

Microeconomics

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The Theory of the Firm and Market Structure: Production

Production Function I

What is production?

- Any activity that creates present or future utility
- A process that transforms inputs (factors of production) into outputs.
- Factors of production:
 - Traditionally: land, labor, capital, entrepreneurship.
 - More recent: knowledge, technology, organization, governance, energy etc.

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Definition

Production function – the relationship that describes how inputs are transformed into output.

- Production function is a recipe!

Production Function II

- Mathematical representation:

$$Q = F(K, L)$$

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- E.g.

$$Q = 2KL$$

		Labor (person-hours/wk)				
		1	2	3	4	5
Capital (equipment-hours/wk)	1	2	4	6	8	10
	2	4	8	12	16	20
	3	6	12	18	24	30
	4	8	16	24	32	40
	5	10	20	30	40	50

Inputs I

Definition

Long run – the shortest period of time required to alter the amounts of all inputs used in a production process.

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Definition

Short run – the longest period of time during which at least one of the inputs used in a production process cannot be varied.

Inputs II

Definition

Variable input – an input that can be varied in the short run.

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Fixed input – an input that cannot vary in the short run.

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Inputs II

Definition

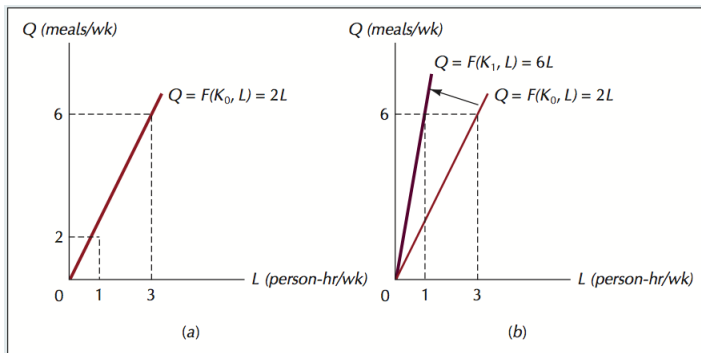
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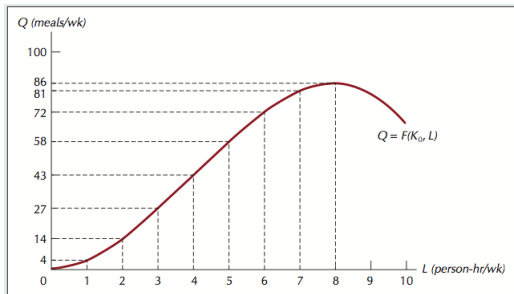
- E.g. Buildings, production lines etc.
- In the long run, all inputs are variable inputs, by definition.

Production in the Short Run I



Production in the Short Run II

Typical shape of the short-run production function:



Total, Marginal and Average Products I

Definition

Total product curve – a curve showing the amount of output as a function of the amount of variable input:

$$Q = F(K_0, L)$$

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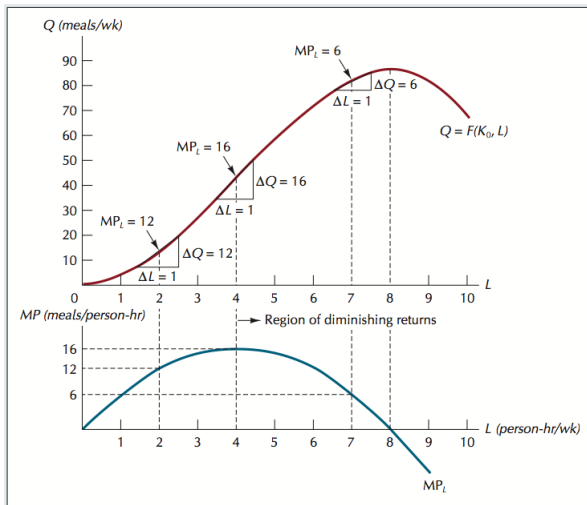
Definition

Marginal product – change in total product due to a 1-unit change in the variable input:

$$MP_L = \frac{\Delta Q}{\Delta L}$$

- Geometrically, the marginal product at any point is the slope of the total product curve at that point.

Total, Marginal and Average Products II



Total, Marginal and Average Products III

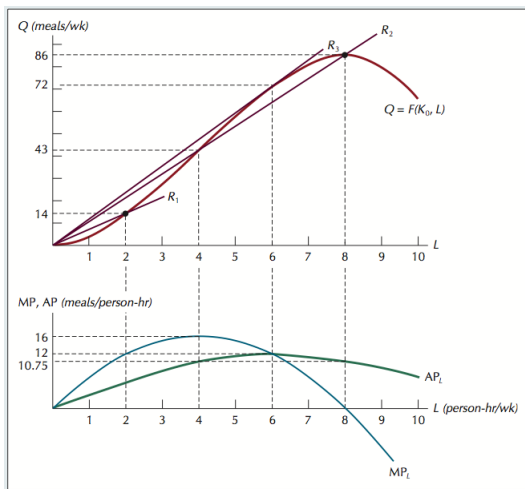
Definition

Average product – total output divided by the quantity of the variable input:

$$AP_L = \frac{Q}{L}$$

- Geometrically, the average product is the slope of the line joining the origin to the corresponding point on the total product curve.

Total, Marginal and Average Products III



Total, Marginal and Average Products IV

- When the marginal product curve lies **above** the average product curve, the average product curve must be **rising**;
- When the marginal product curve lies **below** the average product curve, the average product curve must be **falling**.
- The two curves intersect at the maximum value of the average product curve.

Production in the Long Run I

- In the long run all factors of production are by definition variable
- Consider again:

$$Q(K, L) = 2KL$$

- Solve for $Q = 16$ to get

$$K = \frac{8}{L}$$

For $Q = 32$ it is $K = \frac{16}{L}$ etc.

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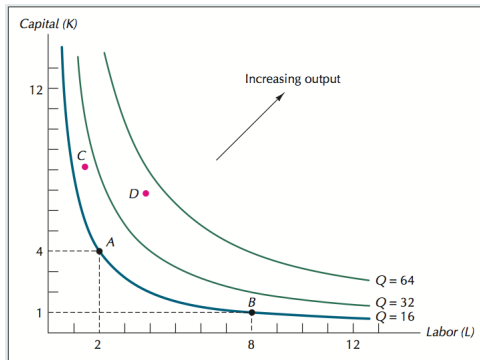
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Definition

Isoquant – the set of all input combinations that yield a given level of output.

Production in the Long Run II



- Movements to northeast on an isoquant map correspond to increasing levels of output.
- Input bundle on an isoquant yields more output than any input bundle that lies below that isoquant.

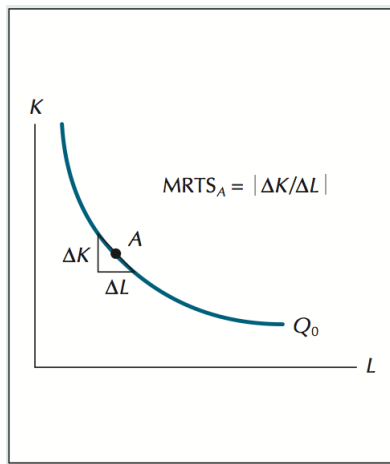
Production in the Long Run III

Definition

Marginal rate of technical substitution (MRTS) – the rate at which one input can be exchanged for another without altering the total level of output:

$$MRTS_x = \left| \frac{\Delta K}{\Delta L} \right|$$

Production in the Long Run IV



Production in the Long Run V

- For small increments we have

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Production in the Long Run V

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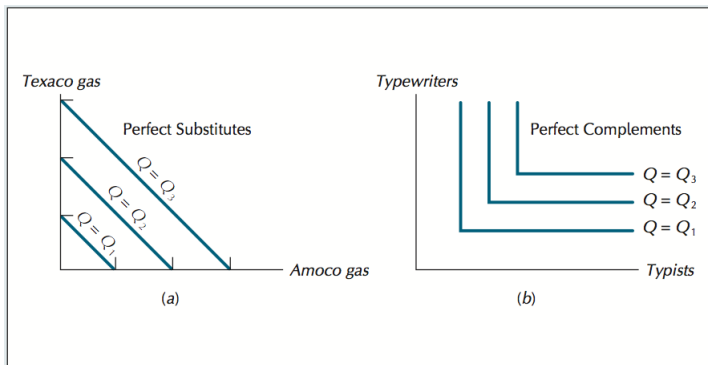
- We have

$$MP_K = \frac{\partial Q}{\partial K} \quad \text{and} \quad MP_L = \frac{\partial Q}{\partial L}$$

- Thus, by chain rule it holds that

$$\frac{\partial K}{\partial L} = \frac{\partial K}{\partial Q} \cdot \frac{\partial Q}{\partial L} = \frac{MP_L}{MP_K}$$

Production in the Long Run VI



Returns to Scale I

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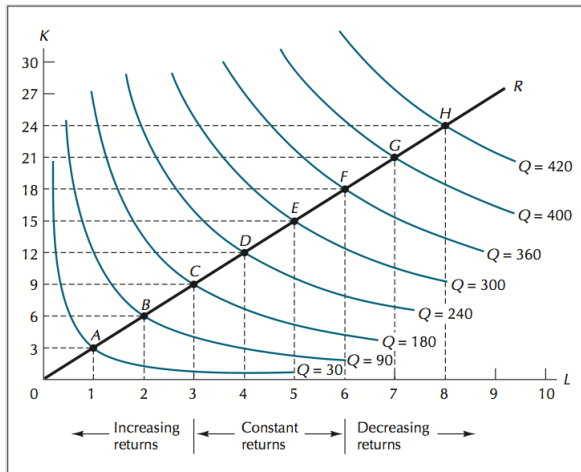
Returns to Scale I

- What happens to output if we increase all factors of production proportionally?

$$Q(tK, tL) = T \cdot Q(K, L)$$

- $T > t$ – increasing returns to scale
- $T = t$ – constant returns to scale
- $T < t$ – decreasing returns to scale

Returns to Scale II

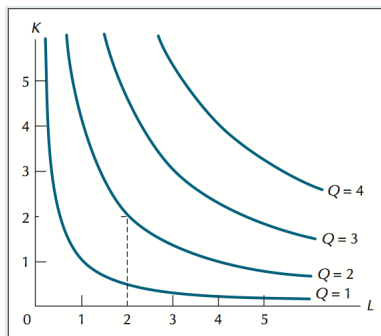


Typical Production Functions I

Cobb–Douglas Production Function:

$$Q = mK^{\alpha}L^{\beta},$$

where $\alpha, \beta \in (0, 1]$ and $m > 0$



Typical Production Functions II

Leontief Production Function:

$$Q = \min[aK, bL]$$

