ISCTE IUL
Instituto Universitário de Lisboa
Lisbon University Institute

Mathematics (Year 1)

BSc in Management

27/01/2014

2nd Final Exam

Total time length of the exam: 2h30m

Name *(in full)*: ……………………………………………………………………………………………………

(Block letters)

Student id.: ………………… Class: Ga i ………..

Lecturer: ……

- Use no calculator or other electronic means of calculation.
- Use only black or blue ink ball point pen.
- During the test, all mobile devices must be switched off.
- No doubts will be entertained.
- Maintain intact the booklet. Violating the booklet will disqualify the student to go on.
- Use only the reserved spaces for your answers. Present justification to your results whenever applicable.
- Use no more than the allowed page for drafts. If you need to use the draft page for your answers, please signal it clearly.

Reserved for marking.

1.a ___	5.a ___
1.b ___	5.b ___
2. ___	6. ___
3. ___	
4.a ___	7.a ___
4.b ___	7.b ___
4.c ___	

1. Consider the following system of linear equations:

$$\begin{cases} ax + 2y + (a+1)z = 0 \\ x - y + z = 1 \\ -x + y + (a+1)z = b \end{cases}$$

a) Discuss the solution set based on the parameters of *a* and *b*.

[*2.0 valores*]

b) For $a = 0$ e $b = -1$, solve the system by using the Cramer's Rule

[*2.0 valores*]

2. Let $(I + 2A)^{-1} = \begin{bmatrix} -1 & 2 \\ -4 & 5 \end{bmatrix}$. Determine the matrix A.

[2.0 valores]

3. Consider the set of vectors in $\Re^3$ S= {(1,1,3), (1,0,-2), (2,1,k+1)}. Determine os values of *k* such that S forms a basis in $\Re^3$.

[*1.5 valores*]

4. Consider the linear transformation T: $\mathfrak{R}^3 \rightarrow \mathfrak{R}^3$

$$T(x, y, z) = (ax, x + by, x + y + z)$$

where *a* and *b* are real numbers.

a) Write the transformation matrix of T in the unitary basis.

[*1.5 valores*]

b) Determine *a* and *b* such that the eigenvalues of T would have the algebraic multiplicity of 3. In this situation, determine the respective eigenvectors set.

[*1.5 valores*]

c) Determine the kernel of the linear transformation and indicate the dimension of it. For those who have not solve the previous question, consider $a = b = 2$.

[*1.5 valores*]

5. Consider the function *f:* $D \subset \Re^2 \rightarrow \Re$ defined as follows:

$$f(x,y) = \frac{xy}{x+2} \text{ and } f(0,0) = 1$$

a) Figure out the domain of definition of *f* and represent it graphically.

[*1.5 valores*]

b) Study *f* in terms of continuity and differentiability in the origin of the axes.

[*1.5 valores*]

6. Let $z = x^{\alpha} g\left(\frac{y}{x}\right)$ and α is a constant. Evaluate the $\frac{\partial z}{\partial x}$

[*2.0 valores*]

7. Consider the function $f(x,y) = 3 + \frac{(x^2-y^2)}{(x^2+y^2)}$

a) Determine the degree of homogeneity of f.

[*2.0 valores*]

b) What is the degree of homogeneity of $\frac{df}{dx}$. Justify your answer.

[*1.0 valores*]

Drafts