

Problem Set 5

Key concepts:

- Production function
 - Isoquant
 - Marginal rate of technical substitution (MRTS)
 - Economies of scale and diseconomies of scale
1. Discuss: An isoquant is a collection of input combinations that are equally profitable. (yes or no?)
 2. The productive process of a given firm is represented by:

$$Q = 10KL^2 - (KL)^3$$

Capital is fixed in the short run and is $K_0 = 1$.

- (a) Calculate the average productivity of labour and the marginal productivity of labour.
 - (b) What level of L :
 - i. maximizes production in the short run?
 - ii. maximizes average productivity?
 - (c) Identify the optimal region of production.
3. A daily production function for calculators is

$$Q = 12L^2 - L^3$$

- . Show all your work for the following questions.
- (a) What is the marginal product equation for labor?
 - (b) What is the AP_L function?
4. The production function of a firm is:

$$Q = 2K^{0.5}L^{0.5}$$

- (a) Are we in the short or in the long run?
- (b) Find the general expression of the isoquant map.
- (c) Calculate the MRTS. What is its value when $Q = 2$ and $L = 3$?
- (d) What returns to scale are implicit in the function and do they mean.

(e) What are the returns to scale in following cases?

$$Q = 2K^{0.2}L^{0.5}$$

$$Q = 2K^{0.5}L^{0.7}$$

5. Consider the following production functions (M denotes machinery and L labor):

(a) $Q = M + 2L$

(b) $Q = \min[M; 0.5L]$

- i. Draw the isoquants for each case.
- ii. What is the partial production function of labor? Calculate the marginal and average products of labor.
- iii. What are the returns to scale for each case?

6. The BMW company has doubled the number of workers employed in assembling cars in their factory in Munich. The number of cars produced rose, however only by 50%. Can you conclude that the production of cars is characterized by decreasing returns to scale?