University Institute of Lisbon

Departament of Mathematics

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Group work

1. Being $E = \{e_1, e_2\}$ the set of states characterizing the market conditions of company A, with e_1 corresponding to the market at a *good* state and e_2 corresponding to the market at a *bad* state. Consider

$$p_A(t) = \begin{bmatrix} p_A(e_1|T = t) \\ p_A(e_2|T = t) \end{bmatrix},$$

the probability for each one of the possible states at moment $T = t$, where T is the variable time. It is known that in the initial moment the probability for the market to be at a *good* state or a *bad* state is the following:

$$p_A(e_1|T = 0) = 0.4\,, \qquad p_A(e_2|T = 0) = 0.6.$$

The *probability vector for the two states* at the initial moment is given by:

$$p_A(0) = \begin{bmatrix} 0.4 \\ 0.6 \end{bmatrix}.$$

The transition between the states of company A from moment t to moment $(t+1)$ is characterized by a *Markov Process* with the following *transition matrix*:

$$P_A = \begin{bmatrix} 0.3 & 0.2 \\ 0.7 & 0.8 \end{bmatrix}.$$

(a) Determine the probability vector for the states at $t = 1$.

(b) Determine powers of P_A such that the powers of the transition matrix converge to a constant matrix. Determine that matrix and comment on the result.

(c) Indicate the expression of the transition matrix from moment t to moment $(t + n)$. Compare the result obtained in (b).
(Use the diagonalization of P_A to justify your answer)

(d) What is the stationary state of the probability vector for the states? Interpret the result obtained according the eigenvalues and the eigenvectors.

(e) Suppose now that the *gains* of company A at state e_1 is 1 and at state e_2 is -1, i.e.,

$$g_A(e_1) = 1\,, \qquad g_A(e_2) = -1.$$

Being $g_A = \begin{bmatrix} 1 & -1 \end{bmatrix}$. The *expected value of the gains in each moment* t is determined as:

$$E(G_A|T=t) = g_A(e_1)p_A(e_1|T=t) + g_A(e_2)p_A(e_2|T=t),$$

where G_A represents the gains of company A.

(i) Write $E(G_A|T=t)$ as a product of vectors.

(ii) Determine the expression of $E(G_A|T=t)$.

2. It exists another company B for which the market conditions are associated with the market conditions of company A. Consider $P_B = [b_{ij}]$, where b_{ij} represents the probability of the state of company B to be e_i when the state of company A is e_j $(i, j = 1, 2)$. The matrix is given by:

$$P_B = \begin{bmatrix} 0.5 & 0.4 \\ 0.5 & 0.6 \end{bmatrix}.$$

(a) What is the probability vector of the states of company B at the initial moment?

(b) Consider that the gains of company B in each of the states are given by:

$$g_B(e_1) = \frac{1}{2}\,, \qquad g_B(e_2) = -\frac{2}{3}.$$

Being $g_B = \begin{bmatrix} \frac{1}{2} & -\frac{2}{3} \end{bmatrix}$.

(i) Calculate the expected value of the gains of company B at the initial moment, i.e., $G_B(0)$.

(ii) Determine the expression of the expected value of the gains for company B at moment t, i.e., $G_B(t)$.
(It is not necessary to present the calculations)