.16

An investor is not sure about which of the following investment alternatives he should choose:

1. A deposit account with compounding monthly interest at a stated annual interest rate of 14%;
2. A bond, with a market price of 93.5 euros, that pays 100 euros at maturity in 6 months.

Showing all calculations, determine which is the best alternative for the investor.

Solution

We can compare both alternatives using the effective interest rate for the semester:

$$1:\ r_2 = \left(1 + \frac{r_{(12)}}{12}\right)^6 - 1 = \left(1 + \frac{0.14}{12}\right)^6 - 1 = 0.072$$

$$2:\ r_2 = \frac{100}{93.5} - 1 = 0.0695$$

Since we are dealing with an investment, the investor should choose the alternative with the highest rate. In this case he will choose alternative 1 where he will get an effective interest rate for the semester of 7.21%.

EXERCISE 1.17

The company *Bl&rg* was given the chance to choose between the following alternatives for a 3 year financial investment:

- Financial Institution 1: investment in which the interest is computed quarterly and is automatically compounded at a stated annual interest rate of 5%;
- Financial Institution 2: investment that compounds a value for three years at a growing effective annual interest rate of: 5%, 5.1% and 5.2%, in the 1st, 2nd and 3rd year respectively;
- Financial Institution 3: this investment offers an effective interest rate for a semester of 2.55%.

State, showing all calculations, in which Financial Institution should the company *Bl&rg* invest its savings.

Solution

We will use the effective annual interest rate to compare the three alternatives. For the first institution we need to determine the effective annual interest rate going from a stated annual interest rate with quarterly compounding; for the second one we just need to compute the average annual interest rate; and for the third institution we will compute the effective annual interest rate that is equivalent to the effective interest rate for the semester that is given:

$$1:\ r = \left(1 + \frac{r_{(4)}}{4}\right)^4 - 1 = \left(1 + \frac{0.05}{4}\right)^4 - 1 = 0.0509$$

$$2:\ r = \sqrt[3]{(1 + 0.05) \times (1 + 0.051) \times (1 + 0.052)} - 1 = 0.051$$

$$3:\ r = (1 + r_2)^2 - 1 = (1 + 0.0255)^2 - 1 = 0.0517$$

Company *Bl&rg* should invest its savings on the alternative offered by the Financial Institution 3, since this is the one offering a higher return, with an effective annual interest rate of 5.17%.

EXERCISE 1.18

Mr Doubt has 10,000 euros to invest for 9 months. After consulting the market he was in doubt between two alternatives:

- A deposit for 9 months, in Simple Interest, with a stated annual interest rate of 4%;
- A deposit for 9 months, in Compound Interest, with a stated annual interest rate of 4% with annual compounding.

Note: If necessary, perform the calculations on a monthly basis.

a) Which of the two alternatives should be chosen in order to maximize the future value at the end of the 9 month period? Justify.

b) For both alternatives to return the same accumulated value at the end of the 9 months, which should be the compounding frequency (the period between two due interests) for the second investment? Justify.

Solution

a) Since both alternatives have the same stated annual interest rate and knowing that in the second alternative the effective annual interest rate has the same value of the stated rate, because the compounding period is annual, the investment in Simple Interest will be more beneficial to *Mr Doubt*. This happens because the investment period (9 months) is inferior to the compounding period of the given rate. For numerical confirmation we have:

$$SI: \ M = 10,000 \times \left(1 + 0.04 \times \frac{9}{12}\right) = 10,300$$

$$CI: \ M = 10,000 \times (1 + 0.04)^{\frac{9}{12}} = 10,298.52$$

b) The compounding period should be the same as the investment period, in this case, 9 months. If the compounding period is less than 9 months, for example monthly, the investment in CI will yield a higher accumulated value. If it is more than 9 months (which happens in *a)*) the opposite will happen, that is, the final value obtained will be higher using SI.

EXERCISE 1.19

Company *Dados* is preparing the end of month forecasted cash flows map. There are doubts regarding the value to pay on a credit operation contracted almost two years ago, from which the last of two annuities will be due precisely at the end of the month. The principal amount (capital borrowed/lent) is 40,000 euros and the stated annual interest rate is 6%.

a) Knowing that the first instalment amounted to 21,817.48 euros, what is the amount the company has to pay at the end of the month for the debt to be paid in full?

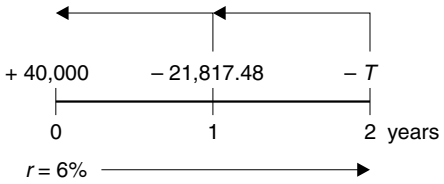
b) Make the cash flow map for the credit operation identifying for each year the amount owed at the beginning and at the end of the period, the amount of interest paid and capital reimbursed, and the consequent instalment value;

c) Assume now that the company had asked for a 3 year loan with a grace period of one year and payment in annuities. Make the new cash flow map and comment on the results obtained;

d) Present the cash flow map assuming that the first year, instead of a grace period, corresponds to a deferral on the debt servicing. What are the major changes in the results obtained?

Solution

a) The 40,000 euros have to be equal to the sum of the present values of all future payments (instalments that are part interest and part principal). Thus, and for the present time:



$$40,000 = \frac{21,817.48}{1 + 0.06} + \frac{T}{(1 + 0.06)^2} \quad \Leftrightarrow \quad T = 21,817.47$$

The final value to pay at the end of the month will be 21,817.47 euros, that is, the loan was paid through constant instalments of principal and interest.[1]

b) Cash flow map:

	***k* (period)**	
	1	**2**
A_{k-1} (amount owed at the beginning)	40,000	20,582.52
$T_k = J_k + R_k$ (instalment)	21,817.48	21,817.47
$J_k = A_{k-1} \times r$ (interest)	$0.06 \times 40,000 =$ $= 2,400$	$0.06 \times 20,582.52 =$ $= 1,234.95$
R_k (reimbursement)	$21,817.48 - 2,400 =$ $= 19,417.48$	20,582.52
$A_k = A_{k-1} - R_k$ (amount owed at the end)	$40,000 - 19,417.48 =$ $= 20,582.52$	0

(1) The value differs in one cent due to rounding.

c) In the first year there is a grace period so the company will only pay interest without doing any reimbursement of capital. The value of the instalment is the same as previously computed in *a)*, since the amount owed at the beginning of year 2 remains the 40,000 euros.

	***k* (period)**		
	1	**2**	**3**
A_{k-1} (amount owed at the beginning)	40,000	40,000	20,582.52
$T_k = J_k + R_k$ (instalment)	2,400	21,817.48	21,817.47
$J_k = A_{k-1} \times r$ (interest)	$0.06 \times 40,000 = 2,400$	$0.06 \times 40,000 = 2,400$	$0.06 \times 20,582.52 = 1,234.95$
R_k (reimbursement)	0	$21,817.48 - 2,400 = 19,417.48$	20,582.52
$A_k = A_{k-1} - R_k$ (amount owed at the end)	40,000	$40,000 - 19,417.48 = 20,582.52$	0

d) In the case of a deferral of the debt servicing for one year, the interest payment and the capital reimbursement is postponed one year and paid at the end of year 2. Since the interest of the first year is not paid at the end of that period, its value is added to the capital owed. We can use the capital equivalence equation to determine the amount of the constant instalments of years 2 and 3. Mathematically this is the same as finding the future value of the 40,000 euros after one year and use this amount to determine the constant annual instalments to be paid the next two years. The capital equivalence equation is presented for the period 0 and thus we have to discount the instalments for 2 and 3 years:

$$40,000 = \frac{T}{(1+0.06)^2} + \frac{T}{(1+0.06)^3} \quad \Leftrightarrow \quad T = 23,126.52$$

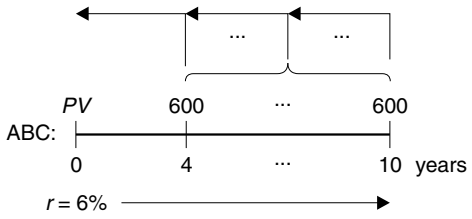
	***k* (period)**		
	1	**2**	**3**
A_{k-1} (amount owed at the beginning)	40,000	42,400	21,817.48
$T_k = J_k + R_k$ (instalment)		23,126.52	23,126.52
$J_k = A_{k-1} \times r$ (interest)		$0.06 \times 42,400 = 2,544$	$0.06 \times 21,817.48 = 1,309.05$
R_k (reimbursement)		$23,126.52 - 2,544 = 20,582.52$	21,817.48
$A_k = A_{k-1} - R_k$ (amount owed at the end)	42,400	$42,400 - 20,582.52 = 21,817.48$	0

EXERCISE 1.20

An investment named *ABC* offers constant revenues of 600 euros per year, from the end of the 4th until the end of the 10th year. Another investment, named *DEF*, offers a lower annual cash flow of 520 euros, but from the end of the 1st year until the end of the 10th year. Both require an initial investment of 10,000 euros and this principal is repaid at the end of the 10th year. Which of the two investments would you choose, given a stated annual interest rate of 6%?

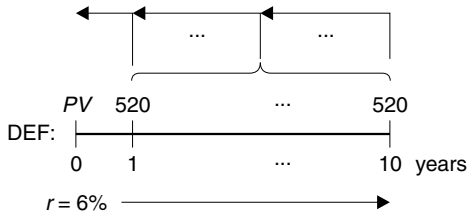
Solution

The two alternative investments will be compared using their present values, that is, through the summation of the present values of all future cash flows. Since the amount invested and the moment of its reimbursement is the same in both alternatives, these values will not be included in the calculations because they do not represent discriminating factors. The calculations are greatly simplified because in both cases we have finite constant annuities:



$$ABC:\ PV = 600 \times a_{\overline{7}|0.06} \times (1+0.06)^{-3} =$$

$$= 600 \times \frac{1-(1+0.06)^{-7}}{0.06} \times (1+0.06)^{-3} = 2,812.25$$



$$DEF:\ PV = 520 \times a_{\overline{10}|0.06} = 520 \times \frac{1-(1+0.06)^{-10}}{0.06} = 3,827.25$$

You should choose investment *DEF* because it has a present value higher than investment *ABC*.

EXERCISE 1.21

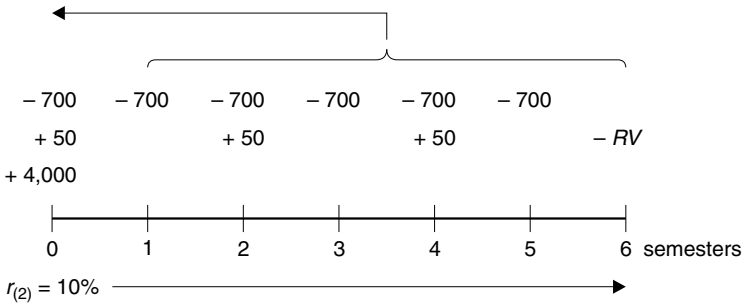
Mr Resíduo is acquiring a machine for his manufacturing business. Given the available alternatives he decided to undertake a 3 year leasing contract with anticipated semi-annual annuity payments. The machine costs 4,000 euros, and the leaser will be in charge of paying the anticipated annual insurance premium of 50 euros. In counterpart, *Mr Resíduo* will be paying a semi-annual payment of 700 euros. Considering a stated annual interest rate of 10% with semi-annual compounding:

a) What should be the residual value of the machine, established by the leaser, for this to be a fair deal to both parties?

b) If the annuities instead of being paid at the beginning of each period were to be paid at the end of those same periods, would the residual value determined in *a)* remain the same? Justify.

Solution

a) To solve this problem we will present the capital equivalence equation for moment 0 (today). On one side of the equation we will have the sum of the present values of the positive cash flows (from *Mr Resíduo's* perspective), and on the other side the sum of the present values of the negative cash flows. Since the payment of the insurance premium and of the leasing annuities represent finite constant annuities, the calculation of their present values is simplified. However, first we need to determine the effective interest rate for the semester (for determining the present value of the leasing annuities) and also the effective annual interest rate (for determining the present value of the insurance premiums). In the latter case, of annual compounding, we could have used directly the given stated annual interest rate, a common practice in financial institutions:



$$r_2 = \frac{r_{(2)}}{2} = \frac{0.1}{2} = 0.05$$

$$r = (1 + r_2)^2 - 1 = (1 + 0.05)^2 - 1 = 0.1025$$

Now we are able to proceed with the capital equivalence equation:

$$4,000 + 50 + 50 \times a_{\overline{2}|0.1025} =$$

$$= 700 + 700 \times a_{\overline{5}|0.05} + RV \times (1 + 0.1025)^{-3} \quad \Leftrightarrow$$

$$\Leftrightarrow \quad 4,000 + 50 + 50 \times \frac{1 - (1 + 0.1025)^{-2}}{0.1025} =$$

$$= 700 + 700 \times \frac{1 - (1 + 0.05)^{-5}}{0.05} + RV \times (1 + 0.1025)^{-3} \quad \Leftrightarrow$$

$$\Leftrightarrow \quad RV = 543.88$$

For this to be a financially fair deal, the residual value should be 543.88 euros. Note that the present value of the RV was obtained using the effective annual interest rate, but could have been done using the stated annual interest rate, as justified previously, or any other effective rate, adjusting the discounting periods accordingly (e.g. if the rate chosen was the effective interest rate for the semester, then the exponent should be – 6, since we would be discounting 6 semesters).

b) No, because of the time value of money. All annuities would be paid 6 months later which, simplifying, corresponds to a deferral of the first annuity from moment 0 (today) to the end of year 3. The present value of all leasing annuities would then be inferior and the only way for the contract to remain fair, that is, to keep the capital equivalence equation valid, would be to increase the residual value. The present value of the leasing annuities would decrease:

$$700 - 700 \times (1 + 0.05)^{-6} = 177.65$$

That would imply an increase in the present value of the residual value in the same amount:

$$177.65 = \Delta RV \times (1 + 0.1025)^{-3} \quad \Leftrightarrow \quad \Delta RV = 238.07$$

The residual value would then be $543.88 + 238.07 = 781.95$ euros.

EXERCISE 1.22

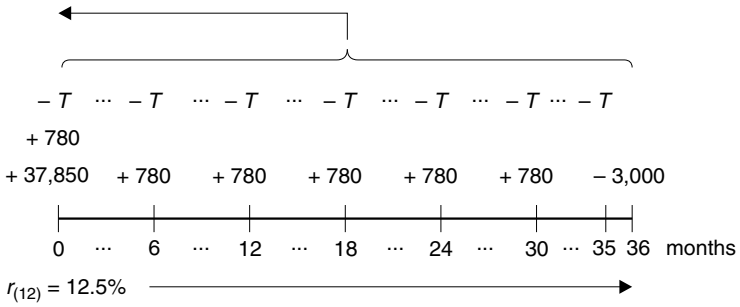
Mrs Fotocópia owns a printing business and wants to acquire a printing machine, valued at 37,850 euros, undertaking a 3 year leasing operation. The residual value at the end of the contract is 3,000 euros and the insurance premium is 780 euros and it is paid at the beginning of each semester by the leaser. Assuming a stated annual interest rate with monthly compounding of 12.5%:

a) Determine the leasing monthly annuity to be paid beginning today;

b) If the insurance premium was to be paid directly by *Mrs Fotocópia*, would the leasing annuity determined in *a)* suffer any change in value? Justify.

Solution

a) The solution to this problem is quite similar to question *a)* of the previous exercise. However, now the goal is to determine the leasing annuity and not the residual value. Before presenting the capital equivalence equation we first need to calculate the effective monthly interest rate (to be able to discount the leasing annuities) and the effective interest rate for the semester (to be able to discount the insurance premiums).



$$r_{12} = \frac{r_{(12)}}{12} = \frac{0.125}{12} = 0.0104$$

$$r_2 = (1 + r_{12})^6 - 1 = (1 + 0.0104)^6 - 1 = 0.064$$

Note that is likely that the financial institutions would have determined the effective interest rate for the semester dividing the stated annual interest rate by two. Fi-

nally, we have all the information needed to present the capital equivalence equation:

$$37,850 + 780 + 780 \times a_{\overline{5}|0.064} =$$
$$= T + T \times a_{\overline{35}|0.0104} + 3,000 \times (1 + 0.0104)^{-36} \quad \Leftrightarrow$$

$$\Leftrightarrow \quad 37,850 + 780 + 780 \times \frac{1 - (1 + 0.064)^{-5}}{0.064} =$$
$$= T + T \times \frac{1 - (1 + 0.0104)^{-35}}{0.0104} + 3,000 \times (1 + 0.0104)^{-36} \quad \Leftrightarrow$$

$$\Leftrightarrow \quad T = 1,317.81$$

The amount of the leasing annuity should be 1,317.81 euros. Again, note that the present value of the RV was obtained using the effective daily interest rate, but could have been obtained using any other effective rate, adjusting the discounting periods accordingly (e.g. if the rate chosen was the effective interest rate for the semester, then the exponent should be – 6, since we would be discounting 6 semesters).

b) The answer is yes. Now the leaser does not replace *Mrs Fotocópia* with her financial obligation, thus she no longer has to pay back to the leaser that amount, which was included in the leasing annuities. The value of the leasing annuity will then be less than it was before. If we take a look at the capital equivalence equation presented in *a)*, by removing the insurance premiums from the left side of the equation, we must reduce that same amount from the right side of the equation to maintain the equality:

$$PV_{Insurance} = 780 + 780 \times \frac{1 - (1 + 0.064)^{-5}}{0.064} = 4,030.2$$

The present value of the insurance premiums is 4,030.2 euros and corresponds to the decrease that the leasing annuities must face:

$$4,030.2 = \Delta T + \Delta T \times \frac{1 - (1 + 0.0104)^{-35}}{0.0104} \quad \Leftrightarrow \quad \Delta T = 133.4$$

The new value of each leasing annuity would be $1,317.81 - 133.4 = 1,184.41$ euros. However, let us not forget that the obligation of paying directly the insurance premiums now belongs to *Mrs Fotocópia*.

EXERCISE 1.23

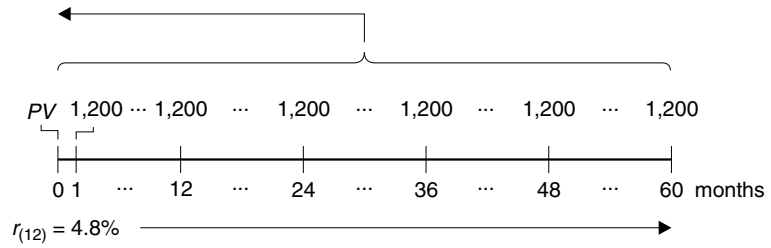
A retired woman has the right to receive a monthly pension of 1,200 euros over the next 5 years. In order to be able to achieve less financial constrains during the last years of her life, she wants to extend the period of the pension to 8 years. Considering a stated annual interest rate of 4.8%:

a) What is the new value for the monthly pension?

b) If, after 5 years, she wants to anticipate the receipt of the remaining amounts, how much would she get?

Solution

a) The purpose of the new deal is to receive less in each month but during more months. The total amount to be received must be financially equivalent. We begin by determining the present value of the base case and then use this value to find the monthly pension for the next 8 years. To determine the present value of the current monthly pension we first need to calculate the effective monthly interest rate from the stated annual interest rate:



$$r_{12} = \frac{r_{(12)}}{12} = \frac{0.048}{12} = 0.004$$

$$PV = 1,200 \times a_{\overline{60}|0.004} = 1,200 \times \frac{1-(1+0.004)^{-60}}{0.004} = 63,898.64$$

Now we have the information to present the capital equivalence equation where the unknown variable is the new monthly pension (here we are assuming that the pension is received at the end of each month):

$$63,898.64 = T \times a_{\overline{96}|0.004} \quad \Leftrightarrow$$

$$\Leftrightarrow \quad 63,898.64 = T \times \frac{1-(1+0.004)^{-96}}{0.004} \quad \Leftrightarrow$$

$$\Leftrightarrow \quad T = 802.88$$

The new monthly pension value is 802.88 euros.

b) At the end of the 5 years there are still 36 months of pensions to receive. To know the value of those 36 pensions we just need to discount and sum them all at the end of year 5, that is, we need to take out the interest part included in each pension amount:

$$PV_{Year\,5} = 802.88 \times \frac{1-(1+0.004)^{-36}}{0.004} = 26,869.08$$

At the end of year 5 the total amount that she would receive is 26,869.08 euros.

EXERCISE 1.24

A couple was repaying a 10 years mortgage loan and today agreed with their bank an increase in the duration of the contract for an additional 10 years after the end date of the original contract, in order to decrease the weight of the loan in their monthly budget. Knowing that the terms of the loan were as follows:

- Monthly instalment of the original contract: 734 euros;
- Stated annual interest rate: Euribor 12 m + 0.9%;
- Original interest rate: 4.25%;
- Euribor 12 m at the moment of the contract change (today): 3.05%;
- Today, there are still 87 instalments to pay from the original contract;

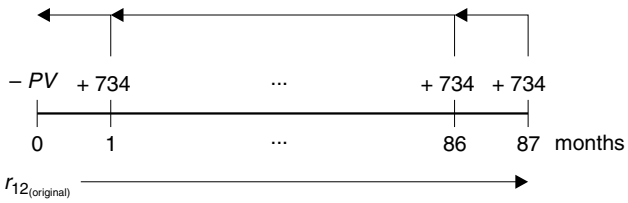
- The change to the contract was made immediately after the payment of that month's instalment;
- In the calculation of the new annuity was considered the interest rate arising from the current market conditions.

a) Compute the new monthly instalment the couple has to pay from now on;

b) What other alternatives (theoretically) are there that could lead to the decrease in the monthly instalment?

Solution

a) To be able to compute the new monthly instalment one must first know the amount owed at the renegotiation date. To determine that amount we must discount to the present day all forthcoming instalments. Since the instalments are an annuity of finite (87 instalments left) and constant terms (734 euros), the calculations are simplified and we must use the effective monthly interest rate (the frequency of the instalments):



$$r_{12} = \frac{r_{(12)}}{12} = \frac{0.0425}{12} = 0.0035$$

$$PV = 734 \times a_{\overline{87}|0.0035} = 734 \times \frac{1-(1+0.0035)^{-87}}{0.0035} = 54,969.43$$

Now that we have the total amount owed, our goal is to determine the new monthly instalment. To proceed with the calculations we first need to determine the new effective monthly interest rate and consider 207 months (87 months remaining from the original contract plus the additional 10 years/120 months):

$$r_{(12)} = \text{EURIBOR}_{12M} + 0.9\% = 0.0305 + 0.009 = 0.0395$$

$$r_{12} = \frac{r_{(12)}}{12} = \frac{0.0395}{12} = 0.0033$$

$$54,969.43 = T \times a_{\overline{207}|0.0033} \quad \Leftrightarrow$$

$$\Leftrightarrow \quad 54,969.43 = T \times \frac{1 - (1 + 0.0033)^{-207}}{0.0033} \quad \Leftrightarrow$$

$$\Leftrightarrow \quad T = 366.92$$

The value of the new monthly instalment is 366.92 euros.

b) In theory on could consider the following alternatives:

- A reduction in the spread used by the financial institution, leading to a reduction in the stated annual interest rate and consequently reducing the effective monthly interest rate to use in the instalment calculations. The spread reduction can be achieve, for example, by giving additional collateral;
- An anticipated reimbursement of capital, which would lead to the reduction in the capital owed;
- Changing the reference rate, for example using the 6 months EURIBOR or one month EURIBOR, which in normal circumstances (lower rates for shorter maturities) would lead to the reduction of the stated annual interest rate and the corresponding reduction in the effective monthly interest rate used in the calculations of the instalments. However, when interest rates increase, using a reference rate of shorter maturity will have a quicker impact on the monthly instalment penalizing the couple's financial situation.

EXERCISE 1.25

Mr Arrenda is leasing his property. Assume that the contract has perpetual duration and that the tenant is willing to acquire the property. *Mr Arrenda* usually makes is investments in the bank at a stated annual interest rate of 3%:

a) What is the minimum amount that *Mr Arrenda* should accept to sell his property, knowing that in the current contract he receives 1,800 euros for the lease at the beginning of each quarter?

b) How much should the amount determined in *a)* be if the value received each quarter increased at a rate of 0.5% quarterly?

Solution

a) The goal is to replace a perpetual lease with an immediate sale. For both alternatives to be financially equivalent, the present value of the perpetual annuity should be the same as the minimum selling value. Since we have a perpetual constant annuity the calculations for the present value are greatly simplified. However, we must first determine the effective quarterly interest rate (annuity frequency) and remember that the values are received at the beginning of each quarter:

$$r_4 = \frac{r_{(4)}}{4} = \frac{0.03}{4} = 0.0075$$

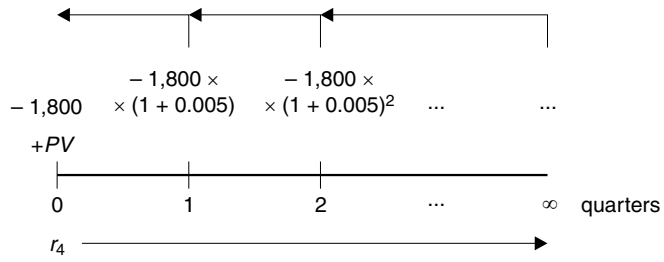
$$PV = 1,800 + \frac{1,800}{0.0075} = 241,800$$

or

$$PV = \frac{1,800}{0.0075} \times (1 + 0.0075) = 241,800$$

The minimum amount that *Mr Arrenda* should ask for selling his property is 241,800 euros.

b) With a constant growth rate of 0.5% in each quarter amount, the present value of the perpetual annuity can be determined by the following expression:



$$PV = 1,800 + \frac{1,800 \times (1 + 0.005)}{0.0075 - 0.005} = 725,400$$

or

$$PV = \frac{1,800}{0.0075 - 0.005} \times (1 + 0.0075) = 725,400$$

The minimum selling value of the property increases substantially, reaching 725,400 euros.

EXERCISE 1.26

An investor named *Today* has agreed to receive 10,000 euros, rejecting an investment (deposit) that would generate the following cash flows:

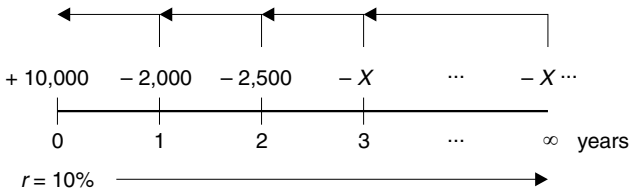
- At the end of the 1st year: receive 2,000 euros;
- At the end of the 2nd year: receive 2,500 euros;
- At the end of each year from the 3rd onwards: receive *X* euros.

a) Considering that the investor used an annual interest rate of 10%, determine the value of *X* that makes both alternatives indifferent;

b) Having agreed to receive the 10,000 euros today means that the value of *X* is in fact superior or inferior to the amount calculated in *a)*?

Solution

a) For both alternatives to be indifferent they should have the same present value. Since the value *X* corresponds to a perpetual annuity with constant terms, its value is equal to:



$$10,000 = \frac{2,000}{1+0.1} + \frac{2,500}{(1+0.1)^2} + \frac{\frac{X}{0.1}}{(1+0.1)^2} \quad \Leftrightarrow \quad X = 740$$

The value *X* that would make both alternatives indifferent is 740 euros.

b) Since the investor chose the 10,000 euros today, it means that the present value of the deposit was inferior (or equal) to this amount. Thus, the value of *X* should have been less than 740 euros.

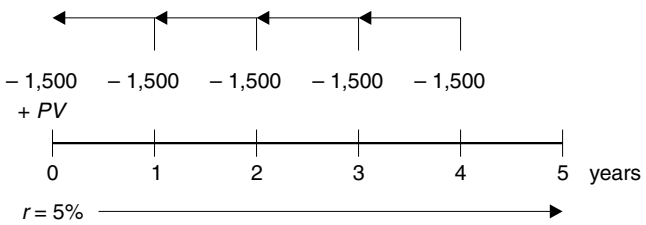
EXERCISE 1.27

Bank *Chuse* is suggesting to a client a loan payable in five annual instalments of principal and interest, anticipated (paid at the beginning of each period), of 1,500 euros each, at a stated annual interest rate of 5%. With the principal (capital) obtained by the loan an investment will be made with the guarantee of receiving, within 5 years, an amount of 12,500 euros. What is the annual rate of return on this investment?

Solution

To determine the annual rate of return on the investment is necessary to know the amount of capital invested. Since we have the information about the number, value and periodicity of the constant instalments of principal and interest, and also know the stated annual interest rate for the loan, we have all the information needed

to calculate the loan amount. Mathematically, we just need to determine the present value of all future instalments:

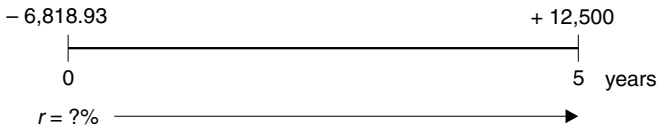


$$PV = 1,500 + 1,500 \times a_{\overline{4}|0.05} =$$

$$= 1,500 + 1,500 \times \frac{1 - (1 + 0.05)^{-4}}{0.05} =$$

$$= 6,818.93$$

Knowing that the capital invested was 6,818.93 euros, the rate of return can be easily obtained by:



$$6,818.93 \times (1 + r)^5 = 12,500 \quad \Leftrightarrow \quad r = 0.1289$$

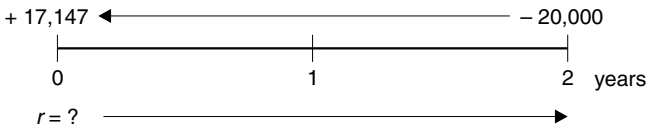
The rate of return obtained is the effective annual interest rate of 12.89%.

EXERCISE 1.28

A cash inflow of 20,000 euros in 2 years' time has a present value of 17,147 euros. If you want to change this value for a constant yearly revenue for 5 years, the first one to be received today, what is the value of each one?

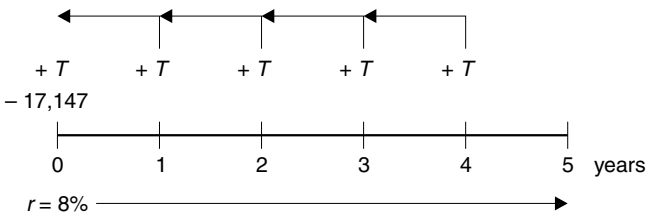
Solution

Since we already know the value, today, to be split in 5 anticipated annual constant instalments, we just need to determine the interest rate to be used in the calculations. This rate can be obtained using the existent relation between the 20,000 euros, in 2 years' time, and the 17,147 euros which correspond to the respective present value:



$$17,147 = \frac{20,000}{(1+r)^2} \quad \Leftrightarrow \quad r = 0.08$$

Note that in the previous calculation we used an exponent of 2 (years) because the interest rate we need to calculate is annual. If the rate needed was semi-annual instead of annual all we had to do was to use an exponent of 4 (referring to 4 semesters). Now we have all the information to determine the value of each annual instalment:



$$17,147 = T \times a_{\overline{5}|0.08} \times (1+0.08) \quad \Leftrightarrow$$

$$\Leftrightarrow \quad 17,147 = T \times \frac{1-(1+0.08)^{-5}}{0.08} \times (1+0.08) \quad \Leftrightarrow$$

$$\Leftrightarrow \quad T = 3,976.46$$

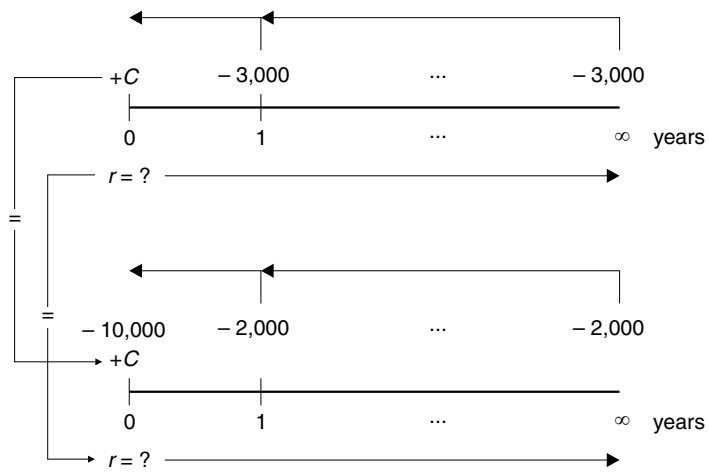
The value of each instalment would be 3,976.46 euros.

EXERCISE 1.29

Consider a perpetual obligation to pay an annual value of 3,000 euros. If you immediately pay 10,000 euros, the perpetual payment is reduced to 2,000 euros. What is the interest rate that is being used in this analysis?

Solution

Both alternatives should be financially equivalent and therefore have the same present value:



$$\frac{3,000}{r} = 10,000 + \frac{2,000}{r} \quad \Leftrightarrow \quad r = 0.1$$

The underlying effective annual interest rate (also stated annual interest rate) is 10%.

EXERCISE 1.30

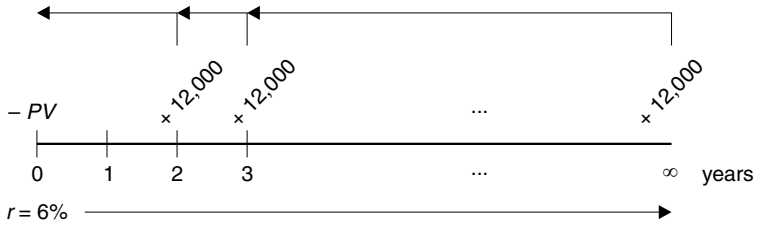
Firm *Sub* undertook today an investment that will be fully subsidized by government funds. These funds will be allocated annually in perpetuity, beginning in 2 years (first payment at the end of the second year). The stated annual interest rate is 6%.

a) Considering that the annual subsidy will be 12,000 euros, what was the amount invested today?

b) Now assume that the subsidy will be paid every two years, how much would firm *Sub* be able to invest today?

Solution

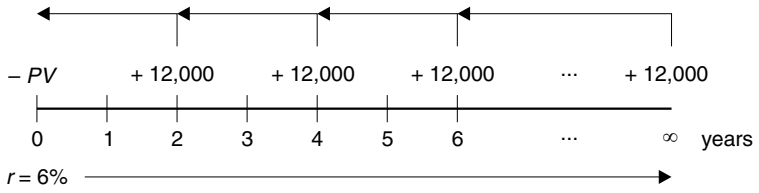
a) The amount invested is equal to the present value of the perpetual subsidy:



$$PV = \frac{12,000}{0.06} \times (1 + 0.06)^{-1} = 188,679.2$$

The amount invested today was 188,679.2 euros. Note that in the previous calculations we needed to make an additional discounting of one period because the subsidy will be paid for the first time in 2 years.

b) Since we are assuming in *a)* that the stated annual interest rate has annual compounding, now we need to compute the effective bi-annual interest rate to determine the new present value:



$$r_{\frac{1}{2}} = (1 + r)^2 - 1 = (1 + 0.06)^2 - 1 = 0.1236$$

$$PV = \frac{12,000}{0.1236} = 97,087.38$$

The amount of the investment would be 97,087.38 euros. Notice that no additional discounting was needed in the calculations. Since the annuity periodicity is now bi--annual, the present value of the annuity is already in the present moment (the first term of the annuity occurs in 2 years' time, the annuity periodicity is bi-annual and the effective interest rate is also bi-annual. We should point out the sharp decrease in the investment value from the new subsidy. Although we are still under a perpetual annuity, decreasing the inflow from the subsidy from annual to bi-annual had a significant influence on the present value, because the first few annuity terms (those that are less affected by the discounting process) are the ones that make up almost the total present value amount of a perpetual annuity.

EXERCISE 1.31

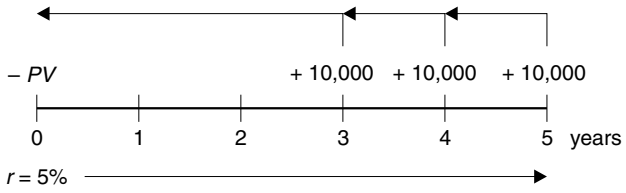
A parent wants to make today a deposit that ensures his son a yearly revenue of 10,000 euros during the 3 years in which he will be taking his higher education course. Each amount will be paid at the end of each academic year and the son will join the University within 2 years. Assuming a stated annual interest rate of 5%:

a) How much must the parent deposit today?

b) If the son received the 10,000 euros at the beginning of each academic year, instead of at the end, the amount the parent should deposit today would be higher or lower than the one determined in *a)*? Justify.

Solution

a) The amount of the investment will be equal to the present value of the yearly revenue of 10,000 euros:

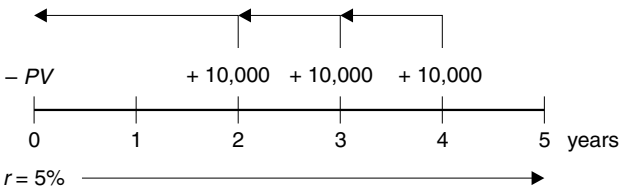


$$PV = 10,000 \times a_{\overline{3}|0.05} \times (1 + 0.05)^{-2} =$$

$$= 10,000 \times \frac{1 - (1 + 0.05)^{-3}}{0.05} \times (1 + 0.05)^{-2} \quad \Leftrightarrow \quad PV = 24,700.66$$

The amount of the deposit should be 24,700.66 euros. Note that in the previous calculations it was necessary an additional discounting of two periods, since the first 10,000 euros are only paid to the son 3 years from now. We should also mention that the effective annual interest rate is the same as the stated annual interest rate.

b) If the 10,000 euros were to be paid at the beginning of each academic year, that is, anticipated, the amount that the parent would have to deposit would be superior. Since we are facing an investment operation, and that the amount invested will have less time to earn interest, one can easily conclude that this amount must be higher. Another way to get to the same conclusion is by verifying that the cash flows are now closer to the present day, thus having a higher present value.



$$PV = 10,000 \times a_{\overline{3}|0.05} \times (1 + 0.05)^{-1} =$$

$$= 10,000 \times \frac{1 - (1 + 0.05)^{-3}}{0.05} \times (1 + 0.05)^{-1} \quad \Leftrightarrow \quad PV = 25,935.7$$

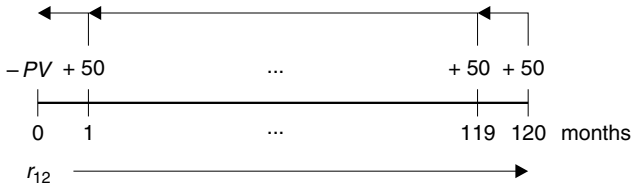
The amount of the deposit would have to be 25,935.7 euros, higher than the one determined in *a)* by 1,235.04 euros.

EXERCISE 1.32

A family has contracted a mortgage loan of 100,000 euros payable in 20 years, with constant monthly instalments of capital and interest. The stated annual interest rate agreed was 6%. At the moment they have only 10 years remaining to repay the loan. How much of the principal should the family reimburse today in order to reduce each of the remaining monthly instalments by 50 euros?

Solution

The goal is to decrease the value of each of the remaining 120 monthly instalments by 50 euros. This 50 euros can be considered an annuity and its present value will correspond to the amount to be reimbursed today by the family. To determine the present value of the annuity we must first calculate the effective monthly interest rate, which corresponds to the annuity frequency:



$$r_{12} = \frac{r_{(12)}}{12} = \frac{0.06}{12} = 0.005$$

$$PV = 50 \times a_{\overline{120}|0.005} = 50 \times \frac{1 - (1 + 0.005)^{-120}}{0.005} = 4,503.67$$

The amount that should be reimbursed is 4,503.67 euros.

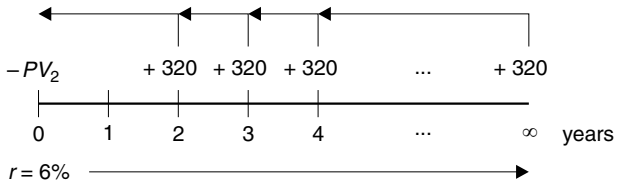
EXERCISE 1.33

When searching for the best way to invest his money, *Mr Portugal* was offered two investment alternatives by his financial institution. The first one guarantees a constant annual cash inflow of 800 euros during 9 years and starts one year from now. The second one provides a constant annual revenue of 320 euros, forever, and begins in 2 years. Which of these two alternatives should *Mr Portugal* choose? Consider a stated annual interest rate of 6%.

Solution

For both alternatives to be comparable we must assume that the initial investment is the same. With this assumption, the alternative with a higher present value should be the chosen one:

$$PV_{\text{Alternative 1}} = 800 \times a_{\overline{9}|0.06} = 800 \times \frac{1-(1+0.06)^{-9}}{0.06} = 5,441.35$$



$$PV_{\text{Alternative 2}} = \frac{320}{0.06} \times (1+0.06)^{-1} = 5,031.45$$

The first alternative is the most attractive to *Mr Portugal* because it is the one with a higher present value, 5,441.35 euros against the 5,031.45 euros of the second alternative.

EXERCISE 1.34

Consider a three year loan of 90,000 euros in which the principal will be amortized in three equal annual amounts. The interest will be paid each semester at a stated annual interest rate of 7%. An annual fee of 0.10% is payable over the outstanding principal and there is an initial contract fee of 400 euros. Moreover, a settlement fee of 180 euros is due at the end of the loan. Present the equation that enables the calculation of the EAR (global rate in the bank's point of view) of the loan.

Solution

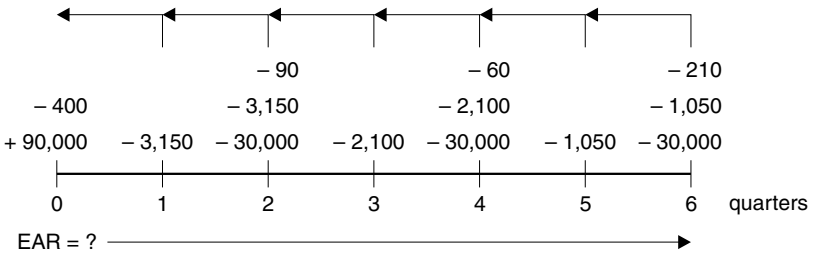
For determining the EAR (effective annual rate) or AER (annual equivalent rate) we must first identify all bank's cash flows in each period. In this case all cash outflows have the bank as recipient and therefore all must be included in the EAR calculations. The following table shows all the cash flows from the client's point of view:

	Semesters						
	0	**1**	**2**	**3**	**4**	**5**	**6**
Principal	+ 90,000		– 30,000		– 30,000		– 30,000
Interest		– 3,150	– 3,150[(1)]	– 2,100	– 2,100	– 1,050	– 1,050
Fees and other expenses	– 400		– 90		– 60		– 210[(2)]
Cash Flow	+ 89,600	– 3,150	– 33,240	– 2,100	– 32,160	– 1,050	– 31,260

(1) $\text{Interest}_2 = 90,000 \times 0.07 \times \frac{1}{2}$

(2) $\text{Fees}_6 = 30,000 \times 0.01 + 180$

The capital owed at the beginning of the first and second semesters is 90,000 euros, on the third and fourth is 60,000 euros and on the fifth and sixth is 30,000 euros. After identifying all cash flows we can construct the capital equivalence equation, in moment zero, that allows us to compute the EAR:



$$89,600 = \frac{3,150}{(1+\text{EAR})^{0.5}} + \frac{33,240}{(1+\text{EAR})^{1}} + \frac{2,100}{(1+\text{EAR})^{1.5}} +$$

$$+ \frac{32,160}{(1+\text{EAR})^{2}} + \frac{1,050}{(1+\text{EAR})^{2.5}} + \frac{31,260}{(1+\text{EAR})^{3}}$$

EXERCISE 1.35

The firm *Black&White*, formerly based in the village of Cascais, has decided to move its manufacturing unit to Lisboa, investing in the new facilities 20,000,000 euros. Needing a funding for 80% of the new plant, the firm borrowed the bank at the effective annual interest rate of 12.5%, having to comply with the payment of quarterly instalments for 40 years. Knowing that the first payment occurs within three months:

a) Determine the amount of each instalment;

b) How much will the firm still owe the bank after 20 years?

Solution

a) The amount of the loan will be equal to $20,000,000 \times 0.8 = 16,000,000$ euros. Since we have quarterly instalments and the given interest rate is effective annual, we first need to determine the effective quarterly interest rate:

$$r_4 = (1+r)^{\frac{1}{4}} - 1 = (1+0.125)^{0.25} - 1 = 0.0299$$

The amount to be financed must equal the sum of all the present values of all instalments. The number of instalments is 160, corresponding to the 40 years multiplied by 4 instalments in each year:

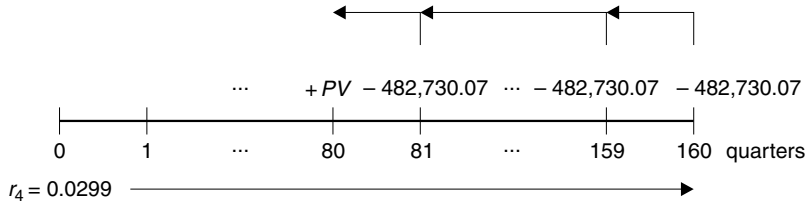
$$16,000,000 = T \times a_{\overline{160}|0.0299} \quad \Leftrightarrow$$

$$\Leftrightarrow \quad 16,000,000 = T \times \frac{1-(1+0.0299)^{-160}}{0.0299} \quad \Leftrightarrow$$

$$\Leftrightarrow \quad T = 482,730.07$$

The amount of each quarterly instalment will be 482,730.07 euros.

b) After 20 years have passed we are exactly at half of the loan maturity. The amount owed at that moment, right after the payment of the 80th instalment, will correspond to the discounting of all forthcoming instalments (we take out the interest part in each one):



$$PV = 482,730.07 \times a_{\overline{80}|0.0299} = 482,730.07 \times \frac{1-(1+0.0299)^{-80}}{0.0299} \Leftrightarrow$$

$$\Leftrightarrow \quad PV = 14,615,744.4$$

After 20 years the firm will still be owing 14,615,744.4 euros, which correspond to 91.35% of the initial loan amount. We can easily conclude that in this type of loan payment the initial instalments are mostly made of interest with almost no capital reimbursement. For example, if we look at the first instalment the capital repaid is less than 1% of the instalment value and about 0.03% of the initial loan amount:

$$\text{Interest}_1 = 16,000,000 \times 0.0299 = 478,400$$

$$\text{Capital}_1 = 482,730.07 - 478,400 = 4,330.07$$

$$\frac{4,330.07}{482,730.07} = 0.009$$

$$\frac{4,330.07}{16,000,000} = 0.0003$$

After 20 years, looking at the value of the 81st instalment, the capital reimbursed now represents 9.47% and about 0.29% of the initial loan amount:

$$\text{Interest}_{81} = 14,615,744.4 \times 0.0299 = 437,010,76$$

$$\text{Capital}_{81} = 482,730.07 - 437,010.76 = 45,719.31$$

$$\frac{45,719.31}{482,730.07} = 0.0947$$

$$\frac{45,719.31}{16,000,000} = 0.0029$$

EXERCISE 1.36

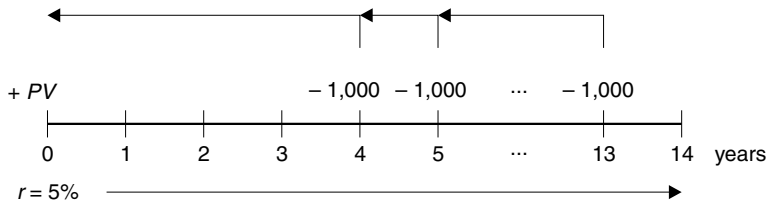
The firm *βenchmark*, after winning the first place in a prestigious competition, was given the opportunity to choose between the following prizes, both with the same present value:

- Prize (*A*): Receive 1,000 euros during 10 years, starting at the end of year 4;
- Prize (*B*): Receive three annual instalments growing at a rate of 2% per year, and the first instalment being paid immediately.

Considering a SAR of 5%, compute the value of each of the three cash inflows associated with prize (*B*).

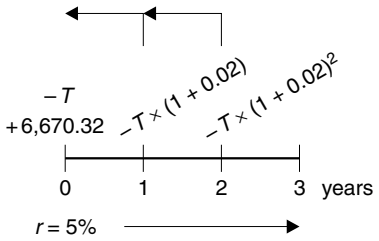
Solution

To determine the value of the instalments in prize (*B*) we need to make equal the present value of both prizes:



$$PV_A = 1,000 \times a_{\overline{10}|0.05} \times (1 + 0.05)^{-3} =$$

$$= 1,000 \times \frac{1 - (1 + 0.05)^{-10}}{0.05} \times (1 + 0.05)^{-3} \quad \Leftrightarrow \quad PV_A = 6,670.32$$



$$6,670.32 = T + T \times (1 + 0.02) \times (1 + 0.05)^{-1} + \\ + T \times (1 + 0.02)^2 \times (1 + 0.05)^{-2} \quad \Leftrightarrow$$

$$\Leftrightarrow \quad T = 2,288.2$$

The amount of the first instalment is 2,288.2 euros, the second is 2,333.96 euros (2% more than the first one) and the third is 2,380.64 euros (2% more than the second).

EXERCISE 1.37

Firm *XYZ* will contract a loan with the following features:

- Amount: 50,000 euros;
- Maturity: 110 days;
- Stated annual interest rate: j%;
- Stamp tax on interest: 4%;
- Open fee: 300 euros paid at the beginning of the funding;
- Settlement fee: 200 euros paid at the end of the loan;
- All in cost (EGAR – effective annual global rate) = 25%.

Compute the stated annual interest rate of the loan.

Solution

Obtaining the EGAR (also stated as GAER – global annual effective rate – or EAPR – effective annual percentage rate) for the loan, 25%, was possible through the identification of all cash flows involved in the operation and the corresponding periods in which they occur. Conversely to the EAR, the computation of the EGAR includes not only the cash flows to the bank, but also the cash flows that the bank will pass through to the fiscal authorities. The following table shows all those cash flows from the firm's point of view:

	Days	
	0	**110**
Principal	+ 50,000	− 50,000
Interest		$-50,000 \times j\% \times \frac{110}{360} = -15,277.78 \times j\%$
Stamp tax		$-15,277.78 \times j\% \times 0.04 = -611.11 \times j\%$
Fees	− 300	− 200
Cash Flow	+ 49,700	$-50,200 - 15,888.89 \times j\%$

$$49,700 = \frac{50,200 + 15,888.89 \times j\%}{(1 + 0.25)^{\frac{110}{360}}} \quad \Leftrightarrow \quad j\% = 0.1892$$

The stated annual interest rate of this loan is 18.92%. This example clearly shows the importance of presenting and knowing the EGAR (EAPR) in credit operations. The effective cost of the loan to firm *XYZ* is quite superior to the stated annual interest rate, 25% instead of 18.92%. The cost associated with the payment of fees, other expenses and taxes may turn out to be a decisive factor when choosing between several financing alternatives.

EXERCISE 1.38

A 65 days loan of 10,000 euros has an all in cost (EGAR) of 16%. The features of the funding are:

- Interest at a stated annual interest rate of 6%, paid at the end of 30 days and at the end of the loan;
- Stamp tax of 4% over interest paid;
- Settlement fee of 1% over the principal amount;
- Initial fee of 0.5% over the principal amount;
- The principal is paid bullet (all the once).

The bank will also charge a management fee in two equal parts. One to be paid at the beginning of the loan and the other on the 30th day. Determine the amount of the management fee.

Solution

Same as in the previous exercise, the value of the EGAR is already given. Using the equation that allows us to determine the EGAR we can determine the value of the management fee. Once again, not only the cash flows to the bank, but also the cash flows to be given to the fiscal entities have to be considered in the computation of the EGAR:

	Days		
	0	**30**	**65**
Principal	$+10,000$		$-10,000$
Interest		$-10,000 \times 0.06 \times \frac{30}{360}$	$-10,000 \times 0.06 \times \frac{35}{360}$
Taxes	$-0.005 \times 10,000$	$-10,000 \times 0.06 \times \frac{30}{360} \times 0.04$	$-10,000 \times 0.06 \times \frac{35}{360} \times 0.04$
Fees	$-\frac{C}{2}$	$-\frac{C}{2}$	$-0.01 \times 10,000$
Cash Flow	$+9,950 - \frac{C}{2}$	$-52 - \frac{C}{2}$	$-10,160.67$

$$9,950 - \frac{C}{2} = \frac{52 + \frac{C}{2}}{(1+0.16)^{\frac{30}{360}}} + \frac{10,160.67}{(1+0.16)^{\frac{65}{360}}} \quad \Leftrightarrow \quad C = 6.68$$

The total amount of the management fee is 6.68 euros, with 3.34 euros being paid at the beginning of the loan and the remaining 3.34 euros at the end of the 30th day.

EXERCISE 1.39

Firm *Contas* contracted a loan under the following conditions:

- Amount: 100,000 euros;
- Maturity: 87 days;
- Stated annual interest rate: 8%;
- Stamp tax over interest paid: 4%;
- Contract fee: 400 euros, paid at the beginning of the loan;
- Settlement fee: 100 euros, paid at the end of the loan with the capital reimbursement.

Determine the all in cost (EGAR) of the loan.

Solution

All cash flows presented are relevant in determining the EGAR. The following table shows all cash flows from the firm's point of view:

	Days	
	0	**87**
Principal	$+100,000$	$-100,000$
Interest		$-100,000 \times 0.08 \times \frac{87}{360} = -1,933.33$
Stamp tax		$-1,933.33 \times 0.04 = -77.33$
Fees	-400	-100
Cash Flow	$+99,600$	$-102,110.7$

$$99,600 = \frac{102,110.7}{(1+\text{EGAR})^{\frac{87}{360}}} \quad \Leftrightarrow \quad \text{EGAR} = 0.1085$$

The value of the EGAR for this loan is 10.85%, about 3 percentage points higher than the stated annual interest rate.

EXERCISE 1.40

A financing operation for one year has an EGAR of 15%. The underlying conditions of this operation are:

- Interest at a stated annual interest rate of 6%, paid at the end of the 182nd day and of the 365th day;
- Initial fee of 2% over the principal;
- Stamp tax of 4% over the interest paid;
- Settlement fee of z% over the principal;
- The principal payment is bullet.

a) Determine the settlement fee z%;
b) Compute the EAR?

Solution

a) To determine the settlement fee z% we just need to identify all cash flows used in the EGAR calculation and perform the corresponding capital equivalence equation:

	Days		
	0	**182**	**365**
Principal	$+C$		$-C$
Interest		$-C \times 0.06 \times \frac{182}{360}$	$-C \times 0.06 \times \frac{183}{360}$
Taxes		$-C \times 0.06 \times \frac{182}{360} \times 0.04$	$-C \times 0.06 \times \frac{183}{360} \times 0.04$
Fees	$-0.02 \times C$		$-z\% \times C$
Cash Flow	$+C - 0.02 \times C$	$-0.0315 \times C$	$-C \times (1.0317 + z\%)$

$$0.98 \times C = \frac{0.0315 \times C}{(1+0.15)^{\frac{182}{360}}} + \frac{C \times (1.0317 + z\%)}{(1+0.15)^{\frac{365}{360}}} \quad \Leftrightarrow \quad z\% = 0.0637$$

The settlement fee is 6.37% over the principal amount.

b) To determine the EAR, and opposite to what is done with the EGAR, we must not include the cash flows to be delivered to the fiscal entities. Thus, the values of the stamp tax should not be included. When discounting the cash flows we should use a day count convention of Actual/360:

$$0.98 \times C = \frac{0.0303 \times C}{(1 + \text{EAR})^{\frac{182}{360}}} + \frac{C \times (1.0305 + 0.0615)}{(1 + \text{EAR})^{\frac{365}{360}}} \quad \Leftrightarrow$$

$$\Leftrightarrow \quad \text{EAR} = 0.1474$$

When determining the EAR, or the EGAR, with existing cash flows for more than two periods, we must use, as previously stated, a spread sheet or a financial calculator. As expected, the value of the EAR on this loan is inferior to the EGAR, although the difference is quite small.

Follow-up Exercises

EXERCISE 1.41

Which of the following alternatives is the best for an investor (consider Compound Interest)?

a) Investment with an effective interest rate for 4 months of 1%;

b) Deposit with interest every two months at a stated annual interest rate of 3%.

EXERCISE 1.42

A highly credited financial institutional publicizes the following product: «Give us 12 monthly amounts of 500 euros, beginning today, and one year from now we will credit your account in 6,136.60 euros». In the current financial and economic environment, does this seem to you like a good investment?

EXERCISE 1.43

A financial institution stated a rate of 2.75% as the maximum effective annual rate to be offered for one year deposits. In your opinion, what is the interest frequency the institution should follow in its deposits in order to offer the highest possible stated annual interest rate?

EXERCISE 1.44

A financial institution wants to draw the attention of some clients to a deposit offering a fixed interest rate for 2 years. Its major competitor is offering a deposit with the same maturity but with a variable interest rate according to the table below. Determine the fixed annual interest rate of the first deposit, stated and effective, that yields the same accumulated interest offered by the competitor's alternative. The interest is determined and compounded quarterly in both cases.

Stated annual interest rate offered by the competitor:

1st Quarter: 1.8%	2nd Semester: 2.5%
2nd Quarter: 2%	2nd Year: 3%

EXERCISE 1.45

Mr A wants to renegotiate his debts. Bank *B* advises *Mr A* to pay back all debts and, in substitution, to ask for a mortgage loan on his apartment, in order to benefit from the favourable conditions now in place for this type of credit. Using the information given below regarding the existing debt and the new loan, determine:

a) The present value of the debt owed by *Mr A*;

b) The amount of the new periodic instalment;

c) The equation that allows the computation of the EGAR (EAPR) for the mortgage loan.

- *Existing debt*

  Personal credit with 9 forthcoming constant instalments (one every four months), with a stated annual interest rate of 9%. Each instalment is 2,140 euros.

- *Features of the mortgage loan*
  - — Maturity of 10 years;
  - — Monthly constant installments of principal and interest with a 6 months deferral;
  - — Stated annual interest rate, fixed, of 4%;
  - — Initial evaluation fee of 200 euros.

EXERCISE 1.46

If you invest today a certain amount, you will receive in 3 years' time 15,000 euros. If instead you decided to replace the present value of that investment with an annuity of 8 constant semi-annual instalments, with the first one being received today, what should be the value of each instalment? Consider a stated annual interest rate of 6% with quarterly compounding.

EXERCISE 1.47

At the present time a couple has failed to comply with its debt obligations and is owing the bank 3 monthly instalments: those from the past two months and the one from the current month that is due today. Each instalment is 550 euros. Besides those, there are still more 40 monthly instalments due, constant of principal and interest in the same amount. The stated annual interest rate is 7%. The Bank has suggested to restructure the debt by extending the maturity. The new loan will have the following features:

- Stated annual interest rate of 9%;
- Maturity (beginning today): 10 years;
- Monthly instalments.

The instalments that are past due will be included in the new loan at the stated annual interest rate of the latter.

a) What is the principal amount of the new loan?

b) What is the new value for the monthly instalment to be paid by the couple?

c) Write the expression that allows for the computation of the EGAR for the new loan;

d) What must have been the argument(s) used by the Bank to increase the interest rate for the new loan?

EXERCISE 1.48

The inflation rate for next year is expected to be 6%. An investor is willing to go ahead with an investment as long as it can guarantee an annual real interest rate of 2.5%. Does a deposit for one year with a stated annual interest rate of 8.38% and quarterly compounding fulfils the investor's demands? Justify.

EXERCISE 1.49

If you had the opportunity to choose between the following alternatives for a two years investment, which one would you choose? Show all calculations.

- Alternative 1: interest is paid every 6 months (at the end of each period) and automatically compounded at a stated annual interest rate of 4%;
- Alternative 2: effective interest rate for 4 months of 1.5%;
- Alternative 3: consists on an accumulation of capital and interest for 2 years, with an increasing effective annual interest rate of 4.5% and 5% for the 1st and 2nd year, respectively.

EXERCISE 1.50

Assume you are beginning your PhD (with a duration of 5 years) in Germany and you are looking for a place to stay. You are still deciding whether you should buy or rent a place:

- Buy: the acquisition value is 140 thousand euros, half paid today and the other half in 6 months. Every year (at the beginning) you pay 600 euros for maintenance expenses. At the end of the 5 years you expect to sell the property for 150 thousand euros;
- Rent: for this alternative to be the most beneficial, how much are you willing to pay monthly for the rent (at the beginning of each month)? In this case the maintenance expenses are paid by the property owner.

Consider a stated annual interest rate of 5% with compound interest.

EXERCISE 1.51

An investor must deposit today a certain amount so that he can receive the following values (at the end of each period):

- A monthly value of 300 euros during the following 8 months;
- A value of 1,000 euros on the 9th month and a value of 1,500 euros on the 10th month;
- A monthly value of 300 euros during an additional 8 months, beginning on the 11th month.

a) Knowing that the Bank offers a stated annual interest rate of 6%, how much should the investor deposit today?

b) How much should the investor deposit today if the bank offers a stated annual interest rate of 6% for the next 10 months and a stated annual interest rate of 6.6% for the remaining periods?

EXERCISE 1.52

An investor owns a perpetual annuity of 100 euros, with a present value of 2,000 euros and where the first amount is received in one year. If he wishes to replace this annuity with a growing annual annuity, with a yearly growth rate of 2%, beginning in 3 years, how much must the first amount be for both annuities to be financial equivalent?

EXERCISE 1.53

Congratulations, you have just won the Specialoto!!! Unfortunately, the prize money will not be given to you today. Instead, you are given two alternatives:

1. Receive a certain constant annual amount for 16 years, with the first amount being credited in your account in 6 months;
2. Receive a certain constant semi-annual amount during 16 semesters, with the first amount being credited in your account in 6 months.

Knowing that the effective annual interest rate for the first 4 years is 4% and for the remaining years 5%, what must be the proportion between the amounts of both alternatives in order for you to prefer the second one?

EXERCISE 1.54

Comment on the following statement: «On a financial operation between a financial institution and a client, if the day counting convention is Actual/365 instead of Actual/360 days, the financial institution is benefited. »

EXERCISE 1.55

Firm *Viajar* as signed a contract with a leasing institution to use a motor vehicle, and agreed on the following conditions:

- Contract value (acquisition value): 25,000 euros;
- Down payment of 2,300 euros, to be paid on the day of the contract;
- Residual value of 6% over the acquisition value, to be paid one month after the last instalment;
- Payment of 24 constant anticipated monthly instalments;
- Deferral of 2 months on the payment of the instalments;
- Stated annual interest rate of 6% with monthly compounding.

Determine the value of each instalment.

EXERCISE 1.56

Consider the following financing alternatives:

1. A loan for three months, with monthly interest and a stated annual interest rate of 6%;
2. A loan for three months, with interest paid at the end of the loan, a stated annual interest rate of 3% and an initial fee of 1% over the principal amount.

Which of the two alternatives has the lowest all in cost (perform the calculations on a monthly basis)?

Chapter 2

FINANCIAL MARKETS

Theory

In the context of the current serious financial crisis, and especially when we have seen the North American rating agencies lowering the rating of several public debts in Europe, we hear strong criticism of some political and even social movements, demonstrating against the existence of the financial markets. However, at the same time, these same voices demand for a better promotion of the economic growth for job creation and wealth. Questioning the existence of markets and, at the same time, wanting growth, seems to be incompatible. So, we should understand the importance of these markets as growth drivers, on one hand, and see if there are correct ways to participate in the financial markets, on the other.

Participants and types of participation in financial markets

In a simplified form we can say that there are two main types of participants in the financial markets: those with need of resources and those with excess of resources. Both are assisted in their activities by several financial institutions.

Let us take a look at the role of each party in each market type: monetary, foreign exchange and capital market.

Money Market

This market consists of the set of financial operations in the short term, up to a year, held between central banks, banking system and private clients or companies. Any one of these entities can participate as debtor or creditor, depending of being a

borrower or an investor, respectively. It is also called liquidity market, since its aim is to enable an efficient and dynamic management of the short-term money of the entities involved.

We can distinguish two types of operations that assume great importance in this market:

- Intervention Operations Market – developed between the Central Bank and the Banks in general;

  Commercial banks are customers of the respective Central Bank, that is, they have a bank account in the latter. If they need money, to finance their structure or to use it in their loan activity with their customers, they can borrow the Central Bank, at the reference interest rate, in case that Central bank decides to conduct liquidity-providing operations. On the other hand, the Central Bank may decide to conduct liquidity-absorbing operations and allow deposits in its accounts, of the commercial banks, also at the reference rate. These operations are extremely important to regulate the level of liquidity and interest rates on the real economy. On the other hand are fundamental to a well-functioning banking system in its daily management of short-term funds and, consequently, to the level of their activity.

- Interbank Money Market (IMM) – Operations performed by banks between each other.

  In relation to the previous, on this Interbank Monetary Market the commercial banks assume the main role and the transactions become more flexible. The institutions negotiate directly with each other the funding or deposit operations. It is an essential mean for institutions to make a good management of their liquidity, being especially important the overnight operations. Note that the flexibility provided by this market proves to be extremely useful, given the uncertainty in the volume of deposits and withdraws done daily in the branches of banks. The interest rates used in this market are necessarily superior to the reference rate of the Central Bank, since we are facing a higher risk level.

  This is the market for the determination of the main interest rate indexes, of which the most widely used in euro is the EURIBOR. This daily index, for various periods, is no more than an average, in each period, of the rates used by the most important banking institutions that participate on the daily operations of supplying funds in euros on the IMM.

  Following the IMM operations we can find the operations carried out between the banking system and the real economy (individuals or companies), the so-called over-the-counter market. The more dynamic the IMM is, the greater the

likelihood of banks to meet the financial needs of the economic agents. Here the banks take a higher risk level, reason enough for them to use interest rates that are always subject to an additive spread over the IMM rates (the most common case is to designate these rates as being the type EURIBOR + spread). The financial institution has to charge a credit margin, which must be in accordance with the risk presented by its customers.

Foreign Exchange Market

Can we imagine a Portuguese company exporting to the USA without being established a relationship between the euro and the US dollar? How many dollars must the customer spend in the purchase, in face of a price in euros? Without the answer to this question, the American client would be unable to buy to a foreign supplier.

The introduction of the euro has resulted in the disappearance of a significant number of currencies but the truth is that we could not imagine the world without the existence of the Forex market, the one in which are carried out exchanges between different currencies.

Many countries consider that their currency is an essential mark of their sovereignty, so we hardly can end up in a situation of having a sole world currency, especially as this would require harmonisation of policies and economic performance and that is impossible to achieve. So, we cannot deny the high importance of this market as an instrument that makes possible all the economic and financial relations between the agents of countries with different currencies.

Also in the foreign exchange market there is a market that consists in the relationships of the financial institutions (FI) with each other, and another with the transactions between these and the real economy. And also here the price conditions of the first are reflected in the second. In this market there isn't the figure of the debtor or the creditor, since we are not facing needs or excesses of resources but only trades in other currencies. The financial institutions, between them, give quotations for the values for the different currencies, giving rise to various exchange rates, which depend on multiple factors. The most important are those relating to the value of the economies (GDP), their interest rates (higher rates attract more capital), the issue of new money (more money means less value per currency) and the currency wars between countries. However, in the final analysis, the dominant factor in the definition of the exchange rates will always be the relationship between supply and demand of a particular currency in relation to another, for which the factors described above represent a major influence.

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Capital Market

This is the market in which individuals, who have resources in excess, can participate in the growth of the companies and contribute decisively to the economic development. In fact, the main participants of this market are companies that need resources, on the one hand, and individuals with available resources, on the other. Here the role of commercial banks is small and the only ones that have been attributed tasks in this market are the investment banks, as helpers in transferring the resources between the participants.

To better understand the main features of this market, we will divide our analysis in stock market and bond market, starting with understanding the difference between the two types of securities.

Shares

It is very difficult to find a large company with only one owner, also because the great investors like, and well, to diversify their investments. In this way, the most common situation is one in which we find large companies to be owned by a large number of investors who have acquired a certain amount of shares in which the companies' capital is divided.

What is the purpose for those who buy these shares? Who purchases shares wants to participate in a particular business, expecting that it will be profitable (equal or more than other alternative) and with good prospects for the future. Finally, ex-

pects that the investment in these shares will pay a return from the business profits, through the payment of dividends. And more, if these future positive perspectives end up occurring, in case the investor needs to sell the shares, he expects to get a profit, that is, sell at a higher price than that paid by purchase. Dividends and capital gains in the price will be then the two components of the return that investors expect to get with the purchase of the shares. That return is required to be equal or higher than the one of alternative investments (with equal risk).

It would be nice if a business never presents surprises in its results but this is impossible. Depending on the industry, with greater or lesser extent, a business presents always a volatility in its revenues and costs. Some businesses are cyclical, others counter-cyclical. Some are more sensitive to extraordinary factors, other less but also. And so, even a company with good perspectives can go through ups and downs. Because of this, investors in shares should be prepared to keep investing in the company for an extended period of time, so they can get, with more probability, the desired return for that investment. Five years is the term pointed to be as the minimum to consider by the investor in his decision making of entry into a firm.

What benefits gets a company that has a solid, stable and diversified base of shareholders? The great advantage of, more easily,[1] making capital raises, if it can't get cheaper resources by another way, when it needs to grow or restructure.

In conclusion: on one hand investors are pleased by the existence of shares, because with them they can manage to own large businesses or diversify their investments. On the other hand, as resource absorbers, companies have advantages in owning a wide range of shareholders, especially in situations when the business needs grow.

Bonds

Like shares, also in the case of bonds there is a benefit to both parties, in this case the debtor and creditor, by the existence of a bond market. The large,[2] company, when needing funding, can extend the range of possibilities of contracting debt, beyond the traditional bank credit. With the help (paid, of course!) of one or more investment banks, the company can ask for a loan to the general public or institutional investors, which lend money by purchasing a number of bonds in which the

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(1) By nature, the stockholders hate to be asked to increase their investments in the same company. They consider that this company must improve its management, using the existing resources, so as not to jeopardise the profitability level that they have been given.

(2) Besides dimension, there are other features that must be met for a company to be able to issue bonds, namely its presence in the stock market, past performance and positive results.

loan is divided. Everyone who buys bonds takes a comparatively smaller risk than the Bank would take by lending the entire amount of debt, so the rates payable by the company to creditors will be relatively lower than those of ordinary bank lending. Even though the rates are lower, bond holders get a better return for their investments than in traditional bank deposit products. This way both sides get what they want: the company, minimize the interest rate paid on financing; creditors, maximize the rate of return obtained or, better, maximize the profitability/risk ratio of their investments.

It is important now to identify what is in common between both the stock and bond markets.

Common features between the two markets, stocks and bonds

- The companies and borrowers (companies or governments) are clearly designated as issuers, because in this market there is always the issue of securities (shares = ownership instruments, or bonds = debt instruments);
- The shareholders and bondholders are clearly named as investors, since both acquire the securities in order to get a return;
- When the securities are issued and sold for the first time, we say that this is a primary market transaction. Note that the resources are transferred directly from investors to the issuer;
- When the securities are sold later, after the primary issue, we say that it occurs in the secondary market. Note that in this case is a securities transaction only among investors: among those who bought the securities in the primary market and those who will now buy the same securities in secondary market. The resources are not transferred to the issuer of the securities but rather from an investor to another. The most common of the secondary markets is the stock exchange, where it is traded, in an open market but with well-defined rules, the stocks and bonds that are listed after the primary market;
- We cannot say that only the primary market is important for economic growth, for being in it that are effectively created the securities. Regarding this fact, the toughest critics argue that the stock exchange, most visible and known side of the financial markets, could disappear for being only a way for trading speculative resources among capitalists. However, analysing in more detail, we can conclude that we can only have a strong primary market if investors who acquire securities for the first time have the certainty that, in case of need, can sell them fast and secure to other investors. Thus, the vitality of the primary market is very dependent on the efficiency of the secondary market;

- To invest in stocks, investors should be prepared to take a long-term investment, what should happen in the case of bonds? In these there is the commitment of the issuer to pay interest, named coupon, defined at the prospectus of the issue, making it appear that there is no uncertainty as to the return which the bondholder will get on his investment. However, the bond issues have a period up to the end of which the issuer repays all the loan capital. That period, since the present moment up to the extinction of the loan, is called maturity. If the investor needs to sell his bonds at a moment before maturity, he will be subject to the price of the securities in the secondary market. This price may be above or below the value that is set at the prospectus of the issue for the issuer to pay at maturity, since the bonds may in the meantime have become more or less attractive than they were at the time of issuance.

  A bondholder will be very pleased if his securities become more attractive, obtaining with them a profit in price. Let's see how this can happen by explaining the opposite case: one in which bonds become less attractive throughout its life. Let's look at the case of the bonds with more weight on the market: the Public Debt at fixed interest rate. If an investor buys today German Government bonds, when the corresponding yields are at all-time lows, what will happen if immediately occurs a widespread rise in market interest rates (by an unexpected rise of the reference rate of the ECB, for example)? If the German Government wants to issue new bonds with the same maturity as held by the investor, after that rise would have to do it at a higher coupon rate. Thus, the investor would like to sell the bonds he holds and purchase the new ones. But to sell those will have to agree to receive a lower price than that of purchase, incurring in losses. The price would be low enough to turn the yield of those bonds equivalent to the yield that the new issue can offer, otherwise no one would be interested in buying the old bonds.

  We conclude that the yield of existing bonds is heavily dependent on changes in market interest rates, by the effect this may have on the prices of these bonds in the secondary market. But let us not forget that this impact is only over the bondholder. The issuer will always repay the bonds only at maturity, according to the conditions defined at the prospectus in primary market. However, for any new issue of bonds, a rise in market rates will mean something negative to the issuer, because he will pay a higher coupon rate on the loan. And that rise in market rates will be positive for investors of new bonds, who will receive that higher coupon.

  We are finally in a position to answer the question posed at the beginning of this section: the possibility of a volatile behaviour in prices of bonds until their maturity, and in particular in the case of fixed rate and long maturities, makes

that investment in bonds should also not be seen as short-term, so that the investor may have some assurance of obtaining the appropriate return for his investment. The term recommended for investing in bonds depends on the maturity[1] of each bond issue but the general idea in the markets is that it is the medium-term;

- Recognizing the influence that interest rates have on investments, we realize why most investors consider monetary, bond and stock markets as a range of possible alternative investments. They transfer their resources either to one of these markets, either to another, depending on the economic conditions and financial context. Running some risk of being too simplistic in this conclusions, we can say that in contexts of low interest rates, we can see a strong appetite for investments in stocks. The returns offered by other alternatives are so low that, if the risk profile allows, somehow compensates putting resources in more profitable assets, although predictably more risky. In addition, the discount rates of the future cash flows of the stocks are so reduced in those low rates contexts, that the present value of these stocks reach relatively high prices[2] Unlike this, in contexts of high interest rates, investors will prefer more liquid investments that offer these high returns explicitly in their contracts. Will be the case of investments such as term deposits or other resulting directly from the money market products. In intermediate contexts, when interest rates have a descending behaviour, the fixed rate bonds become quite desirable, by the effect of the rise in price as we explained above. On the contrary, when interest rates are rising, the floating rate bonds are the ones that can follow the movement of rising coupon.

Throughout this chapter we saw how the financial markets are of the utmost importance to transfer resources from those who have it in excess to the agents, especially companies, who need those resources to boost the economic growth and development. Of course we have seen also that, for investors, there are great advantages in the existence of markets, since without these would be very difficult to be able to make investments of diversified nature and return, transversal to the entire economic-financial spectrum.

Let us now look at the second challenge of the beginning of this chapter: to understand if there are correct ways of participating in the financial markets. If we concede that there are good and bad ways of participating in the markets, we are justi-

(1) More specifically the duration of each bonds issue, a ratio that we will not analyse in this book.

(2) We will understand this reasoning with the study of chapters 1 and 4.

fying the importance of these and being aware of how to avoid more serious problems in the future.

Ways of participating in the markets

Any participant in the markets, whether debtor, creditor, issuer or investor, can choose for one of two possible positions:

- To be aware that all products or securities come with risk, whether credit or price, adapting types of funding or investment to the risk profiles;
- Ignore the need to adapt the products or securities to risk profiles, viewing the markets like a game where the speculative postures win.

We then present some arguments in favour of the first of the postures as the ideal for who is participating in the markets:

- Lend to those who will not spend in profitable or cost saving goods, can mean taking too much default risk (payment of interest and capital);
- Acquire foreign currency only for betting in its recovery, and not because the same is needed to enable economic operations, can lead to losses in a market that is featured for its high volatility;
- Acquire shares or fixed rate bonds with the aim of obtaining capital gains over the short term, when the resources used in this investment have a short period of availability, it may mean having to sell those securities with assumption of losses;
- Even if we can forecast the future, more or less long, of the behaviour of monetary markets, foreign exchange and capital markets, especially by the observation of the interest rate levels in the economic context, the truth is that markets evolve at random, being much influenced also by events of political, social, warlike and even atmospheric (natural disasters) nature;
- The negative consequences, presented in the arguments described, are largely extended if the participating agents choose to massify only one strategy and ignore the enormous benefits of diversification. Diversify is not unaware of where to invest, is instead the possibility that one has to increase the return/risk ratio. Just by diversifying, as mentioned in the introduction, the agents can avoid many of the pitfalls that are posed by volatility and randomness of the markets.

Formulas

- $\text{Interest} = \text{Initial Capital (Principal)} \times \frac{\text{Annual Interest Rate}}{\underbrace{360}_{\text{or } 365}} \times$

$\times \text{number of days of operation}$

- $\text{Future Value at maturity} = \text{Principal} + \text{Interest}$

- $\text{Exchange Rate } A / B = \frac{x \text{ units of currency } B}{1 \text{ unit of currency } A}$

	Bid	**Offer (Ask)**
Exchange Rate *A* /*B*	The financial institution buys currency *A* and sells currency *B*	The financial institution sells currency *A* and buys currency *B*

Solved Exercises

EXERCISE 2.01

A 10,000 euros loan will be contracted on the 13th February and repaid on the 18th of May. The interest rate is 4%. What is the amount of interest to be paid?

Solution

To determine the amount of interest we must first count the number of days in the operation:

Month	Days
February(1)	28 − 13 = 15
March	31
April	30
May	18
Total	94

(1) Assuming that February has 28 days.

$$\text{Interest} = C \times r \times \frac{days}{360} = 10,000 \times 0.04 \times \frac{94}{360} = 104.44$$

The amount of interest will be 104.44 euros.

EXERCISE 2.02

2. An individual deposited today, 20th October, Friday, a check of 1,000 to make a term deposit that will end on the 14th December, Thursday. The Bank will give the deposit the value date of the next business day. The interest rate is 5%. Compute the total interest to be received at the maturity of the deposit.

Solution

To determine the total amount of interest we begin by counting the days of the operation. However, since the value date is not the same as the deposit date, first we need to establish when the deposit started. The start of the deposit will be on Monday, the 23th October, since this is the first business day after the date of the deposit.

Month	Days
October	31 – 23 = 8
November	30
December	14
Total	52

$$\text{Interest} = C \times r \times \frac{\text{days}}{360} = 1,000 \times 0.05 \times \frac{52}{360} = 7.22$$

At the deposit's maturity, the total amount of interest to be received will be 7.22 euros.

EXERCISE 2.03

A deposit of 5,000 euros, carried out between October 19 and November 20, earned an interest of 17.78 euros. Calculate the interest rate of the deposit.

Solution

The procedure to follow is similar to the previous exercises. The difference is that in this case we already know the amount of interest, but we do not know the underlying annual interest rate. We begin by counting the days of the operation:

Month	Days
October	31 – 19 = 12
November	20
Total	32

$$17.78 = 5,000 \times r \times \frac{32}{360} \quad \Leftrightarrow \quad r = 0.04$$

The annual interest rate of the deposit was 4%.

EXERCISE 2.04

Consider that a bank undertook today the following speculation operations in the IMM:

- Borrowed 100,000 euros, for 182 days at an interest rate of 3.40%;
- Invested 100,000 euros, for 91 days at an interest rate of 3.25%.

a) If within 3 months the 91 days rate is 3.45%, will the Bank have a profit or a loss on the speculation operation?

b) If the Bank had a profit of 100 euros, which was the rate that he got for the reinvestment of the funds, after the 91 days, for the next 91 days?

Solution

a) The speculation resides on the fact that today the Bank does not know what the interest rate will be 3 months from now, for operations of 91 days. We will perform our calculations assuming that the latter rate will be 3.45%, and check if the Bank has a profit or loss if that really happens to be the case.

$$\text{Interest } 1^{\text{st}} \text{ Investment} = 100,000 \times 0.0325 \times \frac{91}{360} = 821.53$$

$$\text{Interest } 2^{\text{nd}} \text{ Investment} = 100,821.53 \times 0.0345 \times \frac{91}{360} = 879.25$$

After the 182 days the Bank will earn a total amount of interest of 1,700.78 euros. Let us now focus on the cost of the financing operation.

$$\text{Interest Loan} = 100,000 \times 0.034 \times \frac{182}{360} = 1,718.89$$

The Bank will have a total amount of interest to pay for the loan after 182 days of 1,718.89 euros. We can now determine the total earnings:

$$\text{Earnings} = 1,700.78 - 1,718.89 = -18.11$$

The Bank will have a loss of 18.11 euros on this speculative operation.

b) In order for the Bank to have had a profit of 100 euros it was necessary that the total amount of interest earned in both investments was 100 euros superior to the total amount of interest paid for the loan, thus, the total amount of interest received must have been 1,818.89 euros. Since there was no uncertainty about the interest received in the first investment, the increase in the total amount of interest received must have come from the increase in the amount of interest from the second investment:

$$\text{Interest } 2^{\text{nd}} \text{ Investment} = 1,818.89 - 821.53 = 997.36$$

Now that we know the amount of interest from the second investment we are able to determine the corresponding annual interest rate:

$$997.36 = 100,821.53 \times r \times \frac{91}{360} \quad \Leftrightarrow \quad r = 0.0391$$

For the Bank to have had a profit of 100 euros, the annual interest rate in place after 3 months, for operations of 91 days, must have been 3.91%.

EXERCISE 2.05

A mortgage loan has the following conditions: $EURIBOR_{1M}$ + 0.75%, rounded to 1/8% higher. Knowing that the $EURIBOR_{1M}$ is 3.0154%, what is the interest rate that will be used on the loan in the next month?

Solution

This exercise has the specific goal of bringing into attention some details that may arise from this type of operation and that may lead to wrong choices among the existing alternatives. In this particular case, the rounding of the interest rate to 1/8% higher no longer applies in reality, after the legislation of 2006 and 2007[1] stating new rules about the rounding of interest rates on credit operations. However, and only from an academic perspective, we will show the solution to this case.

The first rate to be determined results from the sum of the index (EURIBOR for 1 month) and the spread (0.75%):

$$SAR_{initial} = 3.0154 + 0.75 = 3.7654$$

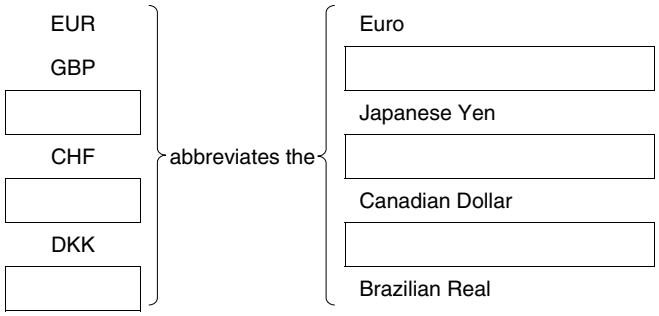
Rounding the decimal part to the one eight percent higher[2] yields a rate of 3.875%. The latter would be the SAR to be used in the mortgage loan.

(1) Decreto-Lei nº 240/2006, of December 22 and Decreto-Lei nº 171/2007, of May 8.

(2) All one eight of a percentage point: 0.125, 0.25, 0.375, 0.5, 0.625, 0.75, 0.875 e 1.

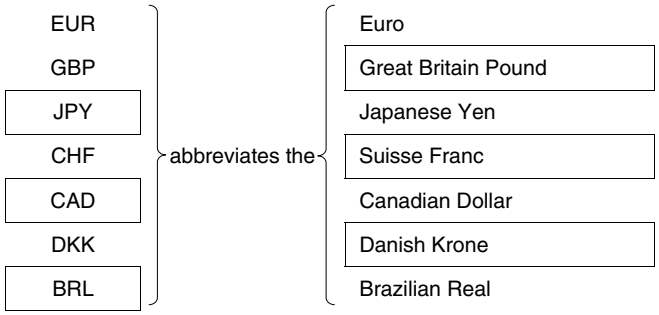
EXERCISE 2.06

Complete the following figure:



Solution

Usually the first two letters represent the name of the country and the third letter the name of the currency, both mainly in English language[1]:



In this example the exceptions to the usual construct of the currencies abbreviations are the EUR (is not relative to a country but to a group of countries), the CHF (CH designates in Latin *Confoederatio Helvetica*) and the BRL (instead of BRR).

(1) The designations of the several currencies are defined in ISO 4217.

EXERCISE 2.07

Put in increasing order the value of the following currencies: euro, United States dollar, Great Britain pound, Suisse franc and the Japanese yen.

Solution

Since the quote for each currency is not fixed, that is, is dependent on market conditions, the relation between them may vary through time. Recently the following relations have been present:

$$\text{JPY} < \text{USD} < \text{CHF} < \text{EUR} < \text{GBP}$$

Although in the current century the relation between the GBP and the JPY with the other currencies has been the same as the one presented above, the relation between the EUR, USD and CHF has suffered some changes. Until December 2002 the USD always was worth more than the EUR, while until September 2010 the USD has always was worth more than the CHF. The value of the EUR has always been above the value of the CHF.

EXERCISE 2.08

A firm wants to exchange 50,000 euros for the corresponding amount in dollars. Knowing that the EUR/USD exchange rate is 1.2500, how many dollars is the firm going to receive?

Solution

The exchange rate EUR/USD being 1.2500 means that one euro is equivalent to 1.25 United States dollars. Thus, with the 50,000 euros the firm is able to get:

$$\text{Total USD} = 50,000 \text{ EUR} \times 1.2500 = 62,500 \text{ USD}$$

The firm will receive 62,500 USD in exchange for the 50,000 EUR.

EXERCISE 2.09

A firm is about to receive 72,400 USD from an export operation. Considering the exchange rate for the EUR/USD to be 1.2500, how many euros will the firm receive?

Solution

The given exchange rate indicates that for each 1.25 United States dollar is possible to get 1 euro. Thus, with the 72,400 USD it is possible to obtain:

$$\text{Total EUR} = \frac{72,400 \text{ USD}}{1.2500} = 57,920 \text{ EUR}$$

The firm will receive 57,920 EUR in exchange for the 72,400 USD.

EXERCISE 2.10

In a given moment the quotes for the EUR/CHF are:

	Bid	Ask
EUR/CHF	1.4010	1.4030

Determine the corresponding quotes for the CHF/EUR.

Solution

We are asked to determine an inverse quotation. The exchange rates given are per each EUR while the ones we need to determine are per each CHF. However, since we have bid and ask quotes, we need to proceed with caution in order for the inverse quotes to be coherent:

$$\text{CHF/EUR}_{\text{bid}} = \frac{1}{\text{EUR/CHF}_{\text{ask}}} = \frac{1}{1.4030} = 0.7128$$

$$\text{CHF/EUR}_{\text{ask}} = \frac{1}{\text{EUR/CHF}_{\text{bid}}} = \frac{1}{1.4010} = 0.7138$$

From the previous calculations one can easily see that the bid quote for the inverted quotation is determined using the ask quote for the initial exchange rate, and vice-versa. Intuitively this must always be the case or otherwise the financial institution would be buying at a higher price than the sale price, and that would make no sense at all. In fact, one can see that the exchange rate $\text{CHF/EUR}_{\text{bid}}$ is the price at which the FI buys CHF and sells EUR, and that will correspond to the $\text{EUR/CHF}_{\text{ask}}$ exchange rate, where the FI sells EUR and buys CHF.

EXERCISE 2.11

An individual is willing to purchase 1,000 USD. The exchange rate presented by the Bank at the counter is:

	Bid	Ask
1 USD	0.8320	0.8340

How many euros will he need to spend to get 1,000 USD?

Solution

The exchange rates are always presented from the financial institution's point of view. Since the quotes are given for each USD, the exchange rates presented are then the USD/EUR. If the individual wants to buy USD, he will do it at the exchange rate that the FI is willing to sell USD, which in this case corresponds to the ask quote:

$$\text{Total EUR} = 1,000 \text{ USD} \times 0.8340 = 834 \text{ EUR}$$

The individual will have to spend 834 EUR to buy the 1,000 USD.

EXERCISE 2.12

A suitcase costs:

- In the US: 228 USD;
- In Switzerland: 245 CHF;
- In Portugal: 180 EUR.

Knowing that the exchange rates are:

- EUR/USD = 1.2430;
- EUR/CHF = 1.4340.

In which country is the suitcase cheaper?

Solution

To get to the answer we first need to determine the price of the suitcase, in the different countries, in the same currency. Using the given exchange rates the easiest option is to put all prices in euros:

$$\text{Price US} = \frac{228 \text{ USD}}{1.2430} = 183.43 \text{ EUR}$$

$$\text{Price Switzerland} = \frac{245 \text{ CHF}}{1.4340} = 170.85 \text{ EUR}$$

The suitcase is cheaper in Switzerland, followed by Portugal and last in the US. If we compare the prices in USD or in CHF the answer will still be the same.

EXERCISE 2.13

The security issued by firm ABC has a maturity of 4 years. This security is a share or a bond? Justify.

Solution

Since the security has a maturity then it must be a bond. Although there are bonds with no maturity, named perpetual, there are no shares with maturity.

EXERCISE 2.14

The dividend paid out by firm MJH represented 4% of the asset's price. Is this asset a share or a bond? Justify.

Solution

The dividend is the return given to the shareholders after positive earnings. Thus, the security is a share. The return associated with a bond is the interest, usually called the coupon.

EXERCISE 2.15

The Portuguese Government issued bonds with a maturity of 3 years. Firm TFR issued bonds with the same maturity. Under normal circumstances, which security will offer a higher return? Justify.

Solution

The return associated with a bond issue is deeply dependent on the inherent risk of the issuing entity and on the specific characteristics of the issue itself. Under normal circumstances, which is not the case of the last years of financial crisis, the risk associated with the Portuguese Government is inferior to the one associated with a specific firm, even more if the latter is unknown to the general public. Thus, the security that should be offering a higher return should be the one issued by the firm TFR. We should note that, justified by the financial crisis, we can find higher returns associated with sovereign debt issues than those offered by a significant number of firms.

EXERCISE 2.16

The takeover bid launched by SONAE over the PT shares is an operation of primary or secondary market? Justify.

Solution

The operation takes place in the secondary market. The goal of the operation is to acquire a number of existing PT shares by SONAE. These shares have been previously issued and exist in the market, owned by other investors/stockholders. With this operation no new securities are issued.

EXERCISE 2.17

An investor holds 95% of the shares of a firm.

a) Do you think that this company has its shares listed on the stock exchange? Why?

b) If the investor instructed an investment bank to sell half of these shares on the market, this would be a primary or secondary market operation?

Solution

a) No. For a firm to be listed on the stock exchange it is necessary to have a minimum percentage of capital dispersion that assures the well-functioning of the market.[1]

b) It is a secondary market operation. The shares already exist and the goal of the operation is just to change ownership.

EXERCISE 2.18

Say which markets are represented by the following indexes and the number of firms included in each one (on the most relevant index):

- PSI;
- NIKKEI;
- DAX;
- IBEX;

(1) Please see "Artigo 229.º" from the "Código dos Valores Mobiliários" in www.cmvm.pt.

- CAC;
- FTSE;
- DJI;
- NASDAQ.

Solution

- PSI – Portuguese market of firms traded on the Euronext Lisbon. The most relevant index is the PSI20, which includes 20 companies;
- NIKKEI – Japanese market of firms traded on the Tokyo Stock Exchange (TSE). The most relevant index is the NIKKEI225, which includes 225 companies;
- DAX – German market of firms traded on the Frankfurt Stock Exchange. The most relevant index is the DAX30, which includes 30 companies;
- IBEX – Spanish market of firms traded on the Madrid Stock Exchange. The most relevant index is the IBEX35, which includes 35 companies;
- CAC – French market of firms traded on the Paris Stock Exchange. The most relevant index is the CAC40, which includes 40 companies;
- FTSE – British market of firms traded on the London Stock Exchange (LSE). The most relevant index is the FTSE100, which includes 100 companies;
- DJI – north-American market of industrial firms traded on the New York Stock Exchange (*NYSE*) and NASDAQ. The index includes 30 companies;
- NASDAQ – north-American market of mainly tech firms traded on the exchange NASDAQ. The most common index is the NASDAQ100, which includes 100 companies.

EXERCISE 2.19

A company contracted today, Tuesday May 6, a loan of 100,000 euros at an interest rate of 6% per year, that will start within 5 days and will mature on August 19. What is the amount of interest that will be paid on the maturity of the loan?

Solution

To determine the amount of interest we begin by counting the number of days in the operation. The interest will begin counting on May 13, which corresponds to the 5th business day after the contract date:

Month	Days
May	31 – 13 = 18
June	30
July	31
August	19
Total	98

$$\text{Interest} = C \times r \times \frac{\text{dias}}{360} = 100,000 \times 0.06 \times \frac{98}{360} = 1,633.33$$

The total amount of interest to be paid at maturity will be 1,633.33 euros.

EXERCISE 2.20

A Portuguese firm contracted 91 days ago a funding of 100,000 USD, at an annual interest rate of 5%. The loan matures today. When the loan began the EUR/USD exchange rate was 1.2540.

a) What is the exchange rate today that makes null the cost of this funding for the Portuguese company?

b) The EUR appreciated or depreciated against the USD? Justify.

Solution

a) For the financing to have a null cost, in EUR, it is necessary that the amount received in euros 91 days ago, be the same as the amount needed to pay the loan in USD. The amount received in euros was:

$$\text{Amount}_{\text{EUR}} = \frac{100,000 \text{ USD}}{1.2540} = 79,744.82 \text{ EUR}$$

At the end of the 91 days it is necessary to pay:

$$\text{Amount}_{\text{USD}} = 100,000 \times \left(1 + 0.05 \times \frac{91}{360}\right) = 101,263.89 \text{ USD}$$

Since the 79,744.82 EUR will have to match the 101,263.89 USD for the financing cost to be null, it is then possible to determine the EUR/USD exchange rate needed for that to happen:

$$\text{EUR/USD} = \frac{101,263.89 \text{ USD}}{79,744.82 \text{ EUR}} = 1.2698$$

The exchange rate must be 1.2698.

Another away to solve the exercise is to realize that the amount owed in dollars increased, per each dollar of loan, at $\left(1 + 0.05 \times \frac{91}{360}\right)$(principal + interest). Thus, for the effect to be null in euros, this increase must be cancelled out by the same change in the initial exchange rate:

$$1.2540 \times \left(1 + 0.05 \times \frac{91}{360}\right) = 1.2698$$

exactly the same result has the one previously obtained.

b) The increase in the EUR/USD exchange rate from 1.2540 to 1.2698 means that with one EUR it is now possible to buy a higher amount of USD, thus the EUR has appreciated relative to the USD. Intuitively, to have a null cost in EUR from a loan contracted in USD, we must benefit from the appreciation of the EUR relative to the USD.

EXERCISE 2.21

A watch is priced at 100 CHF in Switzerland and 11,250 JPY in Japan. A stock is priced at 38,500 JPY in the Tokyo Stock Exchange and 95 USD in the New York Stock Exchange. Compute a possible price for the watch in New York.

Solution

The information about the price of the watch in Switzerland is not necessary to solve the exercise. Using the stock prices from the Japanese and north-American markets it is possible to determine the exchange rate and therefore, the expected price for the watch:

$$\text{USD/JPY} = \frac{38,500}{95} = 405.263$$

$$\text{Price}_{\text{USD}} = \frac{11,250 \text{ JPY}}{405.263} = 27.76 \text{ USD}$$

The expected price for the watch in New York is 27.76 USD.

EXERCISE 2.22

A Portuguese investor made, a month ago, a deposit in USD for a month. On that day the EUR/USD exchange rate was 1.1800. Today, when the deposit matures, the exchange rate is 1.2400. Is the investor happy with the evolution of the exchange rate? Justify?

Solution

No, the EUR appreciated relative to the USD. The same amount of USD, received at the end of the deposit, will be exchange for an inferior amount of EUR, negatively influencing the overall return on the term deposit.

EXERCISE 2.23

An individual exchanged 1,000 USD for 500 GBP. Bought a watch that cost 125 GBP. Exchanged the remaining for EUR. Knowing that the exchange rate EUR/USD is 1.2500, what is the final amount that he obtained in EUR?

Solution

The information regarding the exchange from USD to GBP allows us to determine the underlying exchange rate:

$$\text{GBP/USD} = \frac{1,000}{500} = 2.0000$$

After buying the watch there were only 375 GBP left, which in USD equal:

$$\text{Amount}_{\text{USD}} = 375 \text{ GBP} \times 2.0000 = 750 \text{ USD}$$

Using the given exchange rate for the EUR/USD of 1.2500, it is possible to convert the 750 USD into EUR:

$$\text{Amount}_{\text{EUR}} = \frac{750 \text{ USD}}{1.2500} = 600 \text{ EUR}$$

The final amount obtained was 600 euros.

EXERCISE 2.24

Today the exchange rate *X*/*Y* is 0.6000. If you are a citizen of country *X* and consider, within a month, going on holiday to the country of currency *Y*, will you be happy if at that time the exchange rate is 0.5500? Justify.

Solution

The answer is dependent on the moment when the currency exchange took place. If the exchange between *X* and *Y* occurs todays, then one month from now the tourist will be happy because he obtains a larger amount of currency *Y* than the one he would obtain if he waited until the departure date. Per each unit of *X* he obtains 60 cents of *Y*, instead of the 55 cents he would obtain if he waited. If the currency exchange takes place only at the departure date, then the evolution of the exchange rate is detrimental to the tourist because now he will have to spend more of *X* to buy the same amount of *Y* than he would have one month ago (today).

EXERCISE 2.25

A Portuguese investor exchanged 800 euros into dollars and then exchanged the dollars obtained by British pounds. In the end he was left with 520 pounds. Knowing that the exchange rate USD/GBP is 0.6250, determine the exchange rate EUR/USD.

Solution

To determine the EUR/USD exchange rate it is necessary to have an amount in EUR and the corresponding amount in USD. We can begin by using the given exchange rate for the USD/GBP and convert the 520 GBP back to USD:

$$\text{Amount}_{\text{USD}} = \frac{520}{0.6250} = 832 \text{ USD}$$

Since we already know that the 832 USD correspond to the initial amount of 800 EUR, we can easily determine the inherent exchange rate:

$$\text{EUR/USD} = \frac{832}{800} = 1.0400$$

The exchange rate for the EUR/USD is 1.0400.

EXERCISE 2.26

The current CHF/USD exchange rate is 1.2400. A company will receive an export in USD. This amount converted to EUR, at the EUR/USD exchange rate of 1.4500, generates 10,000 EUR. If instead of converting into EUR the company preferred to repay an existing CHF loan, what is the amount of the loan that could be paid?

Solution

The goal is to convert the 10,000 EUR into CHF. Since we already know the relation between the EUR and USD and between the USD and CHF, the conversion is straightforward:

$$\text{Amount}_{\text{CHF}} = \frac{(10,000 \text{ EUR} \times 1.4500) \text{ USD}}{1.2400} = 11,693.55 \text{ CHF}$$

The loan amount that could be paid would be 11,693.55 CHF.

EXERCISE 2.27

A Portuguese emigrant living in Britain will buy a house in Portugal which costs 250,000 euros. He wants to borrow 10% of that value. A British bank offers a GBP loan for 320 days at an annual interest rate of 2.75% and a Portuguese bank offers a loan in EUR, for the same period at an annual interest rate of 4%. Knowing that today the EUR/GBP rate is 0.9000 and that the emigrant chose the alternative of the Portuguese bank, what is the emigrant's forecast regarding the EUR/GBP rate at the maturity of the loan?

Solution

First we begin by determining the EUR/GBP exchange rate that makes both alternatives indifferent. The first step is to calculate the loan amounts in EUR and in GBP:

$$\text{Loan}_{\text{EUR}} = 250,000 \text{ EUR} \times 0.1 = 25,000 \text{ EUR}$$

$$\text{Loan}_{\text{GBP}} = 25,000 \text{ EUR} \times 0.9000 = 22,500 \text{ GBP}$$

Next we determine the total amount to be paid out at the loan maturity[1]:

$$\text{Loan}_{\text{EUR}} = 25,000 \text{ EUR} \times \left(1 + 0.04 \times \frac{320}{360}\right) = 25,888.89 \text{ EUR}$$

$$\text{Loan}_{\text{GBP}} = 22,500 \text{ GBP} \times \left(1 + 0.0275 \times \frac{320}{365}\right) = 23,042.47 \text{ GBP}$$

(1) Note that the day counting convention to use with the GBP is Actual/365.

Using the previous values it is now possible to determine the exchange rate that makes both financing alternatives indifferent:

$$\text{EUR/GBP} = \frac{23,042.47 \text{ GBP}}{25,888.89 \text{ EUR}} = 0.8901$$

Since the emigrant chose the Portuguese bank, the forecast he did was that the EUR/GBP exchange rate, at the loan maturity, would be inferior to 0.8901, that is, that the amount needed to liquidate the loan in EUR would represent, at most, 23,042.47 GBP (the amount he would paid if he had chosen the British bank).

EXERCISE 2.28

An investor goes to the bank, today Friday 15th December, with a check of 20,000 EUR to make a deposit that will expire on January 24, Wednesday. The bank will give the check the value date of the next business day. If the annual interest rate is 4%, how much interest will be generated at the maturity of the deposit?

Solution

To solve the exercise we begin by counting the days of the operation. The deposit will begin on December 18th, which corresponds to the next business day after the delivery of the check:

Month	Days
December	31 − 18 = 13
January	24
Total	37

$$\text{Interest} = C \times r \times \frac{\text{days}}{360} = 20,000 \times 0.04 \times \frac{37}{360} = 82.22$$

The total amount of interest to be earned with the deposit is 82.22 euros.

EXERCISE 2.29

A company received 30,000 GBP and wants to exchange them for euros. The bank showed the following bid and ask quotes:

	Bid	Ask
EUR/GBP	0.6520	0.6540

Compute the value that the company will receive in EUR.

Solution

Since the quotes are given from the bank's perspective and relative to the base currency (in this case is the EUR), the quote to use is the 0.6540, since it represents the value that the bank is willing to sell EUR in exchange for buying GBP:

$$\text{Amount}_{EUR} = \frac{30,000 \text{ GBP}}{0.6540} = 45,871.56 \text{ EUR}$$

The firm will receive 45,871.56 EUR in exchange for the 30,000 GBP.

EXERCISE 2.30

A Portuguese company has made an export to Japan by the total amount of 1,600,000 JPY. The product sold had been purchased in the US for 12,000 USD and was subject to a value added intervention in Portugal by the amount of 3,000 EUR. In the bottom line, the profit for the whole operation was 1,200 EUR. Knowing that the exchange rate USD/EUR is 0.8000, determine the exchange rate EUR/JPY that allows for that profit of 1,200 EUR.

Solution

Too be able to determine the EUR/JPY exchange rate we need same amounts in both currencies relative to the same good. Since we already know the amount of the export in JPY, we are now going to determine the corresponding amount in EUR:

$$\text{Purchase}_{\text{EUR}} = 12,000 \text{ USD} \times 0.8000 = 9,600 \text{ EUR}$$

$$\text{Sale}_{\text{EUR}} = 9,600 + 3,000 + 1,200 = 13,800 \text{ EUR}$$

Using the sale value in EUR it is now possible to determine the EUR/JPY exchange rate:

$$\text{EUR/JPY} = \frac{1,600,000 \text{ JPY}}{13,800 \text{ EUR}} = 115.942$$

The EUR/JPY exchange rate that allowed for the profit of 1,200 EUR in the export was 115.942.

EXERCISE 2.31

A Portuguese company has just paid an import of 35,400 GBP, relative to the purchase of 5,000 products, at the GBP/EUR exchange rate of 0.9000. These products will be resold in the US after being incorporated with a national device with a unit cost of 3 euros, by a total value of 85,312.5 USD. What would be the EUR/USD exchange rate for the Portuguese company to obtain a unit margin of 2 euros on the export of the products to the US?

Solution

Since we already know the sale amount in USD, to obtain the EUR/USD exchange rate we just need to determine the sale amount in EUR:

$$\text{Purchase}_{\text{EUR}} = 35,400 \text{ GBP} \times 0.9000 = 31,860 \text{ EUR}$$

$$\text{Sale}_{\text{EUR}} = 31,860 + 3 \times 5,000 + 2 \times 5,000 = 56,860 \text{ EUR}$$

Now that we have the sale amount in EUR we are able to compute the EUR/USD exchange rate:

$$\text{EUR/USD} = \frac{85,312.5 \text{ USD}}{56,860 \text{ EUR}} = 1.5004$$

The EUR/USD exchange rate, which allows for the Portuguese company to get a unit margin of 2 EUR, is 1.5004.

EXERCISE 2.32

A deposit began on March 11 (Tuesday) and ended on June 9 (Monday) and generated 120 euros of interest. Knowing that its stated annual rate is the same as the one of another deposit that generated 10 euros of interest in 65 days with a principal of 1.800 euros. Determine the capital of the first deposit.

Solution

Using the information from the second deposit we are able to determine the stated annual interest rate:

$$10 = 1,800 \times r \times \frac{65}{360} \quad \Leftrightarrow \quad r = 0.0308$$

To compute the principal of the first deposit we must begin by calculating its term:

Month	Days
March	31 – 11 = 20
April	30
May	31
June	9
Total	90

$$120 = C \times 0.0308 \times \frac{90}{360} \quad \Leftrightarrow \quad C = 15,584.42$$

The invested capital on the first deposit was 15,584.42 euros.

EXERCISE 2.33

A Portuguese company is negotiating the import of 1.000 tons of raw materials with two suppliers. A Japanese supplier offered a price of 810,000 JPY while a Swiss supplier offered a price of 12,600 CHF. The company decided for the Japanese supplier. Knowing that the current EUR/CHF exchange rate is 1.40, what is the price

(minimum or maximum), for the EUR/JPY exchange rate, that led the company to choose for the Japanese supplier? *Note: you should calculate the asked rate and state whether it represents the minimum or maximum value that the EUR/JPY exchange rate must have, in order to justify the choice for the Japanese supplier.*

Solution

Since we already know the import amount in JPY, to determine the EUR/JPY exchange rate that would make both alternatives indifferent we only need to compute the import amount in EUR for the Swiss supplier:

$$\text{Supplier}_{\text{swiss}} = \frac{12,600 \text{ CHF}}{1.4000} = 9,000 \text{ EUR}$$

$$\text{EUR/JPY} = \frac{810,000 \text{ JPY}}{9,000 \text{ EUR}} = 90.000$$

If the EUR/JPY exchange rate is lower than 90.000, then the cheapest option is to import from Switzerland. Since the firm chose to import from Japan, the EUR/JPY exchange rate of 90.001 represents the minimum rate that makes this option the preferred one. The higher the exchange rate, the more advantageous it becomes to import in JPY, since less EUR are needed to pay for those JPY.

EXERCISE 2.34

A Swiss investor bought 100 shares of PT at the price of 9.80 EUR/each. For this the investor sold 200 shares of UBS, at the price of 2.20 CHF/each, and 400 shares of YAHOO, at the price of 1.80 USD/each. Knowing that the exchange rate CHF/USD is 1.2055, determine the exchange rates:

a) USD/EUR;

b) EUR/CHF.

Solution

a) To solve the exercise we must assume that all the proceedings from the sale of the PT shares were fully used on buying the UBS and YAHOO shares. The investor received 980 EUR with the sale and acquired shares worth:

$$\text{Purchases}_{\text{USD}} = 200 \times 2.20 \text{ CHF} \times 1.2055 + 400 \times 1.80 \text{ USD} \quad \Leftrightarrow$$

$$\Leftrightarrow \quad \text{Purchases}_{\text{USD}} = 1,250.42 \text{ USD}$$

$$\text{USD/EUR} = \frac{980 \text{ EUR}}{1,250.42 \text{ USD}} = 0.7837$$

The USD/EUR exchange rate is 0.7837.

b) Since we already know the relation between the EUR and the USD and between the USD and the CHF, the calculation of EUR/CHF exchange rate is straightforward:

$$\text{EUR/CHF} = \text{EUR/USD} \times \text{USD/CHF} = \frac{1}{0.7837} \times \frac{1}{1.2055} = 1.0585$$

The EUR/CHF exchange rate is 1.0585.

EXERCISE 2.35

The exchange rates in the market are the following:

- EUR/GBP – 0.9000;
- USD/EUR – 0.6600;
- GBP/USD – 1.7000.

According to these exchange rates, and considering that you have 1,000 EUR, can you make money out of exchanging currencies? If so, state the transactions you should do and the profit you would get in EUR.

Solution

Since we have 1,000 EUR to begin with, the idea is to be able to end up with more than that amount through a combination of exchanging currencies operations. We can follow the path: exchange EUR to GBP, GBP to USD and USD to EUR; or exchange EUR to USD, USD to GBP and GBP to EUR:

$$\text{Path}_1 = [(1{,}000 \text{ EUR} \times 0.9000)\,\text{GBP} \times 1.700]\,\text{USD} \times 0.6600 =$$
$$= 1{,}009.8 \text{ EUR}$$

$$\text{Path}_2 = \frac{\left[\frac{\left(\frac{1{,}000 \text{ EUR}}{0.6600}\,\text{USD}\right)}{1.700}\right]\text{GBP}}{0.9000} = 990.3 \text{ EUR}$$

It is possible to get a profit if we begin by exchanging EUR to GBP, GBP to USD and finally USD to EUR. The profit is 9.8 EUR which corresponds to a return of 0.98%.

EXERCISE 2.36

Consider the following exchange rates that occur at the same time in the financial markets in Frankfurt and New York:

	Bid	Ask
EUR/USD (Frankfurt)	1.3850	1.3870
USD/EUR (New York)	0.7330	0.7350

Imagine that you have 1,000,000 EUR. Can you profit from an arbitrage opportunity? If so, identify it and determine the profit that you can achieve in EUR.

Solution

To be able to easily compare the quotes from both financial markets it is convenient that both quotes are presented in the same format. Thus, we begin by determining the EUR/USD in New York:

$$\text{EUR/USD}_{\text{Bid}} = \frac{1}{\text{USD/EUR}_{\text{Ask}}} = \frac{1}{0.7350} = 1.3605$$

$$\text{EUR/USD}_{\text{Ask}} = \frac{1}{\text{USD/EUR}_{\text{Bid}}} = \frac{1}{0.7330} = 1.3643$$

One can easily find an arbitrage opportunity between the quotes in both markets. In the New York market the EUR is depreciated against the USD. Since we own EUR, we want to sell them in the market where the EUR is worth more and buy them back where it is cheaper. The most advantageous option is to exchange the EUR with USD in the Frankfurt market, where we get:

$$\text{Amount}_{\text{USD}} = 1,000,000 \times 1.3850 = 1,385,000 \text{ USD}$$

The sale of the USD and purchase of EUR should take place in the New York market, where we are able to get more EUR with each USD:

$$\text{Amount}_{\text{EUR}} = 1,385,000 \text{ USD} \times 0.7330 = 1,015,205 \text{ EUR}$$

The profit obtained from this arbitrage opportunity is 15,205 EUR, which correspond to a return of 1.52% (immediate and with no permanent capital invested. However, in reality, when these opportunities exist they almost immediately disappear).

EXERCISE 2.37

A Portuguese investor has acquired in the North American stock market 1.000 shares of Microsoft (MSFT) company. The Investor sold the same shares after 2 years. From these operations we know the following:

	At time of purchase	After one year	At time of sale
Price of one share of MSFT in USD	20.00$	22.00$	23.00$
EUR/USD exchange rate	1.4000		
USD/EUR exchange rate		***B)***	
Position total in EUR	***A)***	13,750.00€	14,712.50€

a) Which financial markets are associated with this operation? Justify;

b) Compute ***A)*** – the value, in Euros, invested at the time of purchase;

c) Compute ***B)*** – the USD/EUR exchange rate after one year;

d) The Euro appreciated or depreciated against the dollar in the end of the first year? Justify;

e) To prevent the investor of having losses (break-even) after two years, what is the maximum/minimum percentage of appreciation/depreciation that the Euro can have against the Dollar?

Solution

a) The financial markets present in this operation are the foreign exchange market and the capital market. The foreign exchange market includes all transactions on currency exchange, in this case between the EUR and the USD. The capital market includes operations of medium- long-term financing using financial instruments, in this case shares.

b) The amount invested in EUR will correspond to the 20,000 USD converted to EUR at an EUR/USD exchange rate of 1.4000:

$$\textbf{\textit{A)}} = \frac{20,000 \text{ USD}}{1.4000} = 14,285.71 \text{ EUR}$$

c) The USD/EUR exchange rate is possible to determine using the shares value in EUR and in USD at the end of the first year:

$$\textbf{\textit{B)}} = \frac{13,750 \text{ EUR}}{22 \text{ USD} \times 1,000} = 0.6250$$

The USD/EUR exchange rate at the end of the first year was 0.6250.

d) Using the shares value in USD and the corresponding value in EUR one can easily state that the EUR appreciated against the USD. One can see that the amount invested in USD increased, since each share gained 2 USD in value after the first year. On the other hand, the corresponding amount in EUR fell from 14,285.71 EUR to 13,750 EUR, which is only possible if the USD lost value against the EUR. The previous statement can be confirmed using the exchange rate already computed:

$$\text{EUR/USD}_{1\text{ year}} = \frac{1}{0.6250} = 1.6000$$

The EUR/USD exchange rate went from 1.4000 at the initial moment to 1.6000 after one year. With each EUR it is possible to buy more 20 cents of the dollar, that is, the EUR appreciated against the USD.

e) To prevent the investor from having losses, the EUR amount received from the sale of the shares has to be, at least equal to the amount used to buy those same shares. Since at the moment of sale each share is worth 23 USD, we get:

$$\text{EUR/USD}_{2\text{ years}} = \frac{23\text{ USD} \times 1,000}{14,285.71\text{ EUR}} = 1.6100$$

The EUR/USD exchange rate would have to be 1.6100 tops, thus the EUR cannot appreciate more than 15% $\left(\frac{1.61000}{1.4000} - 1\right)$ against the USD. This value could also be obtained using the increase in the shares' value, which is also 15% $\left(\frac{23}{20} - 1\right)$. To prevent the investor from having losses it is necessary that the appreciation of the EUR against the USD is not higher than the increase in value of the shares.

EXERCISE 2.38

In no more than 10 lines, answer clearly and concisely to the following questions:

a) A bank wants to present to the deposit customers its rates on the 365 days convention. These stated rates will be higher or lower than those previously presented on the 360 days convention? Justify;

b) The Euribor is usually higher than the intervention rates of the European Central Bank. Do you agree? Justify;

c) An investor decided to hold a deposit account in CHF, which offered a lower interest rate than a deposit in EUR. What are the expectations for the investor behind this decision?

d) A company just sold 10 000 of its own shares that had in its self-portfolio to one of its shareholders. Is it an operation of primary or secondary market? Justify;

e) An investor has 1,000 shares of Company ABC. Now is going to buy 100 shares more through the issue that the company is doing to finance a new investment. Is this purchase an operation of the primary or secondary market? Justify.

Solution

a) In order to keep the amount of interest the same the bank has to present higher rates. When using the day counting convention of Actual/365, instead of Actual/360, the interest amount will diminish. The way to cancel out this effect is by increasing the interest rate, multiplying the existing interest rate by a factor of $1.0139\left(\frac{365}{360}\right)$.

b) Yes. The EURIBOR is always higher than the ECB's intervention rates, because it represents an average of the rates at which major financial institutions in the euro area (or that deal with large volumes in euros) are willing to lend to other FI, i.e. the risk of lending money to another FI is always higher than the one carried out in a deposit on the ECB and so, consequently, the return for the capital will have to be higher than the offered by the ECB.

c) The expectation for this investor is that there will be an appreciation of the CHF against the EUR or a depreciation of the EUR against the CHF, in such a way that offsets the lower interest rate offered, i.e. at the end of the term of the deposit, in spite of its capital having been repaid at a lower rate, the exchange rate must be such that when changing the CHF for EUR we can get a higher value than the one we would get if we had made the deposit in EUR.

d) It is a secondary market transaction because it occurs with an asset that already exists, i.e. there was no place for the issuing of a new security, but rather just a sale of securities currently held by the company.

e) This operation falls within the primary market, which deals with the issue and placement of securities in which the proceeds from the sale go to the issuer. In this case is a financing operation through the issuance of new shares, increasing the capital of the company.

Follow-up Exercises

EXERCISE 2.39

An investor deposited in the US an amount in dollars corresponding to 250,000 euros, at an annual interest rate of 3% per year. The operation was made between June 26 (Friday) and October 8 (Tuesday). The Bank gave the deposit the value date of the next business day and the EUR/USD exchange rate was 1.3200:

a) Determine the amount that the investor will receive at the deposit maturity, in USD;

b) If, at the deposit maturity date the Euro has appreciated against the Dollar, the investor will be happy or sad? Exemplify.

EXERCISE 2.40

Comment the following statement: «The PSI20 index is useful, since it gives us detailed information regarding its constituents' performance, for each transaction day in the primary market ».

EXERCISE 2.41

An investor asked today for a loan in Great Britain pounds, at an annual interest rate of 5%. He then invested that amount in a savings product in euros, for the same duration, at an annual interest rate of 7%. Knowing that the price of a BigMac is today 3.2 euros and 2.4 pounds, and that at the end of the operations the investor will have neither gains nor losses with his strategy, what will be the price of the BigMac in pounds 91 days from now, assuming that in euros it will be 3.35 euros?

EXERCISE 2.42

Comment on the following statement: «The EURIBOR is a reference rate for the price asked between Portuguese financial institutions for 6 months operations, published at the beginning of each month.».

EXERCISE 2.43

Company *Finumo,* financed its operations with an amount of 200,000 euros, at an annual interest rate of 2% and for a period of 189 days.

Company *Coumem,* acquired a loan in CHF in an amount equivalent to 200,000 euros at an annual interest rate of 3% and also for a period of 189 days. The EUR/CHF exchange rate was, at the date of the contract, of 0.8910 and currently, at the loan maturity, is 0.8990.

a) Which company got the best deal with its financing operation? Present all calculations and justify them;

b) If the EUR/CHF exchange rate today had the value of 0.8500 would you still feel the same away? Explain your reasoning in detail.

EXERCISE 2.44

An investor bought, at the same time, bonds from two different issuers, some in dollars and some in euros. At the time of purchase each euro was worth 1.4245 dollars and today is worth 1.3125. The bonds in dollars were bought at 10.89 each and today have a sell value of 9.93 each. The bonds in euros were bought at 5.63 each and are worth today 5.02 each.

a) With which of the two bond investments is the investor more pleased with? Justify;

b) What would have to be the current exchange rate between the two currencies in order for the investor to be equally pleased with both investments?

EXERCISE 2.45

A firm has undertook a loan of 10,000 euros that begins on March 3 (Thursday) and ends on September 15 (Thursday). The stated annual interest rate is 6%. The interest is subject to stamp tax of 4%. The firm pays a fee of 75 euros at the beginning of the loan.

a) Compute the amount of interest to be paid at the end of the loan;

b) Determine the EGAR (GAER) and the EAR (AER) of the operation.

EXERCISE 2.46

A firm has received two business proposals from two suppliers of raw materials used in the production of its product:

- The supplier *KJH*, Japanese, asked for a price of 1,200 JPY per kg;
- The supplier *LTY*, Australian, asked for a price of 12 AUD per kg.

a) Knowing that the EUR/JPY exchange rate is 120.00, determine the EUR/AUD exchange rate that makes both alternatives indifferent;

b) The firm sells its products exclusively abroad and has a sell price established in USD. In the last 3 months the profit margin has increased as a conse-

quence of the favourable evolution in the exchange rates. Does that mean that in these last months the EUR has appreciated against the USD? Justify.

EXERCISE 2.47

Consider the following exchange rates:

	Bid	Ask
EUR/USD	1.3520	1.3570
GBP/USD	1.1590	1.1630

An investor has 100,000 euros. He wants to invest 50% of the capital on a deposit in GBP for a period of 180 days at an annual interest rate of 4%. The remaining 50% will be invested on a deposit in USD at an annual interest rate of 3% and with the same duration.

a) State the amount of interest that the investor will receive from each of the deposits;

b) Assume that at the end of the deposit in USD, exchanging the capital and interest amount to EUR, the investor will get an effective annual return (annual equivalent rate) of 5.5%. Determine the EUR/USD exchange rate, at that time, in order for the investor to obtain that 5.5% return.

EXERCISE 2.48

For a company that is about to issue a bond loan, what is the benefit of doing it with the help of an Investment bank underwriter?

EXERCISE 2.49

An investor owns some shares of the firm *ABC* which is about to be acquired by the firm *HGJ*. To pay for the acquisition, the firm *HGJ* is issuing equity capital. These new shares will be used to pay the stockholders of firm *ABC*, leaving them owners of shares of the firm *HGJ.* In your opinion, is this operation included in the primary and/or secondary markets? Justify.

EXERCISE 2.50

Assume you want to buy a music album that is not available in Portugal. You have the options of acquiring it through an English website or from a north-American online music service. In the English website the album costs 30.6 GBP and in the north-American music service you have to pay 45 USD. Knowing that the EUR/GBP exchange rate is 0.8500:

a) What should be the EUR/USD exchange rate in order for both alternatives to be indifferent?

b) Assume that you decided to buy the music album from the English website because it was cheaper. At that time the EUR/USD exchange rate was higher or lower than the one computed in *a)*? Justify.

EXERCISE 2.51

«Every day we hear about changes in the yields (return rates) of Portuguese sovereign bonds. Does this means that every day the Portuguese Government pays its debt at different interest rates?». Answer, stating the associations between the financial markets that you know.

EXERCISE 2.52

A bank loan of 100,000 euros is being contracted on October 22 (Monday) and it matures on February 5 (Tuesday). The stated annual interest rate is 6%. Determine the amount of interest to be paid at the end of the loan.

EXERCISE 2.53

The spot USD/EUR exchange rate is currently 0.8055. If you have 100,000 EUR and want to exchange for USD, how many USD will you get?

EXERCISE 2.54

A firm bought an equipment from an English supplier for 200,000 GBP. It will have to pay that amount in 180 days. In order to accomplish that, the firm made today a deposit in GBP, at a stated annual interest rate of 5%. To get the money for the deposit the firm needed to ask for a loan of 215,000 EUR (which immediately exchanged for GBP) for a period of 180 days, at a stated annual interest rate of 6%.

a) Determine the spot EUR/GBP exchange rate that was in place at the time the deposit was made;

b) What EUR/GBP exchange rate was implicit, at the moment the firm paid for the imported equipment?

EXERCISE 2.55

An IPO done by a shareholder that owns 90% of the company shares is an operation of primary or secondary markets? Justify.

EXERCISE 2.56

A firm just paid 1,722.20 euros of interest for a 100,000 euros loan that ended on May 31 (Thursday). Knowing that the stated annual interest rate was 10%, when did the financing operation began (assume it was a business day)?

EXERCISE 2.57

An investor has two investment options:

- A deposit of 100,000 GBP at a stated annual interest rate of 4%, for a period of 60 days;
- A deposit in euros, at a stated annual interest rate of 3%, for the same duration.

Knowing that currently the EUR/GBP exchange rate is 0.9000 and that the investor chose the deposit in GBP:

a) Determine the EUR/GBP exchange rate that must be in place 60 days from now, in order for both alternatives to be indifferent;

b) Using the result obtained in *a)*, is there an appreciation or depreciation of the EUR against the GBP?

Chapter 3

PROFITABILITY AND SOLVENCY IN THE FIRM

Theory

Profitability

The question that will serve as inspiration to this first part of the chapter is summarized as follows: total Assets in a firm is of 2,500,000 euros. At the end of the year the Net Income of the firm was 500,000 euros. If we are shareholders of this company, are we happy with this result?

At first glance we would say that the company presents a very positive performance by getting a fiscal year net profit of 20% of the total Assets it uses. 20% is a fantastic percentage in absolute terms, especially in the so-called «ordinary» economic contexts in which short-term rates will fluctuate around 3%. However, these conclusions can be quite misleading.

In fact, the answer to the above question cannot be given in a simple and direct way. It contains in itself a set of inconsistencies that we will address throughout this chapter:

- The Net Income is the result that is available for the shareholders and we are comparing it with all the Assets, which are placed by resources that are transferred by shareholders and creditors;
- The Net Income may have resulted from the profit on the sale of an asset that in no way contributed to the main business of the company;
- Is the disclosed profitability sufficient to compensate for the risk presented for those who invest in the company? No information is given, up to now, enough for us to evaluate the return/risk ratio;
- What are the implications of a strategy of maximizing profit in a year, in the face of the solvency situation for the following business years?

- We cannot make the analysis of future prospects for the company because we have no data for comparing with competitors or of external economic factors.

The Different Levels of Profits in the Firm – Operating Profit

The main level of results that we should identify in the activity of a company is without doubt the one which results from existence of the firm itself: the economic result, that is, the profit generated by the business to be carried out every year and the reason why the firm was created or the justification of its continuity.

This main activity can be of extraction and sale, production and sale, services providing, or a combination of these. The business developed by the company, daily and permanently, has to reveal its viability. It is not enough to look only to the final result, which can hide problems at the level of the business profit itself, to reach any conclusions. Are there not any good financing conditions, or even grants, tax benefits or any extraordinary profit,[1] thus non repeatable factors, which may cause a good performance in a company whose business, economic activity, may be dying?

How do we then realize if the business of the company is viable?

We begin by identifying the level of result of the activity, referred to as operating profit, detailing its components:

1. **Gross Margin**, resulting from the difference between revenues and the cost (variable) of goods sold. Very bad must be a business if this margin is negative or low. Several may be the reasons for this but the most likely is that the product or service no longer is wanted by the market, making the selling price insufficient to offset the costs of what is sold. Can the company change the product to make it attractive? Will it be able to create new products? There are so many issues that the company itself should place even before it sees its gross margin going down to unsustainable levels. It is very common to calculate the Gross Margin as a percentage of revenues and compare it with the average presented by the corresponding industry. If this percentage is systematically below what is achieved by competitors, we must question whether it will be just a matter of lower product competitiveness or possibly some negotiating or production problem at the level of cost of sales;
2. **EBITDA (Earnings Before Interest, Taxes, Depreciation, and Amortization)**, is an important operating result and always disclosed in the presentation of the

(1) Note that, according to the current accounting rules, the extraordinary results are not identified separately and are included in the operating result. But the company's analyst should identify separately those exceptional results by reason presented.

firm's results. Of the previous result, gross margin, we subtract the fixed operating expenses. An EBITDA of zero would mean that the value obtained in revenues was used, only to pay the variable costs of production and fixed costs, such as labour costs and external supplies and services. It is clear that the higher this operating result, the best, because it is expected that it can pay much more than the simple production in the business, for a healthy company's sustainability;

3. **EBIT (Earnings Before Interest and Taxes)**, is the most clear operating result because it results of subtracting depreciation from the EBITDA. This means that the EBIT is what remains after considering all the costs of producing what is sold and the part used as capital expenditure (depreciation of property, plant and equipment). Since depreciation expenses are not payable, is the entirety of the previous result, and not this one, that will be reflected in monetary means. However, as we want to ensure the future reinvestment in capital expenditure (plant and equipment), is this EBIT the result with which we should care about to ascertain the state of health of the main activity of the company. Is the EBIT that consists in the source of wealth of a company because is the only repeatable result that depends on the strategy, its implementation and execution, using the physical and human resources of the company itself. If this result is not satisfactory, little can the company do to be saved by good financing conditions from the banks, tax benefits from the government and a lot of patience from the shareholders. In fact, the goal must be to achieve an EBIT high enough to pay correctly the financing structure, taxes and the appropriate return required by shareholders. The several levels of results that reflect these payments are already in the sphere of the financial result that we will look at further on.

For now, and given the importance of EBIT, we shall detail important company analysis ratios that use the data from this result.

The Profitability of the Business

The absolute values convey little or nothing to the analyst. We must turn them into relative data, percentages, in this case to measure profitability, so indispensable and important.

Let us start by an important ratio of wide use, the Gross ROA (Return on Assets):

$$\text{Gross ROA} = \frac{\text{EBIT}}{\text{Assets}}$$

This ratio is a measure of the profitability of the business, as it confronts the operating profit with all assets that are available to the company.

We can only have some conclusion about the profitability level, presented by the company in a given fiscal year, if the percentage obtained in the ratio is compared with future prospects, the economic context data and the average profitability level presented by the industry of the company.

Although this ratio is very useful, in fact we can assure that it contains some inconsistencies. Let's see:

- Part of the total Assets may be property or current assets that in no way contribute to the main activity of the company. Will be the case, for example, of an empty warehouse or financial investments in stock exchange securities. These assets can be excluded, not compromising the sustainability of the business. So, to be more accurate in measuring this profitability, the Assets included for the ratio calculation should be only the operating ones. More, this Operating Invested Capital (IC) should be disclosed in net terms, deducted of operating resources, to represent exactly the capital necessary for the continuity of the current activity of the company;
- On the other hand, we know that who puts the assets at the disposal of the company are the shareholders and the financial institutions, financing the resources needed to purchase those assets. The EBIT includes not only the value that will be used to pay creditors, financial expenses, and the value that will be left to pay the shareholders, but also the taxes payable to the government. Now, if the government does not participate with capital or loans in the company, the result we must work with to assess the profitability of the business should exclude taxes, since we want a full consistency between the profits and the assets that generates that result. This result that excludes taxes is called NOPLAT (Net Operating Profit Less Adjusted Taxes).

Thus, we are led to a ratio of profitability of the business, the ROIC (Return on Invested Capital), that we can look at the same way as the Gross ROA but whose findings are more perfect by the greater accuracy in its definition, as described above.

$$\text{ROIC} = \frac{\text{NOPLAT}}{\text{Invested Capital}}$$

Being, $\text{NOPLAT} = \text{EBIT} \times (1 - t)$ with t representing the Corporate Tax Rate and $\text{IC} = \text{Operating Assets} - \text{Operating Resources}$.

The company's Balance Sheet for the Financial Analyst

To better understand the definition of Invested Capital, let's take a look at how financial analysts subdivide the company's balance sheet in its components.

Assets	Equity and Liabilities (Debt)
Non-current Assets	Permanent Capital
Operating	Equity
Non-operating	Non-current Liabilities
Current Assets	MLT Bank Debt
Operating	Current Liabilities
Non-operating	ST Bank Debt
	Operating Resources
	Other Non-operating Resources

Some important notes:

- Non-current assets vs. current, is due to the time the assets stay in the company: over one year and up to a year, respectively;
- Operating vs. Non-operating, is due to the mandatory presence or not, respectively, in the firm for the pursuit of its business;
- No element of Equity or Liabilities – bank debt – can obviously be considered operating, since both are components of the financing structure.

Thus, the IC can also be defined by:

$$IC = \text{Operating Non-current Assets} + \text{Working Capital}^{1}$$

(1) As we will see later,

Working Capital = Operating Needs – Operating Resources

and also

IC = Total Assets – Non-operating Assets – Operating Resources.

Components of the Business Profitability

Using a simple mathematical property, we can break up the Gross ROA ratio in its Operating Margin and Asset Turnover components:

$$\text{Gross ROA} = \frac{\text{EBIT}}{\text{Revenues}} \times \frac{\text{Revenues}}{\text{Assets}}$$

What is the advantage of analysing the profitability of the business in its detailed formula?

The firm should constantly monitor the profitability of its business and its strategy must go always towards the maximization of profitability. However, if we look at the two components above, we find that certain decisions can increase the margin but, at the same time, can induce a less Assets Turnover, and vice versa. A classic example of this incompatibility of effect on decisions is seen on changes in sales prices. If we raise prices without meet the demand-price elasticity of the product/service in question, we may be increasing the margin but decreasing Turnover significantly. Very differently, innovation decisions with implications on the product/service itself and in its way of production, may have effects that in global will allow a significant improvement of the ratio as a whole. Hence it is very important to analyse the various possible detailed effects of decisions arising from the strategy followed by the company. Further on, when we move to the solvency analysis, we will better assess other incompatibilities of effects on certain decisions when we will realise that goals of accelerated growth in sales may jeopardize the solvency of the company in the short-term, and additionally lead to a decrease in Turnover.

The Different Levels of Profits in the Firm – Net Income or Financial Profit

Downstream of the EBIT we identify the various results that fall into the financial sphere for being dependent of the company's financing structure and taxation on income:

1. **EBT (Earnings Before Taxes)**, income before taxes and therefore will be subject to taxation on income tax. It results of subtracting the finance expenses on the EBIT, or better yet, of subtracting the difference between the finance expenses and the finance income. Thus, for example, if we get a null EBT, it means that the entire result generated by the business is spent to pay the finance result, according to the net external resources used to finance the company. In this case, the company would not pay income tax but its shareholders would be very unsatisfied, and with reason: they are the financers of

the permanent capital owned by the company itself, and their return, in such a case, would be null;

2. **Net Income (NI)**, net result or final outcome, one that is available to the holders of the capital of the company, the shareholders, that in the General Meeting decide whether or not they are distributed in the form of Dividends, relative to the part that is not intended to mandatory reserves. Of everything we have examined in this chapter, we have concluded that the better is the operating performance, the financing conditions and the tax efficiency, the better the outcome for shareholders. Naturally, this conclusion raises a question: wouldn't the shareholders benefit if the company did not have to bear costs of financing, i.e., if in place of the company having external funding (for example, bank debt), the shareholders put a greater amount of resources in the firm? In this case, all the operating result, excluding only taxes, would be available to them...

To assess whether the shareholders would be more satisfied if the company does not contract any debt, we have to meet the following issues which we will study in detail:

1. Is the level of profitability for shareholders the appropriate to the risk that they bear in investing in the company? If not, what is the benefit of putting more money in the same company? How do we evaluate this profitability for shareholders?

Return on Equity (ROE)

To give consistency, in this case also, to the calculation of the ratio, the same will be defined as the fraction between two values that are exclusive of the company's shareholders, as follows:

$$\text{ROE} = \frac{\text{Net Income}}{\text{Equity}}$$

The level of this profitability cannot be seen as absolute. Once again we are dealing with the concept of opportunity cost. A financial model that allows us to evaluate the adequacy of the return on equity, or the financial return in a company, is the CAPM.[1] Consists of a model of simple deduction and of wide use at a global level on the financial markets.

(1) Capital Asset Pricing Model.

With this model we can identify what is the appropriate level of ROE for the level of risk that investors take by putting their capital in the company. This risk, which is obviously the risk inherent of the company's shares, is dependent of the operating risk (weight of fixed capital in the activity) and financial risk (financing structure – Debt/Equity).

In this book we will not address with detail when assessing the value of this return but there is an important summary conclusion to retain: the shareholders expect, demand, a return, for their capital, that has to include a risk premium, that is, a reward for the risk they bear in their investment. Being a business investor is nothing similar to being a holder of German Government bonds, for example. Thus, investors in risk are not happy in obtaining the same return as that of low-risk investments. If this were the case, it would be enough for them to invest with this lower risk.

Not forgetting the question that brought us to this point, we conclude that if the level of ROE is not suitable, we can hardly convince shareholders to raise capital in the company. Thus, the hypothesis of including debt, despite increasing the financial risk of the company, can be considered, in this case, for not having to demand the shareholders unprofitable additional investments. The cost of debt is less than the cost of equity, since creditors are repaid before the shareholders in case of bankruptcy. Lenders assume less risk than shareholders and, as such, require a lower return for the resources that they make available to the company.

But the reason why all companies include debt in their structure of resources goes further than the argument above. To understand why we ask a second question, as follows:

2. How the ROE is influenced by the level of Debt in the structure of resources of the firm? In what situation can we maximize the ROE for the shareholders?

Let us consider the example of the beginning of this chapter, with the additional information that the corporate tax rate is 25%. If the company has not contracted any debt, that is, all the resources are composed by Equity, the total operating profit is used to pay taxes and to give the return to the shareholders.

Thus,

$$\text{NI} = 500,000$$

$$\text{EBT} \times (1 - 25\%) = 500,000 \quad \Leftrightarrow \quad \text{EBT} = 666,667$$

$\text{EBIT} = 666,667$, because as there is no debt, also there are no Finance Expenses and thus the EBIT = EBT

We note that the total operating result is destined to taxes, in 166,667 euros, and to Net Income, in 500,000 euros. The Gross ROA is 26.67% and the ROE reaches 20%.

Imagine now that the shareholders would like to replace part of their capital in the company by debt. Perhaps they would like to diversify their investment to a new business with an expected return/risk ratio that attracts them much or, eventually, explore the effect that the lowest cost of borrowing, in this case let's assume that is 11%, may have in their own profitability, ROE.

Let's see then the result of replacing a 100% in Equity by 50% in Equity and 50% in Debt:

$$\text{EBIT} = 666,667$$
$$\text{Finance expenses} = 1,250,000 \times 11\% = 137,500$$
$$\text{EBT} = 666,667 - 137,500 = 529,167$$
$$\text{NI} = 529,167 \times (1 - 25\%) = 396,875$$

It is clear that the Gross ROA does not change with the funding structure, it remains in 26.67%, because it only depends on the performance of the business itself. Concerning the ROE, it will have to undergo a change which we will explain below. The ROE in this case amounts to 31.75%, surpassing significantly the ROE of non-indebtedness. It seems we can say that, in this situation, shareholders are more satisfied. But attention, never forget the principle that relates return vs. risk. Shareholders have now a higher ROE but also the investment in the company is more risky (higher financial risk because the debt, with its interest, has to be paid back).

What is the reason for the increase in ROE from 20% to 31.75% when we exchange Equity for Debt? If the ROE, in the new situation, stood in 20%, this would have meant that the NI would amount to 250,000 euros (half of Equity, half of NI). But the NI reached a value greater than this, in the 50/50 Debt/Equity situation, totalling 396,875 euros. How were the extra 146,875 euros obtained?

The cost of debt is lower than the ROE in nine percentage points. Hence, $9\% \times 1,250,000 = 112,500$ is what is gained in choosing a cheaper source of funding.

Additionally, and perhaps the most important issue, the finance expenses are considered fiscal costs (note again that the result that is subject to tax is the EBT and not the EBIT). Thus, there is a tax savings by the use of debt: finance expenses are true costs only in $1 - t$, in this case 75%, of its value. 25% of finance expenses is the amount the firm does not pay in taxes. In this case: $25\% \times 137,500 = 34,375$ euros.

Adding the two savings, 112,500 + 34,375, we obtain the total amount of 146,875 euros, precisely the value of the increase in NI when the funding in the firm is shared between Equity and Debt.

We can conclude then that, at least in this company, the ROE increases with the increase of the Debt in relation to Equity. We will understand now the conditions under which this can happen for most companies.

Detailed ROE

To realize well the effects of indebtedness on the profitability for the shareholders, we will break up the ROE ratio itself.

$$\text{ROE} = \left[\text{Gross ROA} + (\text{Gross ROA} - \text{r}) \times \frac{\text{Debt}}{\text{Equity}} \right] \times (1 - \text{t})$$

Note that while the Gross ROA remains above *r* – the average cost of Debt (Liabilities) – the portion $(\text{Gross ROA} - \text{r}) \times \frac{\text{Debt}}{\text{Equity}}$ is positive and the effect on the ROE is always positive. The more we replace Equity by Debt, the higher will be that portion and, consequently, the higher will be the ROE. And that is why to the indicated portion is given the name of Financial Leverage. The fact of replacing equity by debt, in conditions of greater profitability in the business than the cost of funding, works as a lever that raises the profitability for the shareholders.

Of course there are limits to this lever. On the one hand because, as we have seen above, the greater shareholder return can be associated with a much higher risk, undesirable or too high in absolute terms when compared with that return. On the other hand, the Financial Leverage remains positive only while the *r* is less than the Gross ROA and we know that, when a firm increases the weight of Debt, credit institutions are no longer willing to keep the same level of interest rate, increasing it in accordance to the increase in the risk they take.

However, we can realize now, after safeguarded the limitations to this «magic» of Debt, why is very rare that a firm does not use Debt in its funding structure. Would be a tremendous waste not to take advantage, at least, of the benefit of the tax effect on finance expenses.

Solvency

In the first part of this chapter we have studied the profitability of the company. For this we have based all analysis in the firm's results. However, in this second part, we will conclude that often is essential to analyse financial flows rather than results. There is an important difference between these and those. Let's illustrate this difference through a simple example.

Suppose a manager quits his current job and accepts an ambitious challenge in a project that is starting now. He will earn a salary that is three times the previous one. However, each payment of the monthly salary will occur only in the same month of the following year. Thus, with regard to the results obtained, the manager will triple the value, which weighed much in his decision. However, with regard to cash flows, these will not occur simultaneously with the results but with a one-year deferral. How will be the life of this manager's family in the first 12 months of his new challenge? He has been the guarantor of compliance with the financial obligations of the family, house rent, car payment, school of children, etc... Over the next 12 months he will earn a salary that is triple of the previous but, however, for lack of immediate liquidity, he won't be able to comply with the same obligations as he did when he earned much less.

This example was chosen for its simplicity and also because we all, in one way or another, are sensitive to issues of this kind. The most relevant is that difficulties of that nature are also typical of the business context.

We will dedicate this second part of this chapter to examine precisely how the company can manage its short-term solvency situation, i.e., what should or should not do to avoid the impossibility to satisfy its immediate financial obligations.

In the first part we used mainly the Income Statement, in which we have identified the useful data for the analyst in his study of profitability. In this part we will mainly use the Balance Sheet, which will also be subject to some adjustments so that we can identify items very suitable for analysis and decision making with respect to the Solvency of the company.

Balance Sheet analysis

The Balance Sheet is the financial document that shows us, in a moment in time, what is the position of the company in respect to available assets, on the one hand, and the financial commitments to comply, on the other. However, the biggest utility in the analysis of this document is to detail that position according to the different existing deadlines on both sides of the Balance Sheet. As we were able to realize from

the example given above, the balanced situation is only guaranteed when there is liquidity in the assets at the time that they are required to meet the financial commitments. The coincidence of timing, between the financial obligations and liquidity (cash flows or cash), becomes fundamental to the solvency condition and balanced situation of the company in the short-term.

We will detail the components of the Balance Sheet, as much as possible, by the various periods that assets and resources are kept in the company.

The simple separation in Current and Non-current Assets, and of Resources in Operating and Non-operating (within current liabilities), can hide the true deadlines of all: when the assets are available and resources have to be paid. Let's see what we want to show.

The values on Trade Receivables and Inventory, within the Current Assets, seem to have a short stay in the company and, according to the period of receiving or sales speed, can quickly be transformed into liquidity, cash flows. In part this is true but, at the same time, there is nothing more false. In fact, new sales will result in new credits to customers and new needs of product in storage, and it is true that, in terms of value, Trade Receivables and Inventory will remain as Assets in the company in the long run, we could even say while the company exists. Thus, what would seem likely to quickly turn into cash, has actually a feature of long permanence by its continuously renewability;

On the resources side we can establish the same reasoning and conclude that the values in Trade Payables and Tax Payables, although they seem like they have to be paid soon, which is usually the case, as they constantly renew over the several business cycles, they will remain in fact in the company's Balance Sheet for a long time, always as Trade Payables and Tax Payables.

Following this type of careful analysis of what truly is available to pay what is asked, considering the correspondence of deadlines, we will synthesize the fundamental equations for the definition of the Solvency situation in the company.

Permanent Capital (PC)

Permanent Capital = Equity + Medium and Long term Debt, as these are the resources that remain in company in the long run (the Equity is not to be paid back and this type of debt is to be paid in the long-term, more than one year).

Net Permanent Capital (NPC)

Net Permanent Capital = Permanent Capital – Non-current Assets, this still doesn't establish the difference between all the assets and all the resources of long permanence in the company, as we have seen. Thus, this Net Permanent Capital must be in line with the difference between the remaining assets and liabilities of long permanence in the company.

Working Capital (WC)

Working Capital = Current Assets of permanent nature -
– Current Resources of permanent nature

or

Working Capital = Operating Needs – Operating Resources

Being,

- Operating Needs: Trade Receivables, Inventory, and Restricted (Operating) Cash;
- Operating Resources: Trade Payables and Tax Payables.

Once again, the Working Capital reflects the assets, less liabilities, that take a long time to become "free" while the business is rolling. This word «free» is applied as counterpoint of «operating», because the free cash can immediately be used to comply with the financial obligations of the company, if these are immediately required.

Usually the WC is positive but there are cases in which, by the nature of the business, can be negative. A significant example of this type of business is the retail & distribution industry, in which customers pay immediately in cash and the suppliers receive with an extended deadline. Having no value in Trade Receivables, because all Revenues are converted into Cash, it is very probable to have a negative WC. In fact, despite the designation does not make it seem, a negative WC reflects less funding for the operating needs, because it is the business activity that generates immediate net liquidity, and therefore, is, after all, positive to the firm.

Positive, or negative, the important is that, as indicated above, the value of the Net Permanent Capital and the Working Capital be in such harmony to allow the much desired balanced situation in the firm. Through the equality between the level of availability and requirement of assets and liabilities, respectively, it is intended then that the company will be able, at any time, to comply with the equation that we present below.

Liquidity (L)

Liquidity = Net Permanent Capital − Working Capital, known as the Liquidity, this is the representation of the net position (excess or insufficiency) in long-term resources.

If that position is positive, we conclude that the situation of the company is of financial relief in meeting their immediate responsibilities since it has an excess of resources that will only be required to pay in the long-term. If that position is negative, the situation is precisely the opposite and the company may enter into financial distress and even default in payments. The latter certainly is not what we want but the previous one also has its drawbacks. In fact, the long-term resources are almost always more expensive than the short-term. Thus, a positive Liquidity situation, especially if it reaches high values, means that, in order to have a very comfortable situation in terms of short-term solvency, we are, at the same time, undermining the profitability of the company, by wasting an excess in finance expenses.

Therefore, the optimal value will be Liquidity = 0.

Transform the outcome in cash flows – Operating Free Cash Flow (OFCF)

We understand the importance of the Profitability analysis and the Solvency analysis. Until now, it seemed to us that both are completely independent realities, one based in the Income Statement, the other based on the Balance Sheet. We want, for a good performance of the company, to maximize the first and get a balanced situation in the second, realizing that in this goal we can find incompatibilities, as we saw in the preceding paragraph.

To better realize the possible incompatibilities between profitability and solvency and, above all, realize how in fact they are dependent realities, let's find the equation that transforms the Results (profits) in Cash Flows (liquidity).

What is the level of results that best reflects the cash flows generated by the business?

The EBITDA is the one that best reflects the free cash flows of the business, and not the EBIT, because, remember, the depreciation we subtract on the latter is not a cash outflow (didn't give rise to any payment).

But we already know that the values on Trade Receivables and Inventory do not allow a direct transfer from Revenues to Cash. And, conversely, the values in Trade Payables and Tax Payables allow an increase in Free Cash Flow from the business,

because of their non-immediate payment. But beware, these effects only happen by increments under the headings referred to and not by their absolute values. Let us remember the example of the manager who tripled the wage. From the 13th month onwards, the number of wages that the company owes him each month is always 12 (for 12 months of deferral in payment). Thus, the maintenance of the same outstanding value, month-to-month, results in a coincidence between Result and Cash Flow (he receives, from the 13th month on, the exact value of his wage, even being of the previous year).

In this way, the equation that allows us to calculate the Operating Free Cash Flow, based on the Operating Result, is the following:

$$\text{Operating Free Cash Flow} = \text{EBITDA} - \Delta\text{WC}$$

To closely examine the mechanics of this equation is extremely important in any decision making. Note, for example, that decisions can be taken to maximize the EBITDA and, at the same time, those decisions can increase the WC the company uses in the business. Some examples of this are: the increase in the Receivables Average Term to stimulate sales, with effects on the increase of the Operating Needs (increase in the WC by the value in Trade Receivables), or the decision to make massive purchases to take advantage of price promotions of raw materials or products for sale, resulting in an increase of the Operating Needs (increase in the NPC by the value in Inventory). With these kind of decisions, the effect on the Operating Free Cash Flow can be negative, without forgetting that the Liquidity situation (NPC – WC) may become unsustainable.

Formulas

- $\text{Gross ROA} = \dfrac{\text{EBIT}}{\text{Assets}} = \dfrac{\text{EBIT}}{\text{Revenues}} \times \dfrac{\text{Revenues}}{\text{Assets}}$

- $\text{ROIC} = \dfrac{\text{NOPLAT}}{\text{Invested Capital}}$

- $\text{Net ROA} = \dfrac{\text{Net Income}}{\text{Assets}} = \dfrac{\text{Net Income}}{\text{Revenues}} \times \dfrac{\text{Revenues}}{\text{Assets}}$

- $\text{ROE} = \dfrac{\text{Net Income}}{\text{Equity}} =$

$$= \left[\text{Gross ROA} + (\text{Gross ROA} - \text{Average Cost of Debt}) \times \frac{\text{Debt}}{\text{Equity}} \right] \times$$

$$\times (1 - \text{Corporate Tax})$$

- $\text{Average Cost of Liabilities} =$

$$= r = \frac{\text{Financial Expenses} - \text{Finance Income}}{\text{Debt}}$$

- $\text{Debt-to-Equity ratio} = \dfrac{\text{Debt}}{\text{Equity}}$

- $\text{Equity-to-Assets ratio} = \dfrac{\text{Equity}}{\text{Assets}}$

- $\text{Debt ratio} = \dfrac{\text{Debt}}{\text{Assets}}$

- $\text{Net Permanent Capital} = \text{Permanent Capital} - \text{Non-current Assets}$

- Permanent Capital = Equity + Long-term Debt
- Working Capital = Operating Needs − Operating Resources
- Liquidity = Net Permanent Capital − Working Capital
- Operating Free Cash Flow = EBITDA − ΔWorking Capital
- Receivables Average Term (in months) $= \dfrac{\text{Trade Receivables}}{\text{Revenues with VAT}} \times 12$
- Payables Average Term (in months) $= \dfrac{\text{Trade Payables}}{\text{Purchases with VAT}} \times 12$
- Average Duration of Inventory (in months) $= \dfrac{\text{Inventory}}{\text{Cost of Goods Sold}} \times 12$

Solved Exercises

EXERCISE	3.01

From company *Renda,* it was possible to obtain the following information (in million euros): Cost of Goods Sold (40), External Supplies (10), Revenues (120), Other Operating Revenues (10), Labour Costs (20), Operating Taxes (0.5), Finance Income (1.5), Depreciation (20), Other Operating Costs (1.5), Finance Expenses (12), Net Income (22) and Taxes (over profit) (5.5).

a) Compute the following ratios: Gross Profit Margin, Operating Profit Margin, Global Profit Margin, EBITDA/Revenues, Net Finance Expenses/EBIT, Taxes/EBIT e Net Income/EBIT;

b) Based on the previous answers, make some comments on the economic situation of the company *Renda,* for the year *N*;

c) Comment on the following outburst of a financial manager: «if we could pay the employees with depreciation, our economic situation would be much better! ».

Solution

a) The first step is to construct the Income Statement.

Income Statement for company *Renda*

Unit: million euros

	N
Revenues	120.0
Cost of Goods Sold	40.0
Gross Margin	80.0
Other Operating Revenues	10.0
External Supplies	10.0
Labour Costs	20.0
Operating Taxes	0.5
Other Operating Expenses	1.5
EBITDA	58.0
Depreciation	20.0
EBIT	38.0
Finance Income	1.5
Finance Expenses	12.0
EBT	27.5
Income Tax	5.5
Net Income	**22.0**

$$\text{Gross Profit Margin \%} = \frac{80}{120} \times 100 = 66.67\ \%$$

$$\text{Operating Profit Margin} = \frac{38}{120} = 0.3167 = 31.67\,\%$$

$$\text{Global Profit Margin} = \frac{22}{120} = 0.1833 = 18.33\,\%$$

$$\frac{\text{EBITDA}}{\text{Revenues}} = \frac{58}{120} = 0.4833 = 48.33\,\%$$

$$\frac{\text{Finance Income}}{\text{EBIT}} = \frac{1.5 - 12}{38} = -0.2763 = -27.63\,\%$$

$$\frac{\text{Income Tax}}{\text{EBIT}} = \frac{5.5}{38} = 0.1447 = 14.47\,\%$$

$$\frac{\text{Net Income}}{\text{EBIT}} = \frac{22}{38} = 0.5789 = 57.89\,\%$$

b) The company presents overall positive earnings. The operating situation is favourable with a profit margin of nearly 32%. Focusing on the cash flow (considering the EBITDA instead of the EBIT) the situation improves substantially to a profit margin of 48%. The variable operating expenses absorb 33.33% of Revenues.

The financial results are negative, what is quite usual, and absorb about 27.63% of the operating earnings. From the remaining EBIT, 14.47% are used on the Income Tax while 57.89% generate Net Income, improving the companies self-financing. For each euro of revenues the firm manages to get a final earning of slightly more than 18 cents.

To perform a more complete and conclusive analysis it would be useful to have access to the company's data for at least 3 years, and also to the average industry ratios and relevant competitors. It would also be important to assess the macroeconomic environment in which the company's results were obtained.

c) If all operating expenses were relative to depreciation, the economic result (EBIT) would be exactly the same. However, from a cash flow perspective on the result (considering the EBITDA), the situation would actually be much better, since depreciation represents a non-payable expense.

EXERCISE 3.02

Consider the following information for the company *Prevê,* for the next year:

- Revenues → 100,000 euros;
- Gross Profit Margin → 40%;
- Other Operating Expenses (excluding Depreciation) → 20,000 euros;
- Depreciation → 10,000 euros;
- Average Cost of Debt → 6%;
- Corporate Tax Rate → 25%;

- The total investments of the company (Assets) at the end of the year will be 200,000 euros and will be financed by Equity in 40%.

a) You are asked to compute the following ratios: Net ROA, Gross ROA and ROE;

b) Comment the preceding outcomes;

c) If the company predicts that the funding of its assets will be done by Equity in only 20%, will there be any impact on the calculated ratios? (Quantify, assuming that the other variables remain equal to the initial situation).

Solution

a) The first step is to construct the Income Statement.

Income Statement of company *Prevê*

	Unit: euros
Revenues	100,000
Cost of Goods Sold	60,000
Gross Margin	40,000
Other Operating Expenses	20,000
EBITDA	20,000
Depreciation	10,000
EBIT	10,000
Finance Income	0
Finance Expenses	7,200
EBT	2,800
Income Tax	700
Net Income	**2,100**

$$\text{Net ROA} = \frac{2,100}{200,000} = 0.0105 = 1.05\,\%$$

$$\text{Gross ROA} = \frac{10,000}{200,000} = 0.05 = 5\,\%$$

$$\text{ROE} = \frac{2,100}{200,000 \times 0.4} = 0.0263 = 2.63\,\%$$

b) The firm has extremely low returns, although positive. For each euro invested in assets the firm generates only 5 cents of operating results. At the Net Income level the return is only 1 cent. The ROE value is quite small. The maximum potential return on equity is only 2.62%, that is, even if all net income is paid out as dividends, the shareholders do not even get 3 cents for each euro they have invested in the firm. Without data for the industry or for the competitors we are not able to draw conclusions regarding the firm's performance for this year. However, the return for shareholders is so low that is almost impossible to be compensating for the risk taken in this investment.

c) Changing the capital structure does not change the Gross ROA, only the Net ROA and the ROE. The Finance Expenses would increase to 9,600 euros, the EBT would be reduced to 400 euros, the Income Tax would be 100 euros and the Net Income would result in 300 euros. The new values for the Net ROA and for the ROE would be:

$$\text{Net ROA} = \frac{300}{200,000} = 0.0015 = 0.15\,\%$$

$$\text{ROE} = \frac{300}{200,000 \times 0.2} = 0.0075 = 0.75\,\%$$

Both ratios would have a sharp decrease in value. A different capital structure, keeping everything else the same, impacts on the Net ROA level due to the increase in the Finance Expenses and therefore through the decrease in the Net Income. The ROE is affected not only due to the change in the Net Income but also due to the change in the Equity value (with opposite effects). Since the Average Cost of Debt is higher than the Gross ROA, we witness that an increase in the debt level leads to a decrease in the Net ROA and in the ROE. Conversely, if the Average Cost of Debt was lower that the Gross ROA, an increase in the debt level would lead to a decrease in the Net ROA but to an increase in the ROE. If both the Gross ROA and the Average Cost of Debt had the same value, an increase in the debt level would lead to a decrease on the Net ROA but would have no effect on the ROE.

In conclusion, an increase in the debt level, keeping all other things equal, has no impact on the Gross ROA, always impacts negatively on the Net ROA, and the ROE increases or decreases accordingly to the Gross ROA being higher or lower than the Average Cost of Debt.

EXERCISE 3.03

Comment the following dialog:

A: We urgently need to improve our Return on Assets and Return on Equity.

B: With the decrease in the Corporate Tax Rate for this year and with the positive renegotiation of the cost of our debt carried out last month, I think it is going to be piece of cake.

A: This is very good news!

Solution

Changing the Corporate Tax Rate has no impact on the Gross ROA. Regarding the Net ROA and the ROE, the lower the tax rate the higher the Net Income will be and consequently, the higher the values of both returns. The Average Cost of Debt has no influence in the Gross ROA. As in the previous situation, the impact on the Net ROA and on the ROE is positive with a decrease in the Average Cost of Debt, due to the decrease in the Finance Expenses and consequent increase in the Net Income.

EXERCISE 3.04

After the meeting of the directors of the firm *Target,* and after defining the strategies for the next year, the following goals were set:

- Attain Revenues of 100,000 euros;
- Get an Asset Turnover of 2;
- Get a Gross Return on Assets of 15%;
- Negotiate an Average Cost of Debt of 9%;
- Get a Return on Equity of 12%.

Knowing that the Corporate Tax Rate is 25% and that Depreciation shall be up to 20,000 euros, elaborate, to the extent possible, the forecasted Income Statement and Balance Sheet.

Solution

Using the value of Revenues and the Assets Turnover ratio we are able to determine the value of Assets:

$$\text{Assets Turnover} = \frac{\text{Revenues}}{\text{Assets}} \quad \Leftrightarrow \quad \text{Assets} = \frac{100,000}{2} = 50,000$$

Using the Gross ROA we can determine the value of the EBIT:

$$\text{Gross ROA} = \frac{\text{EBIT}}{\text{Assets}} \quad \Leftrightarrow \quad \text{EBIT} = 50,000 \times 0.15 = 7,500$$

The Net Income can be obtained through the ROE or following the construction of the Income Statement:

$$\begin{cases} \text{ROE} = \dfrac{\text{Net Income}}{\text{Equity}} \quad \Leftrightarrow \quad \text{Net Income} = 0.12 \times \text{Equity} \\ \text{Net Income} = (\text{EBIT} - 0.09 \times \text{Debt}) \times (1 - 0.25) \end{cases}$$

We can obtain the Equity value from the previous equations in addition to the Balance Sheet equality between Assets and Equity plus Debt:

$$\begin{cases} \text{Equity} = 46,875 - 0.5625 \times \text{Debt} \\ \text{Equity} = 50{,}000 - \text{Debt} \end{cases} \Leftrightarrow \begin{cases} \text{Debt} = 7,142.86 \\ \text{Equity} = 42,857.14 \end{cases}$$

Using the values computed for the Debt and Equity it is now possible to determine the amounts of the Finance Expenses, of the Income Tax and of the Net Income.

Income Statement of company *Target*

Unit: euros

	N
Revenues	100,000.00
Other Operating Income and Expenses	72,500.00
EBITDA	27,500.00
Depreciation	20,000.00
EBIT	7,500.00
Finance Expenses	642.86
EBT	6,857.14
Income Tax	1,714.29
Net Income	**5,142.86**

Balance Sheet of company *Target*

Unit: euros

Assets	*N* + 1	Equity + Liabilities	*N* + 1
Non-current Assets		Equity	42,857.14
Current Assets		Liabilities	7,142.86
	50,000.00		50,000.00

EXERCISE 3.05

Consider the Balance Sheet and Income Statement for the company *Tri,* for the 3 year period *N/N+2*:

Income Statement of company *Tri*

Unit: euros

	N	*N* + 1	*N* + 2
Revenues	40,000	50,000	70,000
Cost of Goods Sold	20,000	24,000	30,000
Gross Profit Margin	20,000	26,000	40,000
Other Operating Revenues	500	300	250
External Supplies	8,000	9,000	9,500
Labour Costs	6,750	7,000	7,500
Operating Taxes	90	120	125
Other Operating Expenses	110	180	175
EBITDA	5,550	10,000	22,950
Depreciation	6,000	5,000	5,500
EBIT	– 450	5,000	17,450
Finance Income	200	0	100
Finance Expense	2,600	2,300	3,000
EBT	– 2,850	2,700	14,550
Income Tax	0	675	3,613
Net Income	**– 2,850**	**2,025**	**10,938**

Balance Sheet of company *Tri*

Unit: euros

Assets	*N*	*N* + 1	*N* + 2	Funding	*N*	*N* + 1	*N* + 2
Non-current net Assets	25,000	20,000	30,000	Equity	10,000	12,025	22,963
Current Assets	15,000	17,500	25,000	Liabilities	30,000	25,475	32,037
	40,000	37,500	55,000		40,000	37,500	55,000

a) Do the ratio calculation, in their simple and detailed forms, for the Gross ROA, ROE and Net ROA for the three-year period;

b) Analyse the company's profitability and identify the determining factors in the evolution of it;

c) Suppose the company aims to invest in some equipment in *N+3*. The investment amounts to 10,000 euros. With the acquisition of this new equipment the company intends to improve its Gross ROA in 2 percentage points, compared with the previous year. For this non-current asset funding there are 3 scenarios:

i) Full funding by increasing Equity. In this case the cost of the existing funding would decrease in 1 percentage point;

ii) Financing in 60% by Equity and 40% by Debt capital. In this case the cost of the new funding would be at the rate of 10%/year and the existing funding would suffer a further half percentage point increase;

iii) Full financing by Debt capital. The interest rate to be used for the new funding would be 12%/year and the existing funding would suffer an increase of 1.5 percentage points.

Knowing that the Corporate Tax Rate will not change from the previous year, what should be the chosen scenario in order to maximize the ROE?

Solution

a) The requested calculations are presented next:

Indicators	*N*	*N* + 1	*N* + 2
Gross ROA [EBIT/Assets]	– 1.13%	13.33%	31.73%
Net ROA [Net Income/Assets]	– 7.13%	5.40%	19.89%
ROE [Net Income/Equity]	– 28.50%	16.84%	47.63%
Gross ROA	– 1.13%	13.33%	31.73%
Operating Profit Margin	– 1.11%	9.94%	24.84%
Assets Turnover	101.25%	134.13%	127.73%
Net ROA	– 7.13%	5.40%	19.89%
Global Profit Margin	– 7.04%	4.03%	15.57%
Assets Turnover	101.25%	134.13%	127.73%
ROE	– 28.50%	16.84%	47.63%
Gross ROA	– 1.13%	13.33%	31.73%
Debt/Equity	3.00	2.12	1.40
r	8.00%	9.03%	9.05%
t	0.00%	25.00%	24.83%
Financial Leverage	– 27.38%	9.12%	31.64%

b) We can observe a significant improvement on the Gross ROA and on the ROE, going from negative values in *N* to substantially positive ones in *N+2*. In *N*, both the Gross ROA and the Financial Leverage are negative, in particular the latter. The impact on the ROE from the high level of debt was quite negative. The Gross ROA was negative due to the negative Operating Profit Margin (more operating expenses than revenues).

In the following years there is a sharp improvement, both on the Gross ROA, and on the Financial Leverage. The Operating Profit Margin increases 11 percentage points, leveraged by an assets turnover higher than 134%. In *N+2*, and despite the slightly decrease in the assets turnover, the increase of about 150% on the Operating Profit Margin led to a Gross ROA of 32%, well above the 13% registered in the previous year.

The Financial Leverage in *N+1* already contributed positively on the ROE, with an increase of 36 percentage points relative to *N*. On the second year the firm showed less debt levels, despite the fact of achieving in that year a return on assets higher than the average cost of debt. In *N+2* the Financial Leverage is the highest on the period analysed, due exclusively to the increased differential between the Gross ROA and the average cost of debt.

In *N+1* and *N+2* the ROE could have been better if it was not for the fact that in these two years the firm already was subject to the payment of income taxes, the latter capturing about 25% of the return.

c) Assets will increase 10,000 euros from the new investment. The Gross ROA keeps its value at 33.73%, while the Corporate Tax Rate remains unchanged at 24.83%. For all 3 scenarios it is necessary to determine the new Debt-to-Equity ratios and Average Cost of Debt:

i) In the first scenario there is an increase on Equity of 10,000 euros and a reduction on the Average Cost of Debt to 8.05%:

$$\text{ROE} = \left[0.3173 + \frac{32,037}{32,963} \times (0.3173 - 0.0805) \right] \times$$
$$\times (1 - 0.2483) = 0.4411 = 44.11\%$$

ii) In this second scenario both the Debt and the Equity change in value. It is necessary to determine the new Average Cost of Debt:

$$r = \frac{0.0955 \times 32,037 + 0.1 \times 4,000}{32,037 + 4,000} = 0.096 = 9.6\%$$

$$\text{ROE} = \left[0.3173 + \frac{36,037}{28,963} \times (0.3173 - 0.096) \right] \times$$
$$\times (1 - 0.2483) = 0.4792 = 47.92\%$$

iii) On the final scenario only the Debt changes. It is again necessary to determine the new Average Cost of Debt:

$$r = \frac{0.1055 \times 32,037 + 0.12 \times 10,000}{32,037 + 10,000} = 0.109 = 10.9\%$$

$$\text{ROE} = \left[0.3173 + \frac{42,037}{22,963} \times (0.3173 - 0.109) \right] \times$$
$$\times (1 - 0.2483) = 0.5677 = 56.77\%$$

The scenario that maximizes the ROE is the third one, that is, the firm should finance the new investment using only debt funding. Every time the best alternative involves increasing the debt levels of the firm, the solution must be balanced with the fact that the firm will be more financially constrained. Does the improvement in the ROE justifies and compensates for the increasing financial risk?

EXERCISE 3.06

Consider the following information for firm *Alguma*:

- The NOPLAT is 54,000 euros;
- The Average Cost of Debt is 6.0%;
- The firm is subject to a 20% Corporate Tax Rate;
- The Equity value is 400,000 euros;
- The ROE is 12%.

Determine the Gross ROA.

Solution

To determine the Gross ROA it is necessary to know the value for the EBIT and the Assets. The value for the EBIT can be easily obtained from the NOPLAT and from the Corporate Tax Rate:

$$\text{NOPLAT} = \text{EBIT} \times (1 - t) \quad \Leftrightarrow \quad 54,000 = \text{EBIT} \times (1 - 0.2) \quad \Leftrightarrow$$
$$\Leftrightarrow \quad \text{EBIT} = 67,500$$

To determine the value of the Assets we only need to find the value of the Debt, since the Equity value is already given. The Debt value can be obtained from the Average Cost of Debt. However, to know the amount of Finance Expenses we first need to determine the value of the Net Income, which we will do using the calculation formula for the ROE:

$$\text{ROE} = \frac{\text{Net Income}}{\text{Equity}} \quad \Leftrightarrow \quad 0.12 = \frac{\text{Net Income}}{400,000} \quad \Leftrightarrow$$
$$\Leftrightarrow \quad \text{Net Income} = 48,000$$

The value of the Finance Expenses is obtained through the disaggregation of the Income Statement from the EBIT to the Net Income:

$$48{,}000 = (67{,}500 - \text{Finance Expenses}) \times (1 - 0.2) \Leftrightarrow$$
$$\Leftrightarrow \quad \text{Finance Expenses} = 7{,}500$$

The value of Debt is obtained using the calculation formula for the Average Cost of Debt:

$$r = \frac{\text{Finance Expenses}}{\text{Debt}} \Leftrightarrow 0.06 = \frac{7{,}500}{\text{Debt}} \Leftrightarrow$$
$$\Leftrightarrow \quad \text{Debt} = 125{,}000$$

Finally, we are now able to determine the value for the Gross ROA:

$$\text{Gross ROA} = \frac{\text{EBIT}}{\text{Assets}} \Leftrightarrow \text{Gross ROA} = \frac{67{,}500}{400{,}000 + 125{,}000} \Leftrightarrow$$
$$\Leftrightarrow \quad \text{Gross ROA} = 0.1286 = 12.86\,\%$$

The value of the Gross ROA for company *Alguma* is 12.86%.

EXERCISE 3.07

Consider the following information for firm *Aloutra*.

- EBIT is 25,000 euros;
- Gross ROA is 12%;
- Equity value is 100,000 euros;
- ROE is 10%;
- The firm is subject to a Corporate Tax Rate of 25%.

Determine the Average Cost of Debt of the firm.

Solution

To determine the Average Cost of Debt we begin by finding the value for the Debt. Using the Gross ROA it is possible to get to the value of the Assets, to which we then subtract the value of Equity to find the value of Debt:

$$0.12 = \frac{25,000}{\text{Assets}} \quad \Leftrightarrow \quad \text{Assets} = 208,333.33$$

$$208,333.33 = 100,000 + \text{Debt} \quad \Leftrightarrow \quad \text{Debt} = 108,333.33$$

Using the disaggregated formula for the ROE we are now able to determine the value of the Average Cost of Debt:

$$0.1 = \left[0.12 + \frac{108,333.33}{100,000} \times (0.12 - r) \right] \times (1 - 0.25) \quad \Leftrightarrow$$

$$\Leftrightarrow \quad r = 0.1077 = 10.77\,\%$$

The Average Cost of Debt for the firm *Aloutra* is 10.77%.

EXERCISE 3.08

In year *N*, the firm *LPJ* has an EBIT of 100,000 euros, has a Gross ROA of 25% and it is also in a high growth process in its business. For the year *N+1*, the firm forecasts that the EBITDA will be 130,000 euros and that the company will invest on a large scale which consequently will increase by 20% the total Assets when compared with the previous year. Which is the maximum value the Depreciation can reach, so that the company can keep exactly the same level of return on Assets?

Solution

First we determine the current Assets' value using the Gross ROA. After, using the value of the Assets for the next year, we calculate the EBIT and the Depreciation for *N+1*:

$$0.25 = \frac{100,000}{\text{Assets}_N} \quad \Leftrightarrow \quad \text{Assets}_N = 400,000$$

$$\text{Assets}_{N+1} = 400,000 \times (1 + 0.2) = 480,000$$

$$0.25 = \frac{\text{EBIT}_{N+1}}{480,000} \quad \Leftrightarrow \quad \text{EBIT}_{N+1} = 120,000$$

$$\text{Depreciation}_{N+1} \leq 130,000 - 120,000 \leq 10,000$$

The Depreciation value for *N+1* cannot be higher than 10,000 euros, in order for the return on Assets to remain the same.

EXERCISE 3.09

The Gross ROA of firm *Glowe* is 20%. Its EBT is 10,000 euros, the Assets are 100,000 euros and the Liabilities are 40,000 euros.

a) Determine the Average Cost of Debt for the firm;

b) If the Corporate Tax Rate is 20%, what will be the firm's ROE?

Solution

a) Once we already know the Debt value we only need to compute the value for the Finance Expenses. We proceed by computing the value of the EBIT using the Gross ROA:

$$0.2 = \frac{\text{EBIT}}{100,000} \quad \Leftrightarrow \quad \text{EBIT} = 20,000$$

$$10{,}000 = 20,000 - \text{Finance Expenses} \quad \Leftrightarrow \quad \text{Finance Expenses} = 10,000$$

$$r = \frac{10,000}{40,000} = 0.25 = 25\,\%$$

The Average Cost of Debt of firm *Glowe* is 25%.

b) To determine the ROE we need to compute the value for the Net Income and for the Equity:

$$\text{Net Income} = 10,000 \times (1 - 0.2) \quad \Leftrightarrow \quad \text{Net Income} = 8,000$$

$$\text{ROE} = \frac{8,000}{100,000 - 40,000} = 0.1333 = 13.33\,\%$$

The value for the ROE is 13.33%.

EXERCISE 3.10

Firm *Despede* owns investments that are worth 50,000 euros and an Equity-to--Assets ratio of 40%. Knowing that the Gross ROA and the Average Cost of Debt were 10% and that the Income Tax was 500 euros:

a) Determine the ROE;

b) When companies go through financial difficulties, they often choose to fire employees. Considering that the business volume of the company *Despede* does not suffer any change, will a reduction of 2,500 euros in Labour Costs have any impact on the Gross ROA and on the ROE?

Solution

a) Using the value of the Gross ROA and the Assets' value we are able to determine the EBIT. Next, we use the Equity-to-Assets ratio to find the Debt value and the Finance Expenses and from there we can compute the Net Income and finally the value for the ROE:

$$0.1 = \frac{\text{EBIT}}{50,000} \quad \Leftrightarrow \quad \text{EBIT} = 5,000$$

$$\text{Finance Expenses} = 50,000 \times (1 - 0.4) \times 0.1 = 3,000$$

$$\text{NI} = 5,000 - 3,000 - 500 = 1,500$$

$$\text{ROE} = \frac{1,500}{50,000 \times 0.4} = 0.075 = 7.5\,\%$$

The ROE for firm *Despede* is 7.5%.

b) A decrease in the Labour Costs leads to an increase in the EBITDA and in the EBIT on the same amount. Since the value of the Finance Expenses remains unchanged, we only need to determine the Corporate Tax Rate in order to find the new value of the Net Income:

$$t = \frac{500}{5,000 - 3,000} = 0.25 = 25\,\%$$

$$\text{NI} = [(5,000 + 2,500) - 3,000] \times (1 - 0.25) = 3,375$$

$$\text{Gross ROA} = \frac{5,000 + 2,500}{50,000} = 0.15 = 15\,\%$$

$$\text{ROE} = \frac{3,375}{50,000 \times 0.4} = 0.1688 = 16.88\,\%$$

With this change in the Labour Costs, and as expected, both returns show an improvement: the Gross ROA increases to 15% while the ROE increases to 16,88%.

EXERCISE 3.11

Firm *Glory* has a Gross ROA of 16%, an Average Cost of Debt of 8% and a Financial Leverage of 3.5%. Knowing that the Corporate Tax Rate is 20%:

a) Compute the Equity-to-Assets ratio;

b) If the company increases its Equity-to-Assets ratio which would be the effect on the ROE? (Do not use calculations to justify the answer).

Solution

a) Using the formula for the Financial Leverage we can determine the value for the Debt-to-Equity ratio:

$$0.035 = \frac{\text{Debt}}{\text{Equity}} \times (0.16 - 0.08) \quad \Leftrightarrow \quad \frac{\text{Debt}}{\text{Equity}} = 0.4375 = 43.75\,\%$$

The Debt-to-Equity ratio is 43.75%, what is equivalent to an Equity-to-Assets ratio of:

$$0.4375 + 1 = \frac{1}{\text{Equity-to-Assets}} \quad \Leftrightarrow$$

$$\Leftrightarrow \quad \text{Equity-to-Assets} = 0.6957 = 69.57\,\%$$

The Equity-to-Assets ratio for firm *Glory* is approximately 69.57%.

b) Considering that the return on Assets and that the Average Cost of Debt remain unchanged, then the ROE would diminish. Since the Financial Leverage is positive (resulting from a positive difference between the return on Assets and the Average Cost of Debt), the increase in the Equity-to-Assets ratio and consequent decrease on the Debt-to-Equity ratio lead to a deterioration on the return on Equity.

EXERCISE 3.12

Using the financial statements of company *Solvente* for year *N* and the additional information, do the following:

a) Determine the Net Permanent Capital;

b) Determine the Working Capital;

c) Determine the Liquidity indicator;

d) Compute the Liquidity indicator using the Liquid Assets Elements and Liquid Liabilities Elements;

e) Analyse the company's solvency, namely stating the following:

- Identify the solvency situation;
- Causes for the situation identified;
- Consequences or adjustments necessary from the positive/negative solvency situation identified;
- Measures to improve the solvency of the firm.

f) Now assume that 50% of the existing Inventory was acquired with purely speculative objectives. What is the impact in the solvency situation of the firm derived from this new information?

Income Statement for company *Solvente*

Unit: euros

	N
Revenues	200,000
Cost of Goods Sold	120,000
Gross Profit Margin	80,000
External Supplies	30,000
Labour Costs	20,000
Other Operating Expenses	10,000
EBITDA	20,000
Depreciation	6,000
EBIT	14,000
Finance Expenses	6,000
EBT	8,000
Income Tax	2,000
Net Income	**6,000**

Balance Sheet of company *Solvente*

Unit: euros

Assets	***N***	**Equity and Liabilities**	***N***
Non-current net Assets	60,000	Equity	40,000
Current Assets	40,000	Liabilities	60,000
Inventory	25,000	Medium- and Long-term Bank Loans	10,000
Trade Receivables	10,000	Trade Payables	8,000
Other Assets	4,000	Short-term Bank Debt	30,000
Cash	1,000	Tax Liabilities	5,000
		Other Liabilities	7,000
TOTAL	100,000	TOTAL	100,000

Additional Information

- There was no change in Inventory in year *N*;
- The values in Tax Liabilities come from the operations with the exception of the estimated Income Tax;
- The amount in Cash is all considered excess cash.

Solution

a) The Net Permanent Capital results from the difference between the medium- and long-term resources (Equity and Debt) and the investments of similar maturity:

$$\text{Net Permanent Capital (NPC)} = \text{Permanent Capital} - \text{Non-current Assets} \quad \Leftrightarrow$$

$$\Leftrightarrow \quad \text{NPC} = 40,000 + 10,000 - 60,000 = -10,000$$

b) The Working Capital is computed by the difference between the operating financing needs of the Current Assets and the operating financing resources of the Current Liabilities:

$$\text{Working Capital (WC)} = \text{Operating Needs} - \text{Operating Resources} \quad \Leftrightarrow$$

$$\Leftrightarrow \quad \text{WC} = 25,000 + 10,000 - 8,000 - 3,000 = 24,000$$

At the Tax Payables level we should only consider the part relative to the operations, that is, from the value on the Balance Sheet we need to subtract the value of the Income Tax present on the Income Statement. Since we do not have enough information, the Other Assets and Other Liabilities are not considered associated with the operating activities of the firm.

c) The Liquidity corresponds to the difference between the Net Permanent Capital and the Working Capital:

$$\text{Liquidity (L)} = \text{Net Permanent Capital} - \text{Working Capital} \quad \Leftrightarrow$$

$$\Leftrightarrow \quad \text{L} = -10,000 - 24,000 = -34,000$$

d) The Liquidity value can also be obtained through the difference between the Liquid Assets Elements (LAE) and the Liquid Liabilities Elements (LLE). The LAE and the LLE are associated with values of the Current Assets and Current Liabilities (respectively) that are not part of the Working Capital:

$$\text{Liquidity (L)} = \text{LAE} - \text{LLE} \quad \Leftrightarrow$$

$$\Leftrightarrow \quad L = 4,000 + 1,000 - 30,000 - 2,000 - 7,000 = -34,000$$

e) Firm *Solvente* shows a financial deficit since its Liquidity indicator is negative. The deficit value is quite high, corresponding to about a third of the Assets' value. The Net Permanent Capital is negative and its value is not sufficient to cover the financing needs from the Working Capital. The Operating Needs, in which the value of Inventory stands out, are far superior to the Operating Resources. The credit

obtained from suppliers and from the fiscal entities is enough to support the credit given to costumers, but severely inadequate to finance the Inventory in place. The firm shows insufficient medium- and long-term capital in order to compensate for the short-term operating deficit.

To surpass this disequilibrium the firm contracted a short-term bank loan and also benefited from additional credit (Other Liabilities). These were enough to face the Liquidity deficit and also to allow conceding additional short-term credit (Other Assets) and to keep a residual amount in Cash.

Since the Liquidity deficit is severe and assuming that future earnings will be similar to the latest year, the firm must take action with structural measures in order to solve its financial deficiencies. These measures can be sustained with new issuance of Equity in cash or by new medium- and long-term Debt. Since the company is already substantially indebted in the short-term, the best alternative could come from the renegotiation of the maturity of the bank debt from short- to long-term, improving the Net Permanent Capital without loss of financial autonomy. As a last resource, and sustaining some financial risk, the firm may keep things as they are, as long as it is capable to rollover the short-term bank loans.

f) If 50% of Inventory has speculative purposes then the Working Capital will diminish 12,500 euros. The Liquidity deficit will then become 21,500 euros. The financial situation of the company becomes less troublesome, relative to the previous scenario, since the existence of a surplus in Inventory originated on a conscious management decision that can easily be cancelled in the future.

EXERCISE 3.13

Comment on the following statements:

a) «A firm that has high debt values in Trade Payables is a firm clearly financially constrained, since it will have to pay that debt in the near future.»

b) «Financing the operations through medium- and long-term capital makes it easier for a firm to obtain adequate solvency.»

c) «Consecutively obtaining negative earnings can have a serious impact on a firm's solvency.»

d) «A firm can improve its financial condition by outsourcing its production.»

Solution

a) We can look at the sentence from a current/historical perspective or from a future/provisional one:

- From a current perspective, the trade credit obtained from suppliers corresponds to a postponement of cash outflows, what is financially favourable to the company (suppliers are financing the firm's activity with zero interest);
- From a future or provisional perspective, it is a fact that high amounts of debt to suppliers will lead, in the short-term, to cash outflows.

We can also analyse the sentence from another two distinct points of view: the amounts in debt result from the normal activity of the firm or they are due to the delay in payment of the debt.

- On the first one, outstanding huge values of trade payables can be derived from the negotiation of extended payments deadlines. This situation will not be synonymous of any financial distress for the firm, even representing a resource available to it and thereby favourable in obtaining a positive liquidity;
- On the second one, the delay in payment of debts to suppliers can be due to financial difficulties and lack of solvency.

b) The statement is wrong. Using medium- and long-term capital, such is the case of issuing new Equity or contracting Debt with maturity higher than one year, reinforces the Permanent Capital of the firm. This reinforcement leads to the improvement of the Net Permanent Capital and consequently to an improvement in the Liquidity of the firm.

c) The statement is correct. A firm can have several years with negative earnings without entering on a lack of solvency situation. However, negative earnings do lead to a deterioration of Equity and consequently of the Permanent Capital of the firm. The impact on the Net Permanent Capital and on the Liquidity will be negative (the company's solvency will diminish). Most probably, consecutive negative earnings will lead to a short-term financial deficit.

One must be aware of the reasons behind the negative earnings. If they are generated by non-payable expenses (like Depreciation), then the effect previously mentioned can be ignored, since the decrease on the Permanent Capital is followed by a decrease on the Non-current Assets, or, more simply, the negative economic earnings do not translate into negative cash flows.

d) There are different factors to be considered when analysing this statement. Some are derived from scenarios where the firm is facing financial surplus where others are derived from the opposite scenarios. Thus, as factors that contribute to the improvement of the Liquidity situation of the firm we can point out the following:

- Decrease of the Non-current Assets, since the production facilities will no longer be needed. Consequently, this factor leads to an increase of the Net Permanent Capital and the Liquidity indicator;
- Increase of the Net Income, since the firm will use less labour and consequently the Labour Costs will diminish. Increasing the Net Income leads to an increase on the value of Equity and of the Permanent Capital, which then leads to an improvement of the Net Permanent Capital and of the Liquidity indicator;
- Decrease of the Inventory level originated by the inexistence of raw materials. This decrease means less Operating Needs, which leads to a decrease on the Working Capital and to an increase on the Liquidity indicator;
- Increase of the trade credit obtained from suppliers, since the amount of purchases will increase. Increasing this Operating Resource leads to the decrease of the Working Capital and to the improvement of the Liquidity indicator.

As factors that contribute to the deterioration of the company's solvency we point out the following:

- Decrease of the Net Income due to the higher Cost of Goods Sold. The decrease on the Net Income leads to the decrease of the Equity and Permanent Capital, and consequently to the decrease of the Net Permanent Capital and the Liquidity indicator;
- Increase on the Inventory level. Instead of working with inventory of raw materials and finished goods, the firm now works with merchandise. The impact on the Liquidity indicator is negative due to the increase of the Working Capital as a result of the increase of the Operating Needs;
- Decrease of the credit obtained from the fiscal entities. The decrease of the Labour Costs and the increase of the deductible VAT (higher VAT amount from purchases, although a decrease of the VAT from External Supplies) leads to a decrease of the Tax Payables amount and consequently to a reduction of the Operating Resources. The Working Capital will be higher which will result on a deterioration of the Liquidity indicator.

There are still other factors that could be accounted for, as for example the decrease on insurance and tax values on the estate.

A definite answer to this question could only be made in the presence of all the values involved, in particular on those that let us conclude if buying from external suppliers mean high prices on the finished goods.

EXERCISE 3.14

Consider the following information on firm *Clientele*:

- Receivables Average Term: 3 months;
- Change in Inventory: – 5,000 euros;
- Annual Revenues: 120,000 euros;
- External Supplies: 12,000 euros (subject to VAT);
- Payables Average Term: 0 months;
- Tax Payable (VAT Payable): 1,450 euros;
- Gross Profit Margin: 50%;
- Tax Payables Term (VAT): 2 months.

Compute the amount in Trade Receivables.

Solution

To determine the amount of Trade Receivables it is necessary to find the VAT rate on Revenues. Assuming that the VAT rate on Revenues is the same as the VAT rate on purchases, we get:

$$\text{Gross Margin} = 120,000 \times 0.5 = 60,000$$

$$\text{COGS} = 120,000 - 60,000 = 60,000$$

$$\text{Purchases} = 60,000 - 5,000 = 55,000$$

$$1,450 = \frac{120,000 \times \text{VATrate} - (55,000 + 12,000) \times \text{VATrate}}{12} \times 2 \quad \Leftrightarrow$$

$$\Leftrightarrow \quad \text{VATrate} = 0.1642 = 16.42\,\%$$

The amount of Trade Receivables is equal to:

$$\text{Trade Receivables} = \frac{120,000 \times (1 + 0.1642)}{12} \times 3 = 34,926$$

EXERCISE 3.15

Company *Volumich* has annual revenues of 480,000 euros and has a Gross Profit Margin of 20%. Considering a VAT rate of 20% and ignoring the existence of Inventory and External Supplies, determine:

a) The company's Working Capital, knowing that the Receivables Average Term, the Payables Average Term and VAT Payment Term are all the same, which is 2 months;

b) If you are trying to reduce 30% of the Working Capital, how should you adjust the Receivables Average Term, assuming that Revenues will remain the same?

Solution

a) On this exercise the Operating Needs are made up of the Trade Receivables, while the Operating Resources result from the sum of the Trade Payables and the Tax Payables (VAT). We should mention that since the firm works with no Inventory, all purchases are consumed during the year:

$$\text{Trade Receivables} = \frac{480,000 \times (1 + 0.2)}{12} \times 2 = 96,000$$

$$\text{Trade Payables} = \frac{480,000 \times (1 - 0.2) \times (1 + 0.2)}{12} \times 2 = 76,800$$

$$\text{Tax Payables} = \frac{480,000 \times 0.2 - 480,000 \times (1 - 0.2) \times 0.2}{12} \times 2 = 3,200$$

$$\text{WC} = 96,000 - (76,800 + 3,200) = 16,000$$

The value for the Working Capital for company *Volumich* is 16,000 euros.

b) A reduction on WC will be achieved by the decrease on the trade credit given to costumers:

$$\searrow \text{WC} = 16,000 \times 0.3 = 4,800$$

$$\text{Trade Reveivables} = 96,000 - 4,800 = 91,200$$

$$91,200 = \frac{480,000 \times (1 + 0.2)}{12} \times \text{RAT} \quad \Leftrightarrow \quad \text{RAT} = 1.9$$

In order for the company to achieve a reduction of 30% on the WC the RAT should be 1.9 months, instead of the current 2 months.

EXERCISE 3.16

Consider the following data regarding the company *FCR*:

- EBITDA = 22,000 euros;
- Depreciation = 4,000 euros;
- Revenues = 80,000 euros;
- ROIC = 10%;
- Assets Turnover = 50%;
- Corporate Tax Rate = 20%;
- Working Capital items:
  - Trade Receivables = 2,500 euros;
  - Inventory = 1,000 euros;
  - Trade Payables = 1,500 euros;
  - Tax Payables = 1,000 euros.

Compute the value of the Non-operating Assets of company *FCR*.

Solution

To determine the value of the Non-operating Assets we need to know the Assets' value and the value for the Operating Assets. The Assets' value is easy to get since we already have the value of the Turnover ratio and of the Revenues:

$$0.5 = \frac{80,000}{\text{Assets}} \quad \Leftrightarrow \quad \text{Assets} = 160,000$$

To compute the value of the Operating Assets we will use the value of the Invested Capital, since the latter corresponds to the difference between the Operating Assets and the Operating Liabilities:

$$\text{NOPLAT} = 18,000 \times (1 - 0.2) = 14,400$$

$$0.1 = \frac{14,400}{\text{Invested Capital}} \quad \Leftrightarrow \quad \text{Invested Capital} = 144,000$$

$$144,000 = \text{Operating Assets} - (1,500 + 1,000) \quad \Leftrightarrow$$

$$\Leftrightarrow \quad \text{Operating Assets} = 146,500$$

The value of the Non-operating Assets is 13,500 euros, resulting from:

$$160,000 = 146,500 + \text{Non-operating Assets} \quad \Leftrightarrow$$

$$\Leftrightarrow \quad \text{Non-operating Assets} = 13,500$$

EXERCISE 3.17

Below it is presented some data for the year *N* regarding the company *Qué* (in euros):

Revenues	300,000
External Supplies	5,000
EBITDA	55,000
Depreciation	10,000
Financial Expenses	6,000
Net Income	31,200

Other relevant data:

- Gross Profit Margin = 20%;
- Change in Inventory = 5,000 euros;
- Assets = 200,000 euros;
- Equity-to-Assets ratio = 60%;
- Working Capital$_{N-1}$ = 30,000 euros;
- There is no Finance Income.

a) Determine the Average Cost of Debt;

b) Considering a Payables Average Term of one month and VAT rate equivalent to the Corporate Tax Rate, compute the Trade Payables;

c) Knowing that the company *Qué* presented a balanced Liquidity situation and that the Operating Free Cash Flow in the year *N* amounted to 15,500 euros, compute the Net Permanent Capital.

Solution

a) Using the Equity-to-Assets ratio it is possible to determine the value of the Equity. Since we already know the Assets' value, we can easily compute the Debt value and its cost:

$$0.6 = \frac{\text{Equity}}{200,000} \quad \Leftrightarrow \quad \text{Equity} = 120,000$$

$$200,000 = 120,000 + \text{Debt} \quad \Leftrightarrow \quad \text{Debt} = 80,000$$

$$r = \frac{6,000}{80,000} = 0.075$$

The Average Cost of Debt is 7.5%.

b) Let us begin by computing the Corporate Tax Rate, using the value for the EBT:

$$\text{EBT} = 55,000 - 10,000 - 6,000 = 39,000$$

$$t = 1 - \frac{31,200}{39,000} = 0.2$$

To determine the value of the Trade Payables we only need to find the value for the purchases:

$$\text{Purchases} = 300,000 \times (1 - 0.2) + 5,000 = 245,000$$

$$\text{Trade Payables} = \frac{(245,000 + 5,000) \times (1 + 0.2)}{12} \times 1 = 25,000$$

The amount of the Trade Payables is 25,000 euros.

c) If the firm has a balanced Liquidity situation it means that the value of the Net Permanent Capital is equal to the value of the Working Capital. We can obtain the value of the WC through the Operating Free Cash Flow:

$$15{,}500 = 55{,}000 - (WC_N - 30{,}000) \quad \Leftrightarrow \quad WC_N = 69{,}500$$

The Net Permanent Capital in year *N* is 69,500 euros.

EXERCISE 3.18

Regarding the firm *KYU*, it is available the following information:

	Unit: Euros
EBITDA	47,000
Depreciation	30,000
Financial Expenses	3,000
Income Tax	4,200
Net Income	9,800

Also consider the following information:

- Gross ROA = 20%;
- Non-operating Assets = 1,500 euros;
- Operating Needs = 20,000 euros;
- Operating Resources = 12,500 euros;
- There are no other Operating Assets or Liabilities.

a) Compute the firm's NOPLAT;
b) Compute the ROIC.

Solution

a) To determine the NOPLAT it is necessary to compute the EBIT and the Corporate Tax Rate:

$$\text{EBIT} = 47,000 - 30,000 = 17,000$$

$$t = \frac{4,200}{4,200 + 9,800} = 0.3 = 30\,\%$$

$$\text{NOPLAT} = 17,000 \times (1 - 0.3) = 11,900$$

The value for the NOPLAT is 11,900 euros.

b) To determine the ROIC we first need the amount of the Invested Capital. Since we already know the values for the Non-operating Assets and for the Operating Liabilities, we only need to compute the Assets' value. This can be achieved using the Gross ROA ratio:

$$0.2 = \frac{17,000}{\text{Ativo}} \quad \Leftrightarrow \quad \text{Assets} = 85,000$$

$$\text{ROIC} = \frac{11,900}{85,000 - 1,500 - 12,500} = 0.1676 = 16.76\,\%$$

The value of the ROIC for company *KYU* is 16.76%.

EXERCISE 3.19

The company *X-VA*, which is beginning its activity, works in «just in time» (has no Inventory) and presents a Receivables Average Term of 3 months and pays its purchases in cash. The company's Revenues amount to 120,000 euros and the Gross Profit Margin is 20%. The VAT rate is 20% and the Tax Payables have an average Term of 2 months.

a) Calculate the company's Working Capital[1];

b) As an alternative scenario to this first year of activity the management is thinking of reducing the Gross Profit Margin to 10% in order to deal with the in-

(1) Ignore the existence of External Supplies.

creasing competition. If the company wants to keep the same level of Revenues but without changing its Operating Free Cash Flow, what must be the new Receivables Average Term?

Solution

a) Since the company works with no Inventory and pays its purchases in cash, the WC becomes the difference between the amount of Trade Receivables and the amount of Tax Payables (relative to the VAT):

$$\text{Trade Receivables} = \frac{120,000 \times (1 + 0.2)}{12} \times 3 = 36,000$$

$$\text{Tax Payables} = \frac{120,000 \times 0.2 - 120,000 \times (1 - 0.2) \times 0.2}{12} \times 2 = 800$$

$$\text{WC} = 36,000 - 800 = 35,200$$

The value of the WC is 35,200 euros.

b) Changing the Gross Profit Margin will lead to a decrease in the EBITDA of 10% of the Revenues' value, that is, of 12,000 euros. To keep the level of the Operating Free Cash Flow, and knowing that the WC for the previous year was zero (this is the first year of activity), this year's WC must also decrease in 12,000 euros, adjusting for that matter the costumers RAT:

$$\text{WC}_{N+1} = 35,200 - 12,000 = 23,200$$

$$23{,}200 = \frac{120,000 \times (1 + 0.2)}{12} \times \text{RAT}_{N+1} - \\ - \frac{120,000 \times 0.2 - 120,000 \times (1 - 0.1) \times 0.2}{12} \times 2 \Leftrightarrow$$

$$\Leftrightarrow \quad \text{RAT}_{N+1} = 1.97$$

To keep the same level of the OFCF the company should decrease the RAT to 1.97 months, thus reducing the amount invested in WC.

EXERCISE 3.20

In a management meeting the commercial manager proposed an increase in the Receivables Average Term from 1 month to 2 months. Due to this change the Revenues would increase 50%. Acknowledging that the current Revenues are 100,000 euros, that the VAT rate is 20% (the Tax Payables Term is 2 months) and the Gross Profit Margin is 40%:

a) What will be the impact on the Operating Free Cash Flow (OFCF) if that suggestion is accepted (considering everything else unchanged)?

b) Admitting that the Corporate Tax Rate is 25%, point out the impact on the Liquidity situation if the suggestion is accepted.

Solution

a) The change in the OFCF will be equal to the change in the EBITDA subtracting the variation in the change of WC. The EBITDA will increase in 40% of the value of Revenues, that is, will increase in 20,000 euros. At the WC level we have:

$$WC_N = \frac{100,000 \times (1 + 0.2)}{12} \times 1 - \frac{100,000 \times 0.4 \times 0.2}{12} \times 2 = 8,666.67$$

$$WC_{N+1} = \frac{150,000 \times (1 + 0.2)}{12} \times 2 - \frac{150,000 \times 0.4 \times 0.2}{12} \times 2 = 28,000$$

$$\Delta\Delta WC = (WC_{N+1} - WC_N) - (WC_N - WC_0) =$$

$$= (28,000 - 8,666.67) - (8,666.67 - 0) = 10,666.67$$

The change in the OFCF is equal to:

$$\Delta OFCF = \Delta EBITDA - \Delta\Delta WC = 20,000 - 10,666.67 = 9,333.33$$

If the suggestion is accepted the OFCF will increase in 9,333.33 euros.

b) Liquidity is influenced by changes in the Net Permanent Capital and on the Working Capital. The impact on the NPC will come from the change on the Equity value as a result of the change on the Net Income:

$$\Delta\text{EBITDA} = \Delta\text{EBT} = 20{,}000$$

$$\Delta\text{Net Income} = \Delta\text{EBT} \times (1 - t) = 20{,}000 \times (1 - 0.25) = 15{,}000$$

$$\Delta\text{L} = \Delta\text{NPC} - \Delta\text{WC} = 15{,}000 - (28{,}000 - 8{,}666.67) = -4{,}333.33$$

The impact on the Liquidity will be negative in 4,333.33 euros.

EXERCISE 3.21

Consider the following information relative to firm *Quasilast*:

- Gross ROA = 12%;
- ROE = 10%;
- Debt-to-Equity ratio = 1.5;
- Average Cost of Debt = 6%;
- Net Income = 100 thousand euros;
- Operating Needs = 50 thousand euros;
- Operating Resources = 120 thousand euros;
- Non-operating Assets = 200 thousand euros.

Compute the ROIC for the firm.

Solution

To determine the ROIC we need to have the value for the NOPLAT and for the Invested Capital. Let us begin by determining the value for the NOPLAT for which we will need the EBIT. As auxiliary calculations we have to find the Equity value using the ROE simplified formula, we have to determine the Debt level using the Debt-to--Equity ratio, and finally using the Gross ROA we are able to determine the EBIT:

$$0.1 = \frac{100}{\text{Equity}} \quad \Leftrightarrow \quad \text{Equity} = 1{,}000$$

$$1.5 = \frac{\text{Debt}}{1{,}000} \quad \Leftrightarrow \quad \text{Debt} = 1{,}500$$

$$0.12 = \frac{\text{EBIT}}{1,000 + 1,500} \quad \Leftrightarrow \quad \text{EBIT} = 300$$

The Corporate Tax Rate can be obtained from the extended formula for the ROE, which will then allow us to compute the NOPLAT:

$$0.1 = [0.12 + 1.5 \times (0.12 - 0.06)] \times (1 - t) \quad \Leftrightarrow \quad t = 0.5238$$

$$\text{NOPLAT} = 300 \times (1 - 0.5238) = 142.86$$

The amount of the Invested Capital is easy to determine because we already have the values for the Assets, for the Non-operating Assets and for the Operating Liabilities:

$$\text{Invested Capital} = 1,000 + 1,500 - 200 - 120 = 2,180$$

$$\text{ROIC} = \frac{142.86}{2,180} = 0.0655$$

The value of the ROIC for company *Quasilast* is 6.55%.

EXERCISE 3.22

The service company *Service* plans to increase next year (*N+1*) the Receivables Average Term from 1 to 2 months. It is subject to 20% VAT and it is granted a Tax Payables Average Term of 2 months. Currently sells 10,000 euros per month. What will be the increase in its monthly Revenues so that the Operating Free Cash Flow in *N+1* stays the same as in *N*? Consider the year *N* as being the first year of the activity of the firm.

Solution

For simplifying purposes, and since this is a service company, we will assume that the Gross Margin corresponds to the Revenues and that the change in the EBITDA will be equal to the change in Revenues:

$$\text{WC}_N = \frac{120,000 \times (1 + 0.2)}{12} \times 1 - \frac{120,000 \times 0.2}{12} \times 2 = 8,000$$

The change in the Operating Free Cash Flow (OFCF) is the result of the change in the EBITDA subtracting the difference in the change in the WC. Since we want the OFCF to remain unchanged relative to the previous year, then we have:

$$0 = (\text{Revenues}_{N+1} - 120,000) - \\ -\left[\left(\frac{\text{Revenues}_{N+1} \times (1+0.2)}{12} \times 2 - \frac{\text{Revenues}_{N+1} \times 0.2}{12} \times 2\right) - \right. \\ \left. -8,000 - (8,000 - 0)\right] \quad \Leftrightarrow$$

$$\Leftrightarrow \quad \text{Revenues}_{N+1} = 124,800$$

$$\text{Monthly Revenues} = \frac{124,800}{12} = 10,400$$

The monthly Revenues will have to be 10,400 euros, which represents an increase of 400 euros when comparing with the previous year.

EXERCISE 3.23

Answer the following questions clearly and concisely in a maximum of 10 lines:

a) Will a tax exempt company have its ROIC systematically higher than its Gross ROA?

b) The Financial Leverage will be higher with a higher Corporate Tax Rate. Do you agree? Explain.

c) A company has more Net Non-current Assets than Permanent Capital. Under which conditions can the Liquidity be positive?

d) A company has a Working Capital deliberately higher than the Net Permanent Capital. Identify what reason may be underlying this strategy.

e) Imagine that a company is totally financed by Equity and has no Non-operating Assets. Does its ROIC equals its ROE? Explain.

Solution

a) In a tax exempt company, the Gross ROA can be in its maximum equal to the ROIC, never higher, if the Assets are equal to the Invested Capital. Such situation only happens if the company does not possess any Non-operating Assets or Operating Resources.

b) The Income Tax has an impact over the ROE but does not influence the Financial Leverage. Thus, any change on the Corporate Tax Rate will not affect the Financial Leverage.

c) In such conditions, Liquidity may be positive if the company has a negative Working Capital, by a larger amount than the difference between Net Non-current Assets and Permanent Capital.

d) The company may be financing its Assets with short-term Debt, because it can do as much rollovers as it needs and probably gets a lower cost through them comparing with long-term ones. This strategy will work as long as the rollover operation is somehow guaranteed.

e) In this situation it means that the Assets of this company are equal to the Equity. If the company has no Operating Resources it indicates that the Invested Capital matches the Assets, because there are also no Non-operating Assets. Since the company is literally financed by Equity, Finance Expenses equals zero, originating an EBIT equivalent to the EBT. On the other hand, if the company is tax exempt, in other words the Corporate Tax Rate is zero, the EBT and NOPLAT will be equivalent to the Net Income. Only in the conditions described above the ROIC equals the ROE.

Follow-up exercises

EXERCISE 3.24

Mr. Workload is thinking about starting a new business next year. The expected parameters and values are the following:

- Attain Revenues of 1,000 thousand euros;
- Get a Gross Profit Margin of 40%;
- The expenses with External Supplies are expected to be 100 thousand euros;
- Operating Profit Margin of 15%;
- Global Profit Margin of 10%;
- The Finance Expenses will correspond to 10% of the end of year bank debt;
- After accounting for the Depreciation of 50 thousand euros, the Net Non-current Assets will be 320 thousand euros;
- The Average Duration of Inventory will represent 1 month of the expected annual consumption;
- The Receivables Average Term will be 0.75 months;
- The Payables Average Term over the purchase of goods and external supplies will be 2 months;
- The average VAT rate on sales will be 20% while the average VAT rate on purchases will be 16%;
- The Tax Payables liabilities are associated with the Income Tax and also 15,000 thousand euros associated with taxes from operations (from salaries and VAT);

- *Mr. Workload* will put in the business 100 thousand euros in Equity. He will also contract a medium-/long-term bank loan of 100 thousand euros to be paid in 5 constant capital annuities, with the first payment happening at the end of the first year;

a) Using the previous information, analyse the provisional Liquidity situation of *Mr. Workload* new business;

b) Compute the provisional Operating Free Cash Flow.

EXERCISE 3.25

We know the following information regarding firm *Keynesianismo*:

- Revenues = 50,000 euros;
- Gross Profit Margin = 35%;
- Assets Turnover = 0.4;
- ROE = 10%;
- Equity-to-Assets ratio = 80%;
- Corporate Tax Rate = 30%.

a) Compute the Average Cost of Debt;

b) Compute the Net Income for firm *Keynesianismo*.

EXERCISE 3.26

Consider the following data on firm *Simples*:

- EBIT: 100 thousand euros;
- Finance Expenses: 20 thousand euros;
- Income Tax: 8 thousand euros.

a) Compute the firm's Net Income;

b) Compute the firm's NOPLAT.

EXERCISE 3.27

In a certain year firm *Memoex* generated an Operating Free Cash Flow of 400 thousand euros. Its Working Capital was, at the beginning of that year, – 50 thousand euros and it reached – 90 thousand euros at the end of that same year. Compute the EBITDA.

EXERCISE 3.28

Using the following information on firm *Separata*:

- The Receivables/Payables Average Terms are:
  - — Trade Receivables: 3 months;
  - — Trade Payables: 2 months;
  - — Tax Payables: 2 months.
- The annual Revenues are 480 thousand euros, the Cost of Goods Sold is 360 thousand euros the External Supplies are 24 thousand euros;
- The firm works without Inventory;
- The VAT rate is 20% over sales and purchases of goods and services.

Compute the Working Capital.

EXERCISE 3.29

Consider the operating information regarding firm *Elelem* (in thousands of euros):

Income Statement

Revenues	2,000
Cost of Goods Sold	1,200

Balance Sheet

Trade Receivables	600
Inventory	150
Trade Payables	420
Tax Payables	20

Note: the VAT rate is 20%.

a) Identify the Payables Average Term, in months, which the firm is giving to its suppliers. The initial Inventory for the year was 200 thousand euros;

b) Determine the VAT Payable Average Term, in days.

EXERCISE 3.30

Firm *Apumeb* reports an EBIT of 1 million euros, Assets of 10 million euros and Liabilities of 4 million euros. The firm is subject to a Corporate Tax Rate of 20%.

a) Determine its Gross ROA;

b) Knowing that the Net Income is 0.56 million euros, determine the Average Cost of Debt.

EXERCISE 3.31

Firm *Limemo* generated Operating Free Cash Flow of 10,000 euros in year *N*. Knowing that at the end of the previous year its Working Capital was 1,200 euros, that its EBIT in year *N* was 32,000 euros and that the depreciation in year *N* amounted to 500 euros, determine its Working Capital at the end of year *N*.

EXERCISE 3.32

Company *Tecore* has 2,000 euros in Tax Payables (VAT) resulting from 2 months of credit and a VAT rate of 20%. Knowing that the Cost of Goods Sold represents 40% of the Revenues and that the company works with no Inventory, estimate its annual Revenues.

EXERCISE 3.33

Consider the following evolution of company's *Coseev* working capital (in thousands of euros):

	Year 1	Year 2
Trade Receivables	37,000	68,000
Inventory	12,000	12,500
Trade Payables	24,000	25,000
Tax Payables (VAT)	2,000	2,200
WC	23,000	53,300

In your opinion, is the significant increase in Working Capital due to a sharp increase in the company's activity level? Explain.

EXERCISE 3.34

Answer to the following questions:

a) Indicate, justifying, which will be the impact in terms of financial balance (Liquidity) of the following operations:

i) Renegotiation with a Bank, changing a loan amortizing plan: repayment in five years instead of one year;

ii) Selling surplus Inventory;

iii) Repay a non-renewable short-term loan with a cash inflow from revenues;

iv) Changing the Payables Average Term from 3 to 4 months.

b) If the Gross ROA is higher than the Average Cost of Debt, it would be ideal for the company to be as much indebted as possible, possessing the lowest Equity-to-Assets ratio possible! Comment.

c) The stagnancy of the activity level of a company or even its contraction may work as a temporary measure to solve a Liquidity deficit in a short run point of view. Comment.

d) A company has the following Liquidity elements:
— Liquid Asset Elements – Other Assets: 250;
— Liquid Liabilities Elements – Payable Dividends: 400.

How can you describe the financial situation of this company at this moment?

EXERCISE 3.35

Consider the following information on firm *KYG* (amounts in thousands of euros):

- EBIT = 40;
- ROIC = 8%;
- Corporate Tax Rate = 20%;
- Assets = 800;
- Trade Receivables = 200;
- Inventory = 50;
- Other Current Assets = 20 (speculative stock portfolio);
- Non-current Assets = 530, including:
  — Headquarters = 100;
  — Equipment = 250;
  — Wharehouse rented to another company = 80;
  — Service vehicles = 40;
  — Abandoned piece of land = 60.

a) Determine the NOPLAT for the company;
b) Determine the Working Capital for the company;
c) The Financial Leverage for the company is 0.1. Determine its ROE.

Chapter 4

INVESTMENT PROJECT EVALUATION

Theory

In this chapter we will not start with an example or question, we will begin with a statement: the human being is very creative. All of us, to a greater or lesser degree, have already pointed out fantastic ideas for new products, services or business in general. And fortunately there are those who believe so much in a certain concept that end up turning an idea into a true investment project.

However, there are many cases in which wonderful ideas end up succumbing in ephemeral businesses, most of them financially disastrous. The wishful thinking is, in most cases, the cause of such situations: we are confident that the good idea is enough for success.

Unfortunately the ideas require investments. Investments require capital or funding. Capital or funding have to be paid. The implementation of ideas requires labour force. Operating margins have to be enough to pay wages to people and to pay the finance expenses. And this just to indicate two important components of a more complex process, consisting of a series of cash outflows that have to be compensated by cash inflows.

What is the purpose of implementing a great idea, innovative or beautiful, if on the bottom line we end up having successive and accumulated losses?

But how is it possible that a fantastic idea leads to significant losses!?

Multiple are the reasons that serve as an answer to this question. Here are some examples[1] that can be seen individually or in interconnection:

- Limited scale market of the product/service in question, possibly a niche;

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(1) Note that this list will exemplify the cases relating to the product/service. As in the previous chapter, we always give special importance to the business itself, the source of income of the company, of which the product/service plays the main role.

- Product/service with ephemeral survival;
- Expensive product/service;
- Requirement of substantial Capital Expenditure;
- Product with reduced margin;
- Great demand of human resources.

We do not intend to discourage the brilliant minds and sparklers of future business. Our country, the Europe, the world needs entrepreneurs and, especially, needs investors who believe in good ideas. On the contrary, we want to show that it is possible to filter the good business concepts to put in reality those that, with high probability, will be projects or successful companies in the future. The objective of this chapter is to offer a theoretical and methodological tool to investors, for them to have some confidence in real investment projects that they may embrace.

Because business success, in the financial perspective, is measured by profitability and balance situation, as we saw in the previous chapter, the methodology to be used in the evaluation of projects should be able to answer the following question: during its lifetime, will the project be capable of generating sufficient financial outflows, to compensate for the inflows needed, and provide a return for investors to compensate for the risk they assume?

From this question we are able to understand the essential points of the methodology to adopt:

- Identify the cash inflows and cash outflows of the project, ignoring how it will be financed, since we wish to assess the feasibility of the business itself;
- Evaluate the project's profitability to investors.

Cash Flows

Net Cash

We will base the determination of Cash Flows in the expected Results for the activity of the project but, of these results, we will have to identify what is Cash, given the importance of Liquidity in addition to Profitability. So, we will develop the following main tasks to get the final Cash Flow of the project, year by year:

- Calculate the NOPLAT;
- Add the Depreciation (these do not constitute cash outflow);
- Set the Operating Cash Flow with changes in Working Capital;

- Include negative or positive flows of investment or Disinvestment in CAPEX, respectively.

NOPLAT, Depreciation and Working Capital are familiar to us from the previous chapter, but CAPEX can be a novelty. The same means Capital Expenditure and it consists in the investment in Non-current Assets, which we will have to undertake in case we decide to accept the project.

We realize then that the purpose of the calculation of the Cash Flows of the project is of, year after year, determining the final balance of cash which we expect will be generated with the project.

Predict the future and look just at the future

What are the Results, better, Cash Flows, that will be part of our project evaluation? Should we consider allocating costs of other activities? Should we consider sunk costs, for example in market studies? Should we ignore the consequences of a possible cannibalization generated by the project? What is the forecast time horizon to consider?

Almost all of the projects involve a substantial initial Investment (CAPEX). Our intention is to conclude whether we should or not carry out this investment. So, we want to study the size of cash flows that will be generated, or we'll have to invest, just by saying «yes» to the project. Everything that has occurred before this «yes» can no longer be influenced by the decision on the project. In this way, we only care about future Cash Flows and those that are determined or changed by the implementation of the investment.

To the questions posed above, we answer briefly with a practical way to understand which cash flows must make part of our evaluation of the project:

1. We calculate the predicted future cash flows for the situation in which we accept the project;
2. We calculate the predicted future cash flows for the situation in which we do not accept the project;
3. The difference between the first and the second are the incremental cash flows and these are precisely those that we use to conclude whether we make or not the investment.

Let's look at an example. An expansion project will use, among others, an employee who is already in the company since some years ago. His monthly salary appears as an outflow in the situation when the project is accepted but also in the

situation when the project is rejected. Therefore, there is no increment (in this case negative) in Cash Flow caused by this employee, because the difference of the two situations is null. His salary should not be allocated to the new project and, as such, does not constitute cash outflow in our analysis. In fact, the salary of the employee must be paid not because the new project happens, but rather, he continues to be paid even with the new project.

Investment Decision

Net Present Value (NPV)

After we identify the Cash Flows of the project, year by year, we then want to know if the inflows (+) compensate the outflows (-). But, from Chapter 1, we know that we cannot add streams that do not occur in the same moment in time. On the other hand, we know that investors require a certain return to compensate for the risk they assume, as discussed in Chapter 3. In this way, we will have to discount all future Cash Flows to the present moment and, after summing them, we can then compare the total with the CAPEX required. And for this we will need to use a discount rate, which will have to include a premium for the risk taken.

In order to facilitate the conclusion, we calculate the balance between all the discounted Cash Flows and the CAPEX. To this difference we give the name of Net Present Value.

$$NPV = -CAPEX + \frac{CF_1}{1+r} + \frac{CF_2}{(1+r)^2} + ... + \frac{CF_n}{(1+r)^n}$$

where r is the rate of return required by investors (including the risk premium) and n is the number of years of life of the project. Thus, we can conclude that:

- If the NPV results null, it is expected that the project will offset the initial investment and reward investors at a rate that is exactly the minimum required by them. So, if NPV = 0, we should accept the project and the investors shall undertake the investment;
- If the NPV results positive, it is expected that the project rewards the investors at a rate above the minimum required and that is a strong reason to accept it;
- If the NPV results negative, we should reject the project because we anticipate that investors will not be able to obtain the required compensation for the investment and/or reward for their capital.

Internal Rate of Return (IRR)

Before taking a decision on any investment, the investor needs to always know what return can he get with it. A conclusion through the NPV is quite easy to see but does not indicate, in absolute terms, which return is expected for the project. Thus, we are led to determine the overall rate of return on the project, which is known as Internal Rate of Return.

This rate is determined in the same way as an EGAR (APR), as explained at the end of Chapter 1. It couldn't be other way since we are dealing with an operation with several cash flows, of different natures, and spread out in time.

$$0 = -\text{CAPEX} + \frac{CF_1}{1+\text{IRR}} + \frac{CF_2}{(1+\text{IRR})^2} + ... + \frac{CF_n}{(1+\text{IRR})^n}$$

Solving this equation by using, for example, the financial function «IRR» of Microsoft Excel, we obtain the rate of return of the project. This rate should be compared with the minimum rate required by investors, in order to conclude if the project should be accepted.

Mathematically, if the project is conventional, that is, with only a change of sign in the set of year to year Cash Flows, we can conclude that the decision through the analysis of IRR is exactly the same as we would take by using the NPV. Let's see:

- If IRR = *r*, then the NPV is zero and we should accept the project;
- If IRR > *r*, then the NPV is positive and we should accept the project;
- If IRR < *r*, then the NPV is negative and we should reject the project.

Discounted Payback Period (DPP)

The previous methodologies seem to be sufficient to conclude on the feasibility of a project. However, many companies want to go further in the analysis, especially if their activities take place in environments significantly unstable in political, economic and social terms.

For many investors is of extreme importance the knowledge of the period in which the investment is expected to be fully recovered, ensuring that during this period is also generated the minimum required return. This period consists of the Discounted Payback Period, or the period of investment recovery, which is calculated exactly as the NPV but year after year, until we identify the moment when the accumulated of all Cash Flows is zero.

Also here, and for the case of conventional projects, there is a consistency in the conclusions reached by the various methodologies. If the NPV is null, we already know that it is expected that the investment is totally recovered at the end of the life of the project. If the NPV is positive, we can and must calculate the DPP, which will take place before the end of the life of the project. If the NPV is negative, we already know that it is expected that the investment will never be totally recovered during the life of the project, so this should be rejected and it is not even worth trying to calculate the DPP.

Most important to investors will be to define what seems to them to be the ideal period which must not be exceeded in terms of DPP. This ideal period will depend on the expected level of risk for the political, economic and social environment of the business. The higher the risk, the lower should be the deadline they want to take. Thus, in the case of positive NPV, we should compare the DPP obtained by calculation with the deadline defined as ideal. If this is exceeded, the investor will be taken to reject the project, regardless of the NPV being positive.

Profitability Index (PI)

Without any capital constraints, investors are forced to accept all projects that are feasible according to the methodologies studied. However, their resources are limited and, as such, it is necessary to choose the projects with a better relationship between the income and the capital invested. For these cases we need to calculate the Profitability Index, so that we can rank the projects according to their relative profitability.

In a very simple way but without forgetting the time value of money, the PI shall be calculated by dividing the sum of the Cash Flows of the project, without considering any CAPEX, by the total CAPEX. The easiest way to do this is as follows (considering in this simple formula only the initial CAPEX):

$$PI = \frac{NPV}{CAPEX} + 1$$

As we have said, the most important in the use of this methodology is to allow electing, among several projects, the one with the highest profitability.

However, also for conventional projects, the PI is consistent in its findings with the other methodologies:

- $PI = 1 \Rightarrow NPV = 0 \Rightarrow IRR = r \Rightarrow DPP = n \Rightarrow$ accept the project;
- $PI > 1 \Rightarrow NPV > 0 \Rightarrow IRR > r \Rightarrow DPP < n \Rightarrow$ accept the project;
- $PI < 1 \Rightarrow NPV < 0 \Rightarrow IRR < r \Rightarrow DPP > n \Rightarrow$ reject the project.

Practical assessment for the small family projects

In the current context of deep economic and financial crisis, felt not only in our country but also across Europe and the United States, the large percentage of unemployed is challenged to create their own businesses as a means of survival. Those who have some savings can actually do it and they desperately wish that their investments will bear fruit. Certainly they will not be able to hire or develop by themselves the theoretical-practical methodology which we have described in this chapter, in order to assess the feasibility of the project. Thus, we describe here a simple idea that may help better define the way in which individuals must become entrepreneurs.

This idea is summarized in the calculation of a break-even. Knowing what is the monthly expense by the lease of facilities and by spending on personnel, and knowing the expected product margin per unit, we can divide the first by the latter and obtain the minimum units of product or service that we have to sell/provide to not have losses. If, by common sense or practical experience, we realise that this number is unrealistic, for too ambitious, we are not then in the presence of an appropriate project for our intentions. If, to aggravate, the same project requires an initial investment, we have to do the math in terms of the first year (this investment plus the above expenses of one year of activity) and realize how much we would have to sell in a year to compensate for the investment and not suffer losses.

Formulas

- CAPEX = Capital Expenditure
- Cash Flow of the project (year to year) = Operating Cash Flow +
  + Residual value of Working Capital +
  + Disinvestment in CAPEX − CAPEX −
  − ΔWorking Capital
- Operating Cash Flow = NOPLAT + Depreciation =
  = EBIT × (1 − Corporate Tax Rate) + Depreciation
- EBITDA at current prices = EBITDA at constant prices ×
  × (1 + inflation rate)
- Project's discount rate = Risk Free rate + Risk Premium
- Real Interest Rate $= \frac{1 + \text{Nominal Interest Rate}}{1 + \text{Inflation Rate}} - 1$
- Net Present Value (NPV) $= -\text{CAPEX}_0 + \sum_{t=1}^{n} \frac{\text{Cash Flow}_t}{(1+r)^t} \quad \Leftrightarrow$

$$\Leftrightarrow \quad \text{NPV} = \sum_{t=0}^{n} \frac{\text{Cash Flow}_t}{(1+r)^t}$$

- Internal Rate of Return (IRR) :

$$-\text{CAPEX}_0 + \sum_{t=1}^{n} \frac{\text{Cash Flow}_t}{(1+\text{IRR})^t} = 0 \quad \Leftrightarrow$$

$$\Leftrightarrow \quad \sum_{t=0}^{n} \frac{\text{Cash Flow}_t}{(1+\text{IRR})^t} = 0$$

- Discounted Payback Period (DPP) = T when:

$$\sum_{t=1}^{T} \frac{\text{Cash Flow}_t}{(1+\text{discount rate})^t} = \text{CAPEX}_0$$

- Profitability Index (PI) $= \dfrac{\sum_{t=0}^{n} \dfrac{\text{Cash Flow}_t + \text{CAPEX}_t}{(1+\text{discount rate})^t}}{\sum_{t=0}^{n} \dfrac{\text{CAPEX}_t}{(1+\text{discount rate})^t}} =$

$$= \frac{\sum_{t=0}^{n} \dfrac{\text{Cash Flow}_t}{(1+\text{discount rate})^t}}{\sum_{t=0}^{n} \dfrac{\text{CAPEX}_t}{(1+\text{discount rate})^t}} + 1 =$$

$$= \frac{\text{NPV}}{\sum_{t=0}^{n} \dfrac{\text{CAPEX}_t}{(1+\text{discount rate})^t}} + 1$$

Solved Exercises

EXERCISE 4.01

Consider the following investment project, to be evaluated at current prices, assuming an annual inflation rate of 5%.

Unit: thousands of euros

	0	1	2 and following
EBIT		400	500
Working Capital		100	80
CAPEX	800		

The CAPEX includes two equipment:

- Equipment *A*, with a price of 600 thousand euros, 3 years of economic life at the end of which it can be sold for 50 thousand euros;
- Equipment *B*, with a price of 200 thousand euros, 2 years of economic life at the end of which it must be replaced by a similar equipment; this kind of equipment has no second hand market value.

The project is subject to a Corporate Tax Rate of 20%.

In your calculations consider the disinvestment value in CAPEX and the Residual Value in WC on the following year after the end of the activity.

Compute the Cash Flows of the project.

Solution

To determine the Operating Cash Flows and the disinvestment in CAPEX we first need to compute the depreciation values for each year and also the accumulated depreciation. Thus, we begin by creating the depreciation table.

Depreciation table (in thousands of euros)

	AcV.	Year 1	Year 2	Year 3	AcD.	BV	SV	P/L
Equipment *A*	600	200	200	200	600	0	50	50
Equipment *B*	200	100	100	—	200	0	0	0
New Equip. *B*	220.5[1]	—	—	110.25	110.25	110.25	0	– 110.25
Total		300	300	310.25	710.25			– 60.25

Note: AcV. – Acquisition Value; AcD. – Accumulated Depreciation; BV – Book Value; SV – Selling Value; P/L – Profit/Loss.

(1) The reinvestment value at current prices in year 2: $200 \times (1 + 0.05)^2 = 220.5$

Now we are able to proceed with the calculation of the Operating Cash Flows and build the project's Cash Flows table.

Project's Cash Flows table (in thousands of euros)

	Year 0	Year 1	Year 2	Year 3	Year 4
1. EBIT		400	500	500	
2. Income Tax [1. × 20%]		80	100	100	
3. NOPLAT [1. – 2.]		320	400	400	
4. Depreciation		300	300	310.25	
5. Operating Cash Flow [3. + 4.]		620	700	710.25	
6. Residual Value in WC					80
7. Disinvestment in CAPEX					62.05[1]
8. Total of Resources [5. + 6. + 7.]	0	620	700	710.25	142.05
9. Investment in CAPEX	800		220.5		
10. Changes in Working Capital		100	– 20	0	
11. Total of Needs [9. + 10.]	800	100	200.5	0	0
12. Cash Flow of the Project [8. – 11.]	– 800	520	499.5	710.25	142.05

(1) Disinvestment Equip. $A = 50 - (50 - 0) \times 0.2 = 40$

Disinvestment New Equip. $B = 0 - (0 - 110.25) \times 0.2 = 22.05$

EXERCISE 4.02

Consider a project with the following features:

- Initial investment = 80 thousand euros;
- Annual constant Cash Flows from year 1 (inclusively) onwards;
- Project lifetime = 4 years;
- Consider that the investors demand a minimum return of 12%;
- NPV = 72 thousand euros.

Compute the Discounted Payback Period (DPP) of the project to one decimal place. Comment on the acceptance or rejection of the project.

Solution

First we need to determine the constant Cash Flow in each of the four years of the project:

$$72 = -80 + CF \times \frac{1-(1+0.12)^{-4}}{0.12} \quad \Leftrightarrow \quad CF = 50.04$$

Now we have the information that allows us to compute the discounted accumulated value at the end of each year (in thousands of euros):

	Year 0	Year 1	Year 2	Year 3	Year 4
Cash Flow	– 80	50.04	50.04	50.04	50.04
Discounted Cash Flow	– 80	44.68	39.89	35.62	31.80
Accumulated Discounted Cash Flows	– 80	– 35.32	4.57	40.19	71.99

The moment when the accumulated value of the discounted Cash Flows is equal to zero happens between years 1 and 2. To determine the exact moment we use linear interpolation:

$$DPP = 1 + (2-1) \times \frac{0-(-35.32)}{4.57-(-35.32)} = 1.9$$

The DPP is equal to 1.9 years. Since the investment is fully recovered in a period (1.9 years) inferior to the lifetime of the project (4 years), the project should be accepted.

EXERCISE 4.03

A project with only an initial investment in CAPEX of 100 thousand euros has a PI of 1.35.

a) Compute its NPV;

b) Its DPP is 2 years and the lifetime of the project, including the year of disinvestment, is 3 years. Knowing that the discounted Cash Flow in the first year of operating is 60 thousand euros, determine the remaining discounted Cash Flows of the project.

Solution

a) Since the only investment in CAPEX happens at the beginning of the project, finding the NPV is simplified:

$$PI = \frac{NPV}{CAPEX_0} + 1 \quad \Leftrightarrow \quad 1.35 = \frac{NPV}{100} + 1 \quad \Leftrightarrow \quad NPV = 35$$

The NPV is 35 thousand euros.

b) If the DPP is 2 years then the accumulated value of the discounted Cash Flows of years 1 and 2 must be equal to the initial investment, that is, 100 thousand euros. Since we already know that the discounted cash flow of year 1 is equal to 60 thousand euros, then the discounted cash flow of year 2 must equal the difference to the 100 thousand euros, that is, 40 thousand euros. Since, from the preceding question we found the NPV to be 35 thousand euros, then the discounted Cash Flow of year 3 will also be equal to 35 thousand euros, because the project ends after 3 years. In a table format we have (in thousands of euros):

	Discounted Cash Flow	Accumulated Value of the Discounted Cash Flows
0	– 100	– 100
1	60	– 40
2	40	0
3	35	35

EXERCISE 4.04

Consider the following information about an investment project:

Unit: thousands of euros

	1	2	3	4
EBITDA	200	250	300	
EBIT	100	150	200	
NOPLAT	80	120	160	
CAPEX Selling Value				120

The project will be analysed using a discount rate of 10%. The disinvestment in CAPEX generates an accounting profit of 20 thousand euros. The project does not have values of Working Capital.

Determine the NPV of the project and comment on its acceptance or rejection.

Solution

The Depreciation value is determined by the difference between the EBITDA and the EBIT.

The annual Corporate Tax Rate (*t)* can be obtained using the relation between the EBIT and the NOPLAT:

$$80 = 100 \times (1 - t_1) \quad \Leftrightarrow \quad t_1 = 0.2 = 20\%$$

$$120 = 150 \times (1 - t_2) \quad \Leftrightarrow \quad t_2 = 0.2 = 20\%$$

$$160 = 200 \times (1 - t_3) \quad \Leftrightarrow \quad t_3 = 0.2 = 20\%$$

The Corporate Tax Rate is constant throughout the project and is equal to 20%. The disinvestment in CAPEX is equal to:

$$\text{Disinvestment in CAPEX} = 120 - 20 \times 0.2 = 116$$

The initial investment in CAPEX is computed through the sum of all the depreciation values until the end of the project, adding the Book Value at that time. The Book Value can be determined using the Selling Value of the CAPEX of 120 thousand euros and the generated accounting profit of 20 thousand euros, resulting in a final value of 100 thousand euros.

Project's Cash Flows table (in thousands of euros)

	Year 0	Year 1	Year 2	Year 3	Year 4
1. NOPLAT		80	120	160	
2. Depreciation [EBITDA – EBIT]		100	100	100	
3. Operating Cash Flow [1. + 2.]		180	220	260	
4. Residual Value in WC					0
5. Disinvestment in CAPEX					116
6. Total of Resources [3. + 4. + 5.]	0	180	220	260	116
7. Investment in CAPEX	400				
8. Changes in Working Capital		0	0	0	
9. Total of Needs [7. + 8.]	400	0	0	0	0
10. Cash Flow of the Project [6. – 9.]	– 400	180	220	260	116

$$NPV = -400 + \frac{180}{1+0.1} + \frac{220}{(1+0.1)^2} + \frac{260}{(1+0.1)^3} + \frac{116}{(1+0.1)^4} = 220.03$$

The NPV is equal to 220.03 thousand euros. Since the NPV is positive the project should be accepted. The investors will received the required annual return of 10% plus an additional amount, in present values, of 220.03 thousand euros.

EXERCISE 4.05

Consider the following investment project, to be evaluated at current prices:

Unit: thousands of euros

	0	1	2 and following
EBITDA		800	1,000
NOPLAT		450	600
Working Capital		(50)	(20)
CAPEX	450		

The CAPEX includes two equipment:

- Equipment *A*, with a price of 300 thousand euros, 2 years of economic life at the end of which it can be sold for 20 thousand euros;
- Equipment *B*, with a price of 150 thousand euros and 3 years of economic life. This kind of equipment has no second hand market value.

In your calculations consider the disinvestment value in CAPEX and the Residual Value in WC on the following year after the end of the activity.

Compute the Cash Flows of the project.

Solution

Since the lifetime of the project is not given, we will assume that is equal to the economic life of the most relevant equipment (in value), which in this case is equipment *A*. We begin by building the depreciation table:

Depreciation table (in thousands of euros)

	AcV.	Year 1	Year 2	AcD.	BV	SV	P/L
Equipment *A*	300	150	150	300	0	20	20
Equipment *B*	150	50	50	100	50	0	– 50
Total	450	200	200	400			– 30

Now we have all the information needed to proceed with the calculation of the Operating Cash Flows and build the complete table of the project's Cash Flows.

Project's Cash Flows table (in thousands of euros)

	Year 0	Year 1	Year 2	Year 3
1. NOPLAT		450	600	
2. Depreciation		200	200	
3. Operating Cash Flow [1. + 2.]		650	800	
4. Residual Value in WC				– 20
5. Disinvestment in CAPEX				27.5[1]
6. Total of Resources [3. + 4. + 5.]	0	650	800	7.5
7. Investment in CAPEX	450			
8. Changes in Working Capital		– 50	30	
9. Total of Needs [7. + 8.]	450	– 50	30	0
10. Cash Flow of the Project [6. – 9.]	– 450	700	770	7.5

(1) To determine the disinvestment in CAPEX it is necessary to find the Corporate Tax Rate. To do so, and using the values of year 2 and the existing relation between the NOPLAT and the EBITDA, we get:

$$\text{EBITDA} = \text{EBIT} + \text{Depreciation} \Leftrightarrow 1{,}000 = \text{EBIT} + 200 \Leftrightarrow \text{EBIT} = 800$$

$$\text{NOPLAT} = \text{EBIT} \times (1 - t) \Leftrightarrow 600 = 800 \times (1 - t) \Leftrightarrow t = 0.25 = 25\%$$

And the disinvestment value for each equipment is equal to:

$$\text{Disinvestment Equip. } A = 20 - (20 - 0) \times 0.25 = 15$$

$$\text{Disinvestment Equip. } B = 0 - (0 - 50) \times 0.25 = 12.5$$

EXERCISE 4.06

An investment project, besides the initial CAPEX, has three annual Cash Flows. Knowing that the value of the last Cash Flow is 100 thousand euros, that the DPP is 2.2 years and that the investors require a minimum return of 12%, determine the NPV for the project.

Solution

Using the DPP and the discounted Cash Flow of year 3 it is possible to determine the NPV. To do so we begin by computing the discounted Cash Flow of year 3:

$$DCF_3 = \frac{100}{(1 + 0.12)^3} = 71.18$$

From the value obtained we know that 20% (the 0.2 beyond the 2 years) is needed to reach the DPP, while the remaining 80% correspond to the NPV:

$$NPV = 0.8 \times 71.18 = 56.94$$

The NPV is 56.94 thousand euros.

EXERCISE 4.07

Consider an investment project with the following features:

- Initial (and only) investment = 150 thousand euros;
- PI = 2.5;
- Annual Cash Flows;
- Lifetime = 2 years;
- Value of Cash Flow 1 = 90 thousand euros;
- Required minimum return = 5%.

Show the calculation formula for the IRR.

Solution

To determine the IRR we need to find the value of all Cash Flows. Since we already have the discounted Cash Flow 1 and the discount rate to use in the project (5%), it is possible to determine the value of Cash Flow 1:

$$CF_1 = 90 \times (1 + 0.05) = 94.5$$

From the PI expression we are able to compute the value of Cash Flow 2:

$$2.5 = \frac{-150 + \frac{94.5}{1+0.05} + \frac{CF_2}{(1+0.05)^2}}{150} + 1 \quad \Leftrightarrow \quad CF_2 = 314.21$$

Finally, we have all the information needed to write down the expression to determine the IRR:

$$0 = -150 + \frac{94.5}{1+IRR} + \frac{314.21}{(1+IRR)^2}$$

EXERCISE 4.08

Consider the following information for an investment project:

Unit: thousands of euros

	1	2	3	4
Revenues	500	400	400	
COGS	200	200	200	
EBIT	200	100	100	
CAPEX Selling Value				100

The project will be evaluated based on a minimum required return for the shareholders of 20%. It is known that the CAPEX Selling Price is equal to the residual Book Value. Ignore in the analysis the existence of Other Revenues, of Other Operating Costs and Working Capital. Assume a Corporate Tax Rate of 25%.

Proceed with the NPV calculation and comment on the acceptance or rejection of the project.

Solution

We begin by finding the initial investment in CAPEX. Since the book value is the same as the Selling Value, then we know that at the end of the 3 years of the project's life the CAPEX still has a Book Value of 100 thousand euros. Adding the Book

Value to the accumulated depreciation after 3 years, we obtain the initial investment in year zero:

$$\text{Depreciation} = (\text{Revenues} - \text{COGS}) - \text{EBIT}$$

$$\text{Depreciation}_1 = 500 - 200 - 200 = 100$$

$$\text{Depreciation}_2 = 400 - 200 - 100 = 100$$

$$\text{Depreciation}_3 = 400 - 200 - 100 = 100$$

$$\text{Investment in CAPEX}_0 = 100 + 3 \times 100 = 400$$

Project's Cash Flows table (in thousands of euros)

	Year 0	Year 1	Year 2	Year 3	Year 4
1. EBIT		200	100	100	
2. Income Tax		50	25	25	
3. NOPLAT [1. – 2.]		150	75	75	
4. Depreciation		100	100	100	
5. Operating Cash Flow [3. + 4.]		250	175	175	
6. Residual Value in WC					0
7. Disinvestment in CAPEX					100
8. Total of Resources [5. + 6. + 7.]		250	175	175	100
9. Investment in CAPEX	400				
10. Changes in Working Capital		0	0	0	
11. Total of Needs [9. + 10.]	400	0	0	0	0
12. Cash Flow of the Project [8. – 11.]	– 400	250	175	175	100

$$\text{NPV} = -400 + \frac{250}{1 + 0.2} + \frac{175}{(1 + 0.2)^2} + \frac{175}{(1 + 0.2)^3} + \frac{100}{(1 + 0.2)^4} = 79.36$$

The NPV is 79.36 thousand euros. The investors must be willing to go ahead with the project because they get the required return on the invested capital of 20% and also an additional amount, at today's prices, of 79.36 thousand euros.

EXERCISE 4.09

Consider a project with an IRR of 12% and a minimum required return by shareholders of 5%. Knowing that the lifetime of the project is two years, that the Cash Flow of year 1 is 100 thousand euros, and that the initial investment is 300 thousand euros, determine:

a) The value of the Cash Flow of year 2;

b) The DPP.

Solution

a) From the IRR expression it is straightforward to obtain the requested value:

$$0 = -300 + \frac{100}{1+0.12} + \frac{CF_2}{(1+0.12)^2} \quad \Leftrightarrow \quad CF_2 = 264.32$$

The value of the Cash Flow of year 2 is 264.32 thousand euros.

b) To determine the DPP let us build the following auxiliary table (in thousands of euros):

	Year 0	Year 1	Year 2
Cash Flow	– 300	100	264.32
Discounted Cash Flow	– 300	95.24	239.75
Accumulated Discounted Cash Flows	– 300	– 204.76	34.99

The moment when the accumulated discounted Cash Flows is equal to zero happens between years 1 and 2. To determine the exact moment we will use linear interpolation:

$$DPP = 1 + (2-1) \times \frac{0 - (-204.76)}{34.99 - (-204.76)} = 1.85$$

The DPP is 1.85 years.

EXERCISE 4.10

Consider a project with the following features:

- Initial investment = 70 thousand euros;
- Reinvestment in CAPEX on year 2 = 40 thousand euros;
- Discounted value of Cash Flow 1 = 40 thousand euros;
- Discounted value of Cash Flow 2 = 30 thousand euros;
- Project lifetime = 3 years;
- Consider that investors require a minimum rate of return of 7%;
- Consider an IRR of 12%.

Compute the PI and comment on the acceptance or rejection of the project.

Solution

We begin by determining the value of the Cash Flow of year 3. To do so we will use the IRR expression. However, we first need to find the values of the Cash Flows of years 1 and 2, by compounding their present values at the required rate of return of 7%:

$$CF_1 = 40 \times (1 + 0.07) = 42.8$$

$$CF_2 = 30 \times (1 + 0.07)^2 = 34.35$$

$$0 = -70 + \frac{42.8}{1 + 0.12} + \frac{34.35}{(1 + 0.12)^2} + \frac{CF_3}{(1 + 0.12)^3} \quad \Leftrightarrow \quad CF_3 = 6.18$$

Knowing the value of all the Cash Flows of the project we are now able to compute the PI. Considering the existence of the reinvestment in CAPEX on year 2 and the fact that this value is already included on the project's Cash Flow for that year, we get:

$$PI = \frac{-70 + 40 + 30 + \dfrac{6.18}{(1 + 0.07)^3}}{70 + \dfrac{40}{(1 + 0.07)^2}} + 1 = 1.05$$

The PI equals 1.05. The project should be accepted because the PI is greater than 1. At today's prices, and already considering the minimum rate of return demanded by the investors of 7%, the investor will get, for each euro invested, 5 cents of additional return.

EXERCISE 4.11

Firm *ATO* wishes to undertake an investment in a business of producing surf boards. The management has asked your opinion about the correct decision to make, undertaking or not this project, considering the following information:

CAPEX:

Unit: thousands of euros

	Price	Straight-line Depreciation rate	Market Value (Selling Value)
Production Line *A*	30	33.33%	15
Production Line *B*	60	25.00%	5
Intangible Assets	9	33.33%	

The CAPEX will be fully considered at the beginning of the project (year 0). For simplifying purposes consider the disinvestment in CAPEX and the Residual Value in Working Capital in the last year of activity.

The operating forecasted values at constant prices are:

Unit: thousands of euros

	Year 1	Year 2	Year 3 and following
EBIT	– 10	35	45
Working Capital	5	4.5	6

The Corporate Tax Rate is 25% (assume that the taxes are paid in the corresponding year). Consider that the Treasury Yield is 5%, that the risk premium of the project is 6% and that the expected annual rate of inflation is 2.778%. This project is being undertaken by a company with large positive earnings.

You are asked to:

a) Compute the Cash Flows of the project at constant prices;

b) Compute the NPV for the project (at constant prices);

c) Compute the IRR for the project and comment on its acceptance or rejection.

Solution

a) Before advancing with the project´s Cash Flows table we need to determine the annual Depreciation and the CAPEX Book Value at the end of the project. The project will have 4 years of activity corresponding to the economic life of the most relevant equipment (Line *B*):

Depreciation table (in thousands of euros)

	AcV.	Year 1	Year 2	Year 3	Year 4	AcD.	BV	SV	P/L
Line *A*	30	10	10	10	—	30	0	0	0
Line *B*	60	15	15	15	15	60	0	5	5
Intangibles	9	3	3	3	—	9	0	0	0
New Line *A*	30[1]	—	—	—	10	10	20	15	– 5
Total		28	28	28	25				0

(1) The reinvestment value of the new Line *A* will be the same has the value considered on year 0, since we are performing a constant prices analysis.

Project's Cash Flows table (in thousands of euros)

	Year 0	Year 1	Year 2	Year 3	Year 4
1. EBIT		– 10	35	45	45
2. Income Tax [1. × 25%]		– 2.5[2]	8.75	11.25	11.25
3. NOPLAT [1. – 2.]		– 7.5	26.25	33.75	33.75
4. Depreciation		28	28	28	25
5. Operating Cash Flow [3. + 4.]		20.5	54.25	61.75	58.75
6. Residual Value in WC					6
7. Disinvestment in CAPEX					20[3]
8. Total of Resources [5. + 6. + 7.]		20.5	54.25	61.75	84.75
9. Investment in CAPEX	99			30	
10. Changes in Working Capital		5	– 0.5	1.5	
11. Total of Needs [9. + 10.]	99	5	– 0.5	31.5	0
12. Cash Flow of the Project [8. – 11.]	– 99	15.5	54.75	30.25	84.75

(2) Since the project is included in a highly profitable firm, the negative value of the EBIT on the first year will reduce the overall operating earnings of the firm, yielding a decrease of 2.5 thousand euros in the Income Tax.

(3) Since the sum of the accounting profits and losses obtained through the sale of the equipment in zero, the disinvestment value of the CAPEX is equal to the sum of the selling values.

b) To determine the NPV it is first necessary to compute the discount rate to be used in discounting the project's Cash Flows. Since we are performing an economic analysis using constant prices, the discount rate to be used is the real rate:

$$r = r_f \text{ (risk free rate)} + rp \text{ (risk premium)} =$$
$$= 0.05 + 0.06 = 0.11 = 11\%$$

$$r_{real} = \frac{1+r}{1+i} - 1 = \frac{1+0.11}{1+0.02778} - 1 \cong 0.08 \cong 8\ \%$$

with i = inflation rate

$$\text{NPV} = -99 + \frac{15.5}{1+0.08} + \frac{54.75}{(1+0.08)^2} + \frac{30.25}{(1+0.08)^3} + \frac{84.75}{(1+0.08)^4} \quad \Leftrightarrow$$
$$\Leftrightarrow \quad \text{NPV} = 48.6$$

The NPV is 48.6 thousand euros.

c) Using the IRR expression, we get:

$$0 = -99 + \frac{15.5}{1+\text{IRR}} + \frac{54.75}{(1+\text{IRR})^2} + \frac{30.25}{(1+\text{IRR})^3} + \frac{84.75}{(1+\text{IRR})^4} \quad \Leftrightarrow$$
$$\Leftrightarrow \quad \text{IRR} = 0.2437 = 24.37\%$$

To determine the value of the IRR we can use, for example, Microsoft Excel, and use the function IRR. In this case the project's IRR is 24.37%. The project should be accepted. The rate of return implicit in the project (24.37%) is higher than the minimum rate of return demanded by investors (8%). From the preceding solution we already knew that the project should be accepted since the NPV is positive. In the presence of conventional cash flows (sequence of cash outflows followed by a sequence of cash inflows), as such is the case, the IRR will be consistent with the NPV, that is, if the NPV is positive then the IRR is higher than the discount rate and the project should be accepted. The opposite is also true. Finally, if the NPV equals zero, then the IRR equals the discount rate.

EXERCISE 4.12

Consider that projects *A* and *B* have the same initial investment. Project *A* has a lifetime of one year and a PI of 1.50. Project *B* has, besides the initial investment, a single cash flow of 300 thousand euros at the end to the second year, an IRR of 25% and a NPV of 40 thousand euros. Considering that both projects have the same level of risk, determine:

a) The value of the initial investment of project *A*;

b) The value of the Cash Flow 1 of project *A*.

Solution

a) To find the value of the initial investment of project *A* we just need to find that same value for project *B*. Using the expression for the IRR we get:

$$0 = -CAPEX_0 + \frac{300}{(1+0.25)^2} \quad \Leftrightarrow \quad CAPEX_0 = 192$$

Since the initial investment of project *A* is the same as project's *B*, its value is 192 thousand euros.

b) Since the risk of project *A* is the same as project's *B*, then both have the same discount rate. The discount rate for project *B* can be obtained through the expression for the NPV:

$$40 = -192 + \frac{300}{(1+r)^2} \quad \Leftrightarrow \quad r \cong 0.1371 \cong 13.71\%$$

The value of the Cash Flow of year 1 for project *A* can be obtained using the PI:

$$1.5 = \frac{-192 + \dfrac{CF_1}{1+0.1371}}{192} + 1 \quad \Leftrightarrow \quad CF_1 = 327.48$$

The value of the Cash Flow of year 1 for project *A* is 327.48 thousand euros.

EXERCISE 4.13

Consider an investment project to produce a new product, with the following features:

- The NOPLAT is 400 thousand euros on the first year and 600 thousand euros on the following years;
- The Working Capital is 100 thousand euros on the first year and 90 thousand euros on the following years;
- The CAPEX includes 3 equipment:
  - EqA with a price of 100 thousand euros, 2 years of economic life and without any second hand market value;
  - EqB with a price of 300 thousand euros, an economic life of 5 years and a Selling Value of 80 thousand euros;
  - EqC with a price of 600 thousand euros, an economic life of 4 years and a Selling Value of 100 thousand euros.
- The Corporate Tax Rate is 25%.

Compute the Cash Flows for the project, considering the disinvestment in CAPEX and the Residual Value in Working Capital on the year after the end of the activity.

Solution

To determine the project's cash flows we first need to compute the annual Depreciation, and the Book Values of the CAPEX at the end of the project. The project will have 4 years of activity, corresponding to the economic life of the most relevant equipment (EqC):

Depreciation table (in thousands of euros)

	AcV.	Year 1	Year 2	Year 3	Year 4	AcD.	BV	SV	P/L
EqA	100	50	50	—	—	100	0	0	0
EqB	300	60	60	60	60	240	60	80	20
EqC	600	150	150	150	150	600	0	100	100
New EqA	100	—	—	50	50	100	0	0	0
Total		260	260	260	260		60	180	120

Project's Cash Flows table (in thousands of euros)

	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
1. NOPLAT		400	600	600	600	
2. Depreciation		260	260	260	260	
3. Operating Cash Flow [1. + 2.]		660	860	860	860	
4. Residual Value in WC						80
5. Disinvestment in CAPEX						150[1]
6. Total of Resources [3. + 4. + 5.]	0	660	860	860	860	230
7. Investment in CAPEX	1,000		100			
8. Changes in Working Capital		100	– 20	0	0	
9. Total of Needs [7. + 8.]	1,000	100	80	0	0	0
10. Cash Flow of the Project [6. – 9.]	– 1,000	560	780	860	860	230

(1) Disinvestment in CAPEX = $180 - (180 - 60) \times 0.25 = 150$

EXERCISE 4.14

A project has an investment of 600 thousand euros and a lifetime of 3 years. At the end of year 1, a third of the investment is recovered, and at the end of year 2, two thirds are recovered. However, its NPV is negative in 40 thousand euros. Knowing that the Cash Flow of year 3 is 200 thousand euros, determine the remaining Cash Flows.

Solution

From the existing information we know that the discounted Cash Flow of year 1 is 200 thousand euros and that the discounted Cash Flow of year 2 is also 200 thousand euros. Using the expression for the NPV it is possible to determine the value for the discount rate of the project:

$$-40 = -600 + 200 + 200 + \frac{200}{(1+r)^3} \quad \Leftrightarrow \quad r = 0.0772 = 7.72\%$$

Having the discount rate we can now find the values of the Cash Flows of years 1 and 2:

$$CF_1 = 200 \times (1 + 0.0772) = 215.44$$

$$CF_2 = 200 \times (1 + 0.0772)^2 = 232.08$$

The Cash Flow of year 1 is 215.44 thousand euros and the Cash Flow of year 2 is 232.08 thousand euros.

EXERCISE 4.15

Consider the following investment project:

Unit: thousands of euros

	0	1	2 and following
EBIT		400	500
Working Capital		100	80
CAPEX	600		

The CAPEX consists of a machine that has 3 years of economic life, at the end of which it can be sold for 50 thousand euros. The project is subject to a Corporate Tax Rate of 20%. Compute the Cash Flows of the project, considering the disinvestment in CAPEX and the Residual Value in Working Capital on the year after the end of the activity.

Solution

The project will have 3 years of activity, corresponding to the economic life of the machine.

Project's Cash Flows table (in thousand of euros)

	Year 0	Year 1	Year 2	Year 3	Year 4
1. EBIT		400	500	500	
2. Income Tax [1. × 20%]		80	100	100	
3. NOPLAT [1. – 2.]		320	400	400	
4. Depreciation		200	200	200	
5. Operating Cash Flow [3. + 4.]		520	600	600	
6. Residual Value in WC					80
7. Disinvestment in CAPEX					40[1]
8. Total of Resources [5. + 6. + 7.]	0	520	600	600	120
9. Investment in CAPEX	600				
10. Changes in Working Capital		100	– 20	0	
11. Total of Needs [9. + 10.]	600	100	– 20	0	0
12. Cash Flow of the Project [8. – 11.]	– 600	420	620	600	120

(1) Disinvestment in CAPEX = 50 – (50 – 0) × 0.2 = 40

EXERCISE 4.16

A project has an initial investment of 1,000 thousand euros. Its Discounted Payback Period is exactly 3 years. On the 4th and 5th years its Cash Flows, already discounted, are respectively 200 thousand euros and 100 thousand euros. Is it possible to compute the NPV using only this information? Justify.

Solution

Yes, because the NPV represents the added value obtained after the full recovery of the investment made. If the payback period is exactly 3 years (which means that the accumulated value of the discounted Cash Flows is zero at that moment), the sum of the remaining discounted Cash Flows will be the NPV. In this case the NPV is 300 thousand euros.

EXERCISE 4.17

A project has, after the year of investment, only two annual Cash Flows of 100 thousand euros each. It generates an IRR of 8%, 2 percentage points less than the minimum rate of return required by investors, reason why the project was rejected. Determine its NPV.

Solution

Using the expression of the IRR it is possible to compute the value of the initial investment, which will allows us to know all Cash Flows of the project and consequently be able to determine the NPV:

$$0 = -CAPEX_0 + \frac{100}{1+0.08} + \frac{100}{(1+0.08)^2} \quad \Leftrightarrow \quad CAPEX_0 = 178.33$$

$$\text{NPV} = -178.33 + \frac{100}{1+0.1} + \frac{100}{(1+0.1)^2} = -4.78$$

The NPV is negative in 4.78 thousand euros.

EXERCISE 4.18

Company *ALFIN* has in its growth opportunities portfolio an investment project with a lifetime of 5 years. The project includes the acquisition of one machine, with a price of 80 thousand euros and an economic life of 4 years. It is expected that the market value for the machine at the end of the project corresponds to 80% of its Book Value. Not considering inflation, the expected EBITDA for the 1st year is 50 thousand euros and 60 thousand euros on the following years. The Corporate Tax Rate is 20%. The amount of Working Capital will correspond to 10% of the EBITDA. The expected annual inflation rate is 2.5%. The shareholders demand a minimum rate of return (at constant prices) of 8%/year. For simplifying purposes consider the disinvestment in CAPEX and the Residual Value in Working Capital in the last year of activity.

a) Compute the NPV at constant prices;

b) Compute the NPV at current prices.

Solution

a) Since all given values are in constant prices, we have all the information needed to build the Depreciation table, the Working Capital table and the project's Cash Flows table:

Depreciation table (in thousands of euros)

	AcV.	Year 1	Year 2	Year 3	Year 4	Year 5	AcD.	BV	SV	P/L
Machine	80	20	20	20	20	—	80	0	0	0
New Machine	80[1]	—	—	—	—	20	20	60	48	– 12

(1) The reinvestment value of the new machine is the same as the one considered in year zero, since the analysis is in constant prices.

Working Capital table (in thousands of euros)

	Year 1	Year 2	Year 3	Year 4	Year 5
1. EBITDA	50	60	60	60	60
2. Working Capital [1. × 10%]	5	6	6	6	6
3. Changes in WC	5	1	0	0	0

Project's Cash Flows table (in thousands of euros)

	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
1. EBITDA		50	60	60	60	60
2. Depreciation		20	20	20	20	20
3. EBIT		30	40	40	40	40
4. Income Tax [3. × 20%]		6	8	8	8	8
5. NOPLAT [3. – 4.]		24	32	32	32	32
6. Operating Cash Flow [5. + 2.]		44	52	52	52	52
7. Residual Value in WC						6
8. Disinvestment in CAPEX						50.4[2]
9. Total of Resources [6. + 7. + 8.]	0	44	52	52	52	108.4
10. Investment in CAPEX	80				80	
11. Changes in Working Capital		5	1	0	0	0
12. Total of Needs [10. + 11.]	80	5	1	0	80	0
13. Cash Flow of the Project [9. – 12.]	– 80	39	51	52	– 28	108.4

(2) Disinvestment in CAPEX = 48 – (48 – 60) × 0.2 = 50.4

$$NPV = -80 + \frac{39}{1+0.08} + \frac{51}{(1+0.08)^2} + \frac{52}{(1+0.08)^3} +$$

$$+ \frac{-28}{(1+0.08)^4} + \frac{108.4}{(1+0.08)^5} \Leftrightarrow NPV = 94.31$$

The NPV for the project, using constant prices, is 94.31 thousand euros.

b) To perform the analysis using current prices we need to apply the annual inflation rate where necessary. Thus, we need to determine the new values for the EBITDA (and consequently for the Working Capital), and also the value of the reinvestment on the new machine:

Depreciation table (in thousands of euros)

	AcV.	Year 1	Year 2	Year 3	Year 4	Year 5	AcD.	BV	SV	P/L
Machine	80	20	20	20	20	—	80	0	0	0
New Machine	88.31[1]	—	—	—	—	22.08	22.08	66.23	52.98	– 13.25

(1) The value of the reinvestment on the new machine is equal to the value considered on year zero compounded 4 years at the annual inflation rate: $80 \times (1+0.025)^4 = 88.31$.

Working Capital table (in thousands of euros)

	Year 1	Year 2	Year 3	Year 4	Year 5
1. EBITDA	51.25	63.04	64.61	66.23	67.88[2]
2. Working Capital [1. × 10%]	5.13	6.3	6.46	6.62	6.79
3. Changes in WC	5.13	1.17	0.16	0.16	0.17

(2) The annual value of the EBITDA is determined by compounding the values at constant prices, at the annual inflation rate, during the corresponding years. For example, to determine the EBITDA of year 5, we have: $60 \times (1+0.025)^5 = 67.88$

Project's Cash Flows table (in thousands of euros)

	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
1. EBITDA		51.25	63.04	64.61	66.23	67.88
2. Depreciation		20	20	20	20	22.08
3. EBIT		31.25	43.04	44.61	46.23	45.8
4. Income Tax [3. × 20%]		6.25	8.61	8.92	9.25	9.16
5. NOPLAT [3. – 4.]		25	34.43	35.69	36.98	36.64
6. Operating Cash Flow [5. + 2.]		45	54.43	55.69	56.98	58.72
7. Residual Value in WC						6.79
8. Disinvestment in CAPEX						55.63[3]
9. Total of Resources [6. + 7. + 8.]	0	45	54.43	55.69	56.98	121.14
10. Investment in CAPEX	80				88.31	
11. Changes in Working Capital		5.13	1.17	0.16	0.16	0.17
12. Total of Needs [10. + 11.]	80	5.13	1.17	0.16	88.47	0.17
13. Cash Flow of the Project [9. – 12.]	– 80	39.87	53.26	55.53	– 31.49	120.97

(3) Disinvestment in CAPEX = $52.98 - (52.98 - 66.23) \times 0.2 = 55.63$

In order to be consistent with the estimation of the project's Cash Flows, we now have to compute the NPV using the nominal discount rate:

$$r = (1 + 0.08) \times (1 + 0.025) - 1 = 0.107 = 10.7\ \%$$

$$\text{NPV} = -80 + \frac{39.87}{1 + 0.107} + \frac{53.26}{(1 + 0.107)^2} + \frac{55.53}{(1 + 0.107)^3} +$$

$$+ \frac{-31.49}{(1 + 0.107)^4} + \frac{120.97}{(1 + 0.107)^5} \quad \Leftrightarrow \quad \text{NPV} = 92.21$$

The NPV for the project, using current prices, is 92.21 thousand euros.

EXERCISE 4.19

Comment/answer in a clear and concise manner the/to the following statements/questions:

a) Determining the disinvestment in CAPEX and the Residual Value in Working Capital is irrelevant to the computation of the Discounted Payback Period;

b) The assumption of the reinvestment of the Cash Flows, implicit on the IRR calculation, may lead to an undue acceptance of an investment project;

c) In an investment project, what is the effect on the NPV from increasing simultaneously in one month the Receivables Average Term, the Payables Average Term and the Tax Payables Average Term (VAT)? Justify;

d) An investment project has a NOPLAT of zero in every year. Consider a discount rate of zero. Is it still possible for the NPV to be positive?

e) Consider that the disinvestment value of the CAPEX is equal to its Book Value. In this situation, is it indifferent to consider the disinvestment in the last year of activity or in the next one?

f) In a project was chosen, for the calculation of the depreciation, the reducing--balance method (the value of the depreciation decreases every year) instead of the straight-line depreciation. What is the effect of this decision on the NPV of the project?

g) Imagine a project without investment (CAPEX) and a positive EBIT in every year. Are these conditions enough to ensure a positive NPV? Justify;

h) In an existing firm it is known the following information regarding an investment project for the launch of a new product:

— For the store it will be hired a new sales person with an annual cost of 8,000 euros. He will be busy with the promotion of this new product in only 70% of his time, having the remaining 30% available for another possible product;

— It was performed some months ago a market test, with a cost of 10,000 euros, to 300 potential consumers for the choice of the final color for the product.

Regarding the described information, what is the total value that you would include in the project's evaluation? Explain briefly why;

i) A project with a positive NPV very close to zero should be, to be cautious, rejected, in the case it was assessed on the evaluation process that it is a project with a very high risk;

j) The DPP is one of the most used profitability indicators in the project's evaluation process, because it allows you to calculate, with no margin of error, the number of years required to equal the NPV to zero.

Solution

a) It is not irrelevant. The payback period of the investment only includes the project's Cash Flows until the end of that period. If the payback period falls within the last year of the project, when the Residual Value in Working Capital and the disinvestment in CAPEX are considered, then those values become relevant in determining the Discounted Payback Period;

b) The statement is true. The IRR assumes that all intermediate project's Cash Flows are reinvested at the IRR itself. In many cases, particularly when the IRR is high, that may not be possible and the reinvestment rate can be quite lower than the IRR. What initially may seem like a highly profitable project due to this calculation process in the IRR, may in reality lose most of its estimated profitability and become an average project;

c) It depends. By increasing simultaneously in one month the RAT, the PAT and the Tax PAT, there will occur a change in the Working Capital and consequently also a change in the NPV. This change can be:

- Negative (decreasing the NPV) if the company's gross margin is positive;
- Positive (increasing the NPV) if the company's gross margin is negative.

d) Yes, if the disinvestment in the CAPEX presents an accounting profit. Considering a discount rate of zero the Working Capital has no relevance for the project's Cash Flows. The investment value in CAPEX (a cash outflow) is balanced by the sum of the Operating Cash Flows (with an annual value equal to the Depreciation value) and the disinvestment value in CAPEX. If the Selling Value is higher than the Book Value (higher than the amount not yet depreciated), then the NPV is positive. If the Selling Value is equal to the Book Value then the NPV is zero and, if the Selling Value is lower than the Book Value, then the NPV is negative.

e) No, because the present value of the disinvestment would be smaller (decreasing also the NPV) if it is considered in the year following the last year of activity, when compared with the present value of the disinvestment in the last year of activity. Would only be indifferent if the discount rate happened to be zero.

f) Depreciation is only relevant in the evaluation of an investment project by the tax benefit associated with them, allowing the company to pay less taxes. In this case, the reducing-balance method makes the value of the Depreciation to be higher in the early years, decreasing over the following years. This makes the Income Tax lower in the early years and higher in the following. Thus, the Cash Flows during the first years are higher. Such situation will increase the NPV as the discounting factor is lower in the early years.

g) No, due to the Working Capital that can have a negative effect on the value of the NPV. The present value of the changes in WC in the first years is greater than the present value of its Residual Value (due to the discounting factor). Thus, if the present value of the EBIT is not enough to compensate for this difference, the NPV will not be positive.

h) In the first case the value to be included would be the full 8,000 euros, because the project needs this new employee and the other project, where 30% of his time can be allocated, is only a possibility. In the second case, these costs should not be included and are considered sunk costs. These costs happened in the past and thus, their occurrence is independent of whether or not the project will be accepted.

i) The entire risk directly associated with the project has already been previously accounted for in the calculation of the discount rate (higher rates are used when the risk is higher). Therefore, when we get a NPV higher than zero, the project must be accepted regardless of the risk.

j) The statement is incorrect. On one hand the DPP is not a profitability indicator but an indicator of risk. On the other hand, its calculation is done on an annual base and thus is just an approximated value. Moreover, in the case of non-conventional cash flows (more than one change in sign), prudence is required in the calculation of the DPP because there may exist more than one moment when the accumulated discounted Cash Flows is zero.

Follow-up Exercises

EXERCISE 4.20

Comment, justifying, the following statement: «If the Corporate Tax Rate is zero, the NPV determined at constant prices is the same as the one determined in current prices.»

EXERCISE 4.21

Two projects have the same initial investment (only investment) of 100 thousand euros. Project *A* has a NPV of 134 thousand euros, a lifetime of 2 years and constant Cash Flows. Project *B* has a PI of 1.25, a lifetime of 2 years and also constant Cash Flows. Knowing that the required minimum rate of return for both projects is 8%:

a) Which of the two projects is the best to pursuit?

b) Write down the expression that allows you to determine the IRR of project *A*;

c) What is the DPP of project *B*?

EXERCISE 4.22

Consider the following information regarding an investment project (at constant prices and in thousands of euros):

	0	1	2
EBIT		600	700
Working Capital		200	90
CAPEX	600		

Investors demand a nominal rate of return for the project of 9.14%. The annual estimated inflation rate is 2% and the Corporate Tax Rate is 40% (consider that the tax is paid in full in the corresponding year). The CAPEX includes a machine with an economic life of 2 years, after which it can be sold for 60 thousand euros.

a) Identify the Cash Flows for the project at constant prices, considering the Residual Value in Working Capital and the disinvestment value of the CAPEX on the year after the end of the activity;

b) Analyse the economic viability of the project at constant prices.

EXERCISE 4.23

In an investment project, the Cash Flow of year 1, discounted at the IRR, has a value of 200 thousand euros. The Cash Flow of year 2, also discounted at the IRR, is 5% higher than the previous one. The lifetime of the project is 3 years and the Cash Flow of year 3 is 240 thousand euros. The IRR is 10% and the NPV is positive in 50 thousand euros. Write down the expression that allows you to determine the minimum rate of return required by the shareholders.

EXERCISE 4.24

Assume that the Operating Cash Flows and the changes in Working Capital of an investment project, at constant prices (in thousands of euros), are the following:

	1	2	3	4
Operating Cash Flows	100	120	150	200
Changes in WC	10	2	3	5

Regarding the estimated Cash Flows for the project (at constant prices) you know the following:

	0	1	2	3	4	5
Cash Flows for the Project	(60)	?	?	?	?	68

The initial investment consist of a single equipment with an economic life of 3 years and that can be depreciated using the straight-line depreciation method. The Corporate Tax Rate is 20%.

a) Determine the remaining Cash Flows of the project;

b) What is the market value (Selling Value) of the equipment at the end of the project?

c) Analyse the project's economic viability at current prices, considering an estimated annual inflation rate of 2% and a required minimum real rate of return of 10%.

EXERCISE 4.25

Suppose that you are at the middle of an investment project's execution phase and that the Payables Average Term is now higher than the one initially estimated. In your opinion, this deviation has a positive or negative impact on the project's evaluation? Justify.

EXERCISE 4.26

Consider an investment project with the following features (amounts in thousands of euros):

- The initial investment is 210, with an economic life of 3 years;
- Investment undertook three months ago: 40, with an economic life of 2 years;
- Additional investment to be done on year 2: 50, with an economic life of 2 years;
- There is no Working Capital;
- The DPP is 2 years;
- The NOPLAT of the first year is 40 and on the third year is 60;
- Risk free rate: 5%;
- Risk premium: 6%.

a) Compute the NPV for the project;
b) Compute the PI for the project;
c) Write down the expression that allows you to compute the IRR.

EXERCISE 4.27

Consider the following investment project, at constant prices:

Unit: thousands of euros

	Year 1	Year 2	Year 3 and following
EBITDA	50	80	125
Working Capital	(5)	(10)	(8)

The CAPEX consist of the following elements:

Unit: thousands of euros

	Acquisition Value	Depreciation Rate	Selling Value
Equipment *A*	70	50%	20
Equipment *B*	90	33.33%	10
Intangibles	30	33.33%	0

- The project is subject to a Corporate Tax Rate of 20%;
- Consider the residual value in Working Capital and the disinvestment in CAPEX on the year after the end of the activity;
- The project will be analysed using a minimum rate of return required by investors of 15%;
- In case of a loss the firm may carry those losses forward to the following years.

Analyse the economic viability of the project.

EXERCISE 4.28

State if the following sentences are true or false, justifying.

a) Every investment project that, after the evaluation, shows an IRR higher than a Bank's deposit rate, should be accepted;

b) An expansion project to a Middle East country may not be accepted, even with a NPV significantly high;

c) We should consider as a positive Cash Flow of a new project the amount received two months ago from the sale of an obsolete machine. With the new project this machine will be replaced.

EXERCISE 4.29

Complete the following Cash Flows table of an investment project (in thousands of euros):

	2013	2014	2015	2016	2017
Revenues		1,627	1,824	1,965	
Operating Expenses (excludes Depreciation)		432	568	*f)*	
Depreciation		150	*c)*	150	
NOPLAT		836	*d)*	974	
CAPEX	450				*h)*
Changes in WC		*b)*	– 110	60	*i)*
Cash Flow of the Project	*a)*	866	*e)*	*g)*	94

EXERCISE 4.30

A firm has an investment project with a lifetime of 3 years. We have the following information:

- CAPEX on year 0 of 1,000 euros;
- The discounted value of the Cash Flow of year 1 is 370.37 euros and of year 3 is 158.77 euros;
- The DPP is 1.918 years.

a) Compute the NPV for the project;

b) In addition to the previous information we now know the following:

— The PI is 1.19803;

— The discount rate is 8%;

— The Cash Flow of year 2 is 800 euros and this includes an additional investment in CAPEX.

Determine the additional investment in CAPEX to be undertaken on year 2.

c) Assume that under an optimistic scenario there is a strong probability that the Labour Costs at the end of the first year will be lower than the estimated value in 300 euros. What would be the change in the project's NPV, relative to the initial setting, assuming a Corporate Tax Rate of 40% and the inexistence of Working Capital?

The main goal of this book is to explain the solutions to several exercises on relevant financial topics, from corporate financial analysis and financial markets, project investment analysis and the corresponding evaluation tools, to the mechanics of time value of money. The restrictions imposed by the intended academic focus lead to a simplification of the included exercises relative to real life, which allows for their resolution in less time. In accordance, we felt the need to complement the exercises with a theoretical overview, in each chapter, of the main financial principles that give support to most decision making in a financial environment. The financial theory must avoid certain deformed ideas and concepts of easy gains and greed that usually are imbedded in the human nature. We focus on providing the fundamentals behind the true financial logic and we add to the exercises practical alerts that complement the conclusions and calculations provided.

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