ISCTE IUL
Instituto Universitário de Lisboa
Lisbon University Institute

Mathematics (Year 1)

BSc in Management

13/01/2014

Final Test	**Part II** (Questions 4, 5, 6 e 7)	Max. Time length allowed: 1H 30m
Final Exam	**Part I + Part II** (Questions 1,2,3,4,5,6 e 7)	Max. Time length allowed: 2H 30m

Name (in full): ....
(Block letters)

Student id.: ....   Class: Ga i ....

Lecturer: ....

- Use no calculator or other electronic means of calculation.
- Use only black or blue ink ball point pen.
- During the test, all mobile devices must be switched off.
- No doubts will be entertained.
- Maintain intact the booklet. Violating the booklet will disqualify the student to go on.
- Use only the reserved spaces for your answers. Present justification to your results whenever applicable.
- Use no more than the allowed page for drafts. If you need to use the draft page for your answers, please signal it clearly.

Reserved for marking.

Part I

1) a)	
b)	
c)	
2) a)	
b)	
c)	
3) a)	
b)	

Part II

4) a)	
b)	
c)	
5) a)	
b)	
c)	

6) a)	
b)	
c)	
7) a)	
b)	

PART I (FINAL EXAM ONLY)

[4.0 valores] 1. Consider the matrices:

$$A = \begin{bmatrix} -1 & 1 & 2 \\ 1 & -1 & -a \\ 1 & 0 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ b \\ 1 \end{bmatrix} \quad \text{e} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix},$$

where $a, b \in \mathbb{R}$.

(1.5) a) Discuss the nature of the solution system of $AX = B$ according to the parameters a and b.

(1.5) b) For what values of a the matrix A is invertible? Consider $a = 1$ and calculate the inverse matrix of A.

(1.0) c) Consider that $a = 1$ and $b = 0$, determine the solution of the system $AX = B$, using the method of your preference.

[4.0 valores] 2. Let S a subspace of $\mathbb{R}^3$ spanned by the vectors:

$$\{(1,1,2),\ (1,2,0),\ (1,5,-6),\ (1,3,-2)\}.$$

(1.5) a) Present a basis for S and indicate its dimension.

(1.5) b) What are the coordinates of the vector $(1,3,-2)$ in this basis?

(1.0) c) Using the above mentioned vectors, find a basis for $\mathbb{R}^3$. Justify your answer.

[2.0 valores] 3. Let B a matrix 20×20. B is regular.

(1.0) a) What is the result of the product of row 7 of matrix B by the column 4 of the matrix B^{-1}. Justify your answer.

(1.0) b) Determine $\left|\hat{B}\right|$.

PART II (FINAL TEST/ EXAM)

(the quotation for final test is 2x the printed figure)

[2.5 valores] 4. Consider the linear transformation $T\colon \mathbb{R}^3 \longrightarrow \mathbb{R}^3$:

$$T(x,y,z) = (x+z, y, x+z).$$

(0.5) a) Write the matrix that represents the transformation T in the unitary basis of $\mathbb{R}^3$.

(1.0) b) Consider the following basis in $\mathbb{R}^3$:

$$\mathcal{B} = \{(1,0,1), (-1,0,1), (0,1,0)\}.$$

Determine the matrix of transformation T relatively to the basis $\mathcal{B}$.

(1.0) c) Can T be diagonalized? What can you conclude about the eigenvalues and eigenvectors of T?

(Suggestion: In case you have not solved the question b) you may solve from the question a))

[3.0 valores] 5. Consider the function $f: D \subset \mathbb{R}^2 \longrightarrow \mathbb{R}$ defined as follows:

$$f(x,y) = \begin{cases} \dfrac{\sin x - \sin y}{x+y} , & (x,y) \neq (0,0) \\ 0 , & (x,y) = (0,0) \end{cases}$$

(1.0) a) Present the domain of definition of f. Provide also a graphic representation of the same.

(1.0) b) Calculate, if it exists, the limit of $f(x,y)$ at the point $(0,0)$.

(1.0) c) Study f about its continuity and differentiability at the point $(0,0)$.

[3.0 valores] 6. Consider the function $f: \mathbb{R}^2 \longrightarrow \mathbb{R}$ defined as follows:

$$f(x, y) = x \sin y + y \cos x.$$

(1.0) a) Evaluate the gradient of f at $(0,0)$.

(1.0) b) Using the value of $f(0,0)$, calculate by approximation the image of the point $(0.01;\ -0.01)$.

(1.0) c) Calculate the hessian matrix of f at the point $(0,0)$.

[1.5 valores] 7. Consider the function $g\colon \mathbb{R}^2 \longrightarrow \mathbb{R}$ defined as:

$$g(x,y) = x^2h(x) + y^2h(y),$$

where $h\colon \mathbb{R} \longrightarrow \mathbb{R}$ is a differentiable function and homogeneous of degree 2.

(1.0) a) Verifies that $xg'_x + yg'_y = 4g$.

(0.5) b) What can you tell about the equality of the above question?

Drafts