EN358 Ship Structures
Notes for an Undergraduate Course

Spring 2009

(Revised December 2008)

Naval Architecture Program
United States Naval Academy
Annapolis, Maryland

Preface

Ships are the largest manmade moving objects on earth. The tanker SS Jarhe Viking, for instance, is 1502 feet (nearly six city blocks, or twice the distance from Mahan to Bancroft) long! She is longer than the tallest building in the world is high. Her draft makes her too big to pass through either the English Channel or the Suez Canal. She is designed to withstand wind and wave forces found anywhere at sea. Just imagine the forces generated when a structure twice the size of Luce Hall runs into a wave the size of Rickover! As these waves are fairly common in the world's oceans, imagine doing it thousands of times! The paper clip fatigue example takes on a whole new meaning.

While ships are massive, their structures may not be. Hull plating may range from 6" thick steel plating on warships to 1 mm thick carbon fiber laminates on racing vessels. Low weight is often critical to performance and economy so structures are often designed with small margins. Structural failures are relatively common and are, on rare occasions, catastrophic enough to cause the vessel to founder. Due to environmental and economic reasons, ships are rarely in service more than 25 to 30 years (although some Coast Guard's cutters and Navy aircraft carriers buck this trend). These long lives mean that fatigue and corrosion are common reasons for structural failure. Other common reasons are a lack of quality control in material properties and fabrication and insufficient maintenance.

In ship structural design, the naval architect is challenged by the large and uncertain forces, the realities of quality control in a shipyard, the ripple influence of their decisions and economics. A simple decision such as whether a deck plate should be 3/8" or the slightly thicker 10 mm will greatly influence stability and cost, particularly when the main deck of a tanker can be four acres in size. Adding to the naval architects' concern is the applicability of the available analytical models. Traditionally, naval architects use structural design guides and rules formulated by non-governmental Classification Societies. These guides are based on fundamental structural theories combined with empirically derived adjustments. The guides do not cover all the required structure however, and are not considered sufficient for a complete design. The naval architect must use the available theories correctly to avoid either wasting the owner's money or causing a structural failure. More frequently, the guides are used to complement finite element analysis (FEA). This tool allows the naval architect to combine multiple theories simultaneously in a graphical interface. While it has a steep learning curve, FEA can yield more efficient structure more quickly than other methods.

EN358 uses material previously learned in statics, dynamics, material science, strength of materials, stability and buoyancy, and principles of naval architecture. Hopefully the student kept their texts and notes! These course notes are designed to *supplement* material presented on the board and in Chapter 4 of the book, "Principles of Naval Architecture, Vol. 1" (PNA). Included are administrative materials, the assignments, and basic notes. You should bring these notes to every class and lab! The notes also have some questions, blank spaces for equations, and sample exercises which the student should complete as a study aid. You should print these notes and put them in a 3-ring binder. Add in your work and additional notes in the appropriate spots during the semester, and you will have a complete reference for ship structural design!

The material covered in this class is heavily used in the capstone ship design classes and the marine fabrication and salvage electives. There will be times when you are going through the book and this reader that the concepts or techniques are not that easy to understand. Don't forget, a good engineer asks questions when they are not clear on a concept!

Additional thanks to Prof. Greg White of the Naval Academy and to Capt. Bill Simpson, USCG of the Coast Guard Academy for information, guidance and encouragement while putting these notes together.

P.H. Miller J.W. Stettler

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Class Schedule (Spring 2009)

Week	Date	Day	Topics
1	7 Jan	Wed	Course Introduction
	9 Jan	Fri	Ship Structural Components, Ship Structural Design Process
2	12 Jan	Mon	Structural "Failure"
	13 Jan	Tue	Lab Exercise: Ship Structural Design Basics (Weekly Assignment #1)
	14 Jan	Wed	Ship Structural Loads
	16 Jan	Fri	Hull Girder Bending Concept
3	19 Jan	Mon	***ML King Jr. Day (no class)***
	20 Jan	Tue	Weight Curves for Ships ***(Monday schedule)***
	21 Jan	Wed	Lab Exercise: Boundary Conditions & Hull Girder Analysis (Weekly Assignment #2)
	23 Jan	Fri	Buoyancy Curves for Ships
4	26 Jan	Mon	Moment of Inertia and Section Modulus Calculation
	27 Jan	Tue	Lab Exercise: Weight Curve & Hull Section Modulus Calculation for Mariner Class (Weekly Assignment #3)
	28 Jan	Wed	Composite Beam Approach
	30 Jan	Fri	Shear Stress in Small Open Beam Sections
5	2 Feb	Mon	Shear Stress and Shear Flow in Closed and Large Beam Sections
	3 Feb	Tue	Lab Exercise: Hull Bending Analysis for Mariner Class (Weekly Assignment #4)
	4 Feb	Wed	Shear Stress and Shear Flow in Closed and Large Beam Sections (continued)
	6 Feb	Fri	Asymmetric Bending: Asymmetric Loading and Asymmetric Sections
6	9 Feb	Mon	Shear Lag and "Effective Breadth"
	10 Feb	Tue	Lab Exercise: 4-Point Beam Bending Lab & Cable Guide T-Stiffener Analysis (Weekly Assignment #5)
	11 Feb	Wed	Shear Lag and "Effective Breadth" (continued)
	13 Feb	Fri	Hull–Superstructure Interaction
7	16 Feb	Mon	***Presidents' Day (no class)***
	17 Feb	Tue	Lab Exercise: Balsa Beam Design Project (Weekly Assignment #6)
	18 Feb	Wed	Introduction to the Finite Element Analysis Method
	20 Feb	Fri	Stiffness Matrix Method – Bar Elements
8	23 Feb	Mon	In-class Exercise: Propulsion Shaft Analysis Using the Stiffness Matrix Method
	24 Feb	Tue	Lab Exercise: Balsa Beam Testing & FEA Beam Bending Analysis (Weekly Assignment #7)
	25 Feb	Wed	Stiffness Matrix Method – Truss Elements & Coordinate Transformations
	27 Feb	Fri	Plate Bending, Small Deflection Plate Theory
9	2 Mar	Mon	In-class Exercise: Stiffened Plate Salvage Patch Design
	3 Mar	Tue	Lab Exercise: Small Deflection Plate Bending (Weekly Assignment #8)
	4 Mar	Wed	Large Deflection Plate Bending – Membrane Stresses
	6 Mar	Fri	Plates Loaded Beyond Their Elastic Limit – Plasticity, Load Shedding, Hinges
10	9 Mar	Mon	Plates Loaded Beyond Their Elastic Limit – Design for Permanent Set
	10 Mar	Tue	Lab Exercise: Stiffened Panel Bending (Weekly Assignment #9)
	11 Mar	Wed	Stiffened Panels in Bending
	13 Mar	Fri	Stiffened Panels in Bending (continued)
*	16-20 Mar	*	***Spring Break (no class)***
11	23 Mar	Mon	Buckling & Column Design – "Ideal" Columns
	24 Mar	Tue	Lab Exercise: Stiffened Panel Design Project (Weekly Assignment #10)
	25 Mar	Wed	Buckling & Column Design – Eccentricity
	27 Mar	Fri	Beam-Columns
12	30 Mar	Mon	Columns & Beam-Columns – Application to Stiffener Design
	31 Mar	Tue	Lab Exercise: Stiffened Panel Construction, Stanchion Design (Weekly Assignment #11)
	1 Apr	Wed	Elastic Plate Buckling – Uniaxial Compression
	3 Apr	Fri	Other Plate Buckling – Biaxial Compression & Shear
13	6 Apr	Mon	Plates Subject to In-plane Compression and Lateral Loads
	7 Apr	Tue	Lab Exercise: Stiffened Panel Testing, Midship Design Project Procedure (Weekly Assignment #12)
	8 Apr	Wed	In-class Exercise: Ship Structural Design Process
	10 Apr	Fri	Ultimate Strength of Plates
14	13 Apr	Mon	Elastic Buckling of Stiffened Panels
	14 Apr	Tue	Lab Exercise: Midship Design Project, FEA Buckling of Plates/Panels (Weekly Assignment #13)
	15 Apr	Wed	Stiffener Tripping
	17 Apr	Fri	Buckling of a Stiffened Panels
15	20 Apr	Mon	Classification Society Design Guides and Rules
	21 Apr	Tue	Lab Exercise: Midship Design Project (Weekly Assignment #14)
	22 Apr	Wed	Introduction to Welding and Weld Design
	24 Apr	Fri	Introduction to Welding and Weld Design (continued)
16	27 Apr	Mon	Introduction to Structural Reliability
	28 Apr	Tue	Course Wrap-up, Review *(Spring term ends)*

Course Objectives

Upon completion of this course, the student should be able to:

1. Perform a preliminary structural design of a ship. This includes demonstrating a basic understanding of the sources of structural loads, types and control of material stresses, primary and secondary structural failure modes, classification society rules, factors of safety, and materials selection.

2. Apply basic hull girder analysis for the design of a ship structure, including calculations of vertical global hull girder bending loads, section modulus, and bending stresses.

3. Apply basic concepts of shear stresses in ship primary and tertiary structures, including shear flow and shear lag effects.

4. Apply basic concepts for the bending of beams, plates, and stiffened panels as applied to a ship structure.

5. Apply basic concepts for the buckling of columns, plates, and stiffened panels as applied to a ship structure.

6. Apply basic concepts of matrix stiffness and finite element analysis to the design of a ship structure.

Course Policy Statement (Spring 2009)

Instructor:

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1. **General:** One of the basic tenets of naval architecture is that the ship should be strong enough to survive the environment in which it is intended to operate. This course will educate you in the structural theory and practical application to ships and other marine structures. Topics include longitudinal and transverse strength of the hull girder, bending moments in a seaway, beam and plate theory and application, development of a ship's structural design, computer-based methods of structural analysis, and application of shipbuilding materials.

2. **Background:** This course builds on the topics you learned in principles of naval architecture, statics, dynamics, strength of materials and materials science. If you are weak in those areas, you should review that material or stop by for some EI!

3. **Course Notes and Reference Book:** These course notes provide much of what you will need to succeed in this course – including concepts, examples, and assignments. These course notes are posted on the course Blackboard page. Print out the requisite sections (see the class schedule), and bring them to each class (use a 1 ½ inch 3-ring binder, as you will need to add your own notes, assignment materials, etc.). In addition to these course notes, the *required* reference book is *Principles of Naval Architecture, Vol. 1, Chapter 4 (Strength of Ships) (PNA)*, published by the Society of Naval Architects and Marine Engineers (SNAME), Jersey City, NJ. Become a student member of SNAME and get a big discount! You must bring these course notes to each class. The student is expected to read and understand the corresponding sections of the course notes and reference book.

4. **Grading Policy:** The course grading breakdown is as follows:
   Weekly quizzes – 20%
   Final exam – 20%
   Weekly assignment – 20%
   Beam design project – 10%
   Stiffened panel design project – 10%
   Midships section design project – 20%

5. **Assignments:** A hallmark of engineering is cooperation between engineers to solve problems. You are therefore encouraged to work in small groups for assignments. When appropriate, use an engineering problem-solving approach: i.e. Problem Statement, Sketch & Free Body Diagram, Principles & Relations, Given & Assumptions, and Solution. (i.e., a step-by-step method from problems statement to answer – see handout "Naval Architecture Program Homework Format"). For all deliverables, use green engineering paper (or use type-written (computer-generated) print as desired/required) and be sure to box or highlight each answer and perform a common sense check on its magnitude and units. Include a cover sheet on all submittals. Be sure to cite your references, and any collaboration. Reflecting the "real world", late work will incur a penalty at 25% per work day for a maximum of 50%. If you turn in an assignment late, write at the top "that it is late", and include date and time, and a reason. If it is a very good (or creative) reason, you might have the penalty waived.

   Weekly assignments will include work performed during lab periods, during lecture periods, and outside of class. Each student must turn in *individual* submissions of their weekly assignments. Photo copies will not be accepted. This is to ensure that each student has all the material needed for the quizzes and final exam.
   For each of the team projects, each team will turn in one copy of the submittal. However, team members should make sure that each team member has a copy of the submission, as the project concepts may be tested on the quizzes and the final examination.

6. **Exams and Quizzes:** To test your knowledge and comprehension of the course material, examinations will take the form of weekly short quizzes (typically held at the beginning of the lab periods) and a final exam. The weekly short quizzes will focus on the most recent material presented in class, in the course notes, and assigned reading in PNA,

and will replace the traditional 6 and 12 week exams. Some of the quizzes might have cumulative material. The final exam will be cumulative and will occur during the scheduled final exam period. For any quiz or final exam, calculators are permitted (and will be needed). Reflecting actual engineering practice, all quizzes and the final exam will be open-book and open-notes, however keep in mind that there will be a strict time limit – so don't plan on having time to "hunt" for material during the quizzes. Keep up with the reading (course notes and reference book), and you will have little problem.

7. **Design Projects:** You will conduct three structural design team projects. Each project will involve work spread over a number of weeks. The first will be the design, construction and testing of a balsa beam. The second will involve the design and construction and verification of a stiffened panel ship structure! The final project will involve the comprehensive design of a ship's midships structure.

8. **Calculators:** Ensure you have a working calculator *every time* you attend class or lab!

9. **Absences:** If you are absent, it is to your advantage to contact me beforehand. You are responsible for the material discussed, for any assignments handed-out during your absence, and for making arrangements to make-up any missed quizzes, labs or class exercises. Unless prior arrangements are made, you are also responsible for turning in any work due the day of your absence. If you have received permission ahead of time to turn in an assignment late, note that on the top of the submission (on the title page).

10. **Extra Instruction:** Extra instruction is encouraged! If you do not understand a concept, discuss it with one your peers or contact me for EI. For EI, contact me by e-mail or phone, so a mutually agreeable time can be arranged.

EN358 Midship Design Final Submission Evaluation

This is provided for your planning and guidance. Use this in conjunction with the detailed assignments pertaining to the project. Provide a copy of this with your final submission.

Team Members:

____

____

____

Note: the first three grades reflect the earlier submissions.

1.	Submission 1: Preliminary Procedure	15 points	_______
2.	Submission 2: Moments and Section Modulus	15 points	_______
3.	Submission 3: Bottom Plating	15 points	_______
4.	Executive Summary/Principal Dimensions	2 pts	_______
5.	Final Design Procedure	5 pts	_______
6.	Weight Spreadsheet	5 pts	_______
7.	Plots (Hog, Sag, Still Water, Body)	5 pts	_______
8.	Calculations		
	Section Modulus	5 pts	_______
	Global Hull Girder	3 pts	_______
	Frame Spacing	5 pts	_______
	Internals Scantlings	5 pts	_______
9.	Weight Estimate (per foot)	5 pts	_______
10.	Midship Construction Drawing (CAD)	5 pts	_______
11.	Specifications	5 pts	_______
12.	Evaluation Factor (misc)	5 pts	_______

Total _______/100

Comments:

Sample Peer Evaluation Form

Note: these must be submitted electronically. See the course Blackboard page for the web-link.

Please print the names of all of your team members and evaluate (including yourself) the degree to which each member fulfilled their responsibilities to the project. These ratings should reflect each individual's level of participation, effort and sense of responsibility, and NOT their academic ability. Be as honest as possible.

The possible ratings are:

Good: (Note: this is the "standard" category for a group member with no significant plusses or minuses.) Consistently did what they were supposed to do. Acceptably prepared and cooperative. Attended almost all meetings, and had good excuses for those missed.

Very Good: Always did their assignments. Always very well prepared and cooperative. Always at team meetings.

Excellent: Consistently went above and beyond their share of the project. For example, they may have tutored their team mates or went well beyond what was expected of them. (Note: this rating can only be applied to one member of a team, and comments must be included below.)

Satisfactory: Usually did what they were supposed to. Minimally prepared and cooperative.

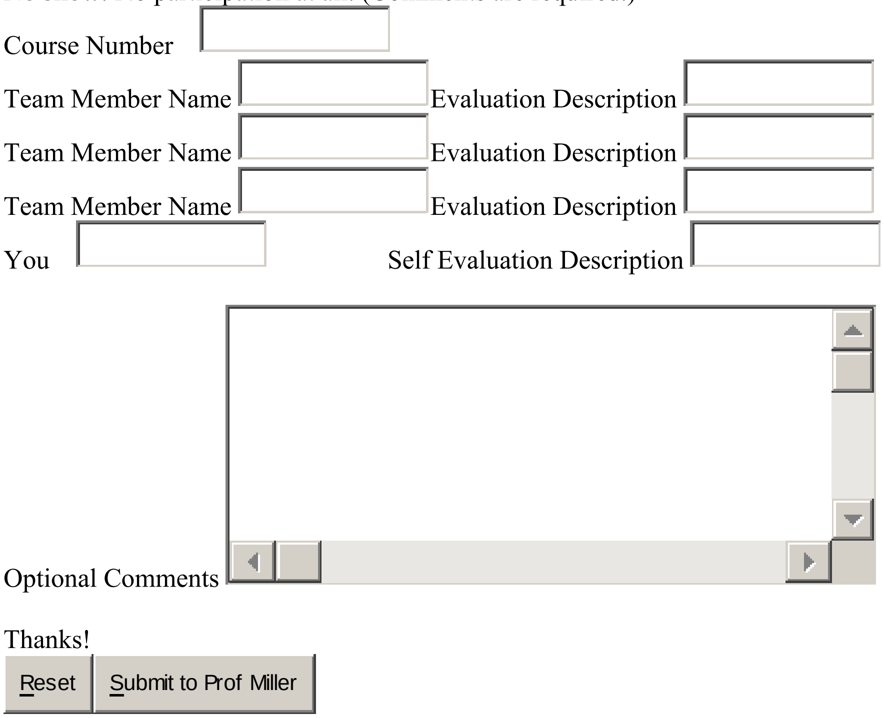
Marginal: Sometimes failed to show up or complete assignments with only lame excuses. Prepared about half the time. Other teammates had to complete some of their assignments on rare occasions.

Deficient: Often failed to show up or complete assignments. Usually unprepared and uncooperative. Required extra work by teammates. (Comments are required.)

Unsatisfactory: Consistently failed to show up or complete assignments, unprepared and uncooperative most of the time. (Comments are required.)

Superficial: Practically no participation. (Comments are required.)

No show: No participation at all. (Comments are required.)



Course Number

Team Member Name Evaluation Description

Team Member Name Evaluation Description

Team Member Name Evaluation Description

You Self Evaluation Description

Optional Comments

Thanks!

Ship Structures: An Overview

The successful ship structure designer learns to efficiently balance the multitude of variables inherent in such an open-ended design project. For instance, thousands of different materials could be used in the vessel's construction. Each would lead to a unique solution, some of which would be better than others. One goal of EN380 was to give you an introduction to marine materials so that you came to this course with an idea of materials trade-offs. In this course we will concentrate on metals as building materials as they are the most plentiful, and easiest to analyze. Vessels are also built of wood, concrete and composites, and students interested in designing ships with these materials are encouraged to consider a number of senior electives, including Marine Fabrication Methods, Composite Materials, and Independent Research Courses.

Loads on ships are relatively uncertain and in many cases are very difficult to predict. Given that, it is not surprising that probabilistic methods of load prediction are becoming more popular in naval architecture! The sizes of structural components are limited due to deck clearances, piping, wiring and the general arrangements. In addition to the loads, plating thickness is often driven by weight, economic and fabrication issues. It is literally impossible to say that the perfect ship is an optimization reality.

The ship structure design process is fairly well developed. Starting with a blank piece of calculation paper, the structural designer begins by estimating the loads. They then select appropriate analysis techniques. They then determine the necessary structural characteristics of the components, such as plating thickness, the moment of inertia and the yield strength. The basic structural arrangement is then developed, and the components are analyzed for acceptable factors of safety. As the optimum is rarely (never) found the first time, the design engineer then iterates by adjusting the numbers of stiffeners, the plating thicknesses and/or possibly the materials until an acceptable or "optimal" design is reached. The final step is to develop the structural drawings, starting with the Midships Construction Drawing. The complete structural design of a large ship may take tens of thousands of man-hours, while a simple small craft may be done in only a few hours.

In this course, we cover enough material that if the student successfully learns it, they will be able to perform a complete preliminary structural design of a medium-sized commercial or military vessel made of metal. The course coverage is also sufficient for the student to complete nearly the entire structural design of a vessel up to 79 feet.

One challenge for anyone learning naval architecture is the terminology. Some of the terms make sense in common modern language, but many defy common logic as their origins are centuries old and span many languages. A "ceiling" for example, is not overhead as in common language, but rather is the inside skin of the vessel's topsides. A "floor" on a ship is not something you usually walk on (which is a deck, tanktop, or sole), but is a frame supporting the hull bottom and possibly the lowest deck.

The basic ship structural components are commonly divided into two general types: *plating* and *stiffeners*. Stiffeners include frames, longitudinals, stringers, deck beams, deck girders, bulkhead stiffeners, and stanchions. Plating includes bulkhead plating, bottom plating, side shell plating, and deck plating, to name only a few. Ship structures are most commonly made of combinations of plating and stiffeners. Analytically we often treat stiffeners as *beams* or 1-D objects, which means that their length is much greater than their width or height (a pencil is an example). We often treat plating as 2-D objects, with two dimensions (length and width) much greater than the thickness (a piece of paper is an example). A third class of structures is "*solids*", which are 3-D objects with their length, width and thickness all of similar dimension. For manufacturing and weight reasons, solids are rarely used in typical vessels and will not be covered in this course.

Ship structures are also characterized by which loads they are intended to resist.

- *Primary structure* resists the global bending of the vessel due to vessel hogging and sagging conditions. Primary structure is evaluated by treating the ship as a very large box beam subjected to bending by the waves.
- *Tertiary structure* resists the local hydrostatic pressures and/or point loads. A common example is side plating with seawater pressure on one side.
- *Secondary structure* either resists large areas of hydrostatic loads or transfers the tertiary loads to primary structure. A secondary structure may be the combined plating and stiffener structure of the hull bottom or side.

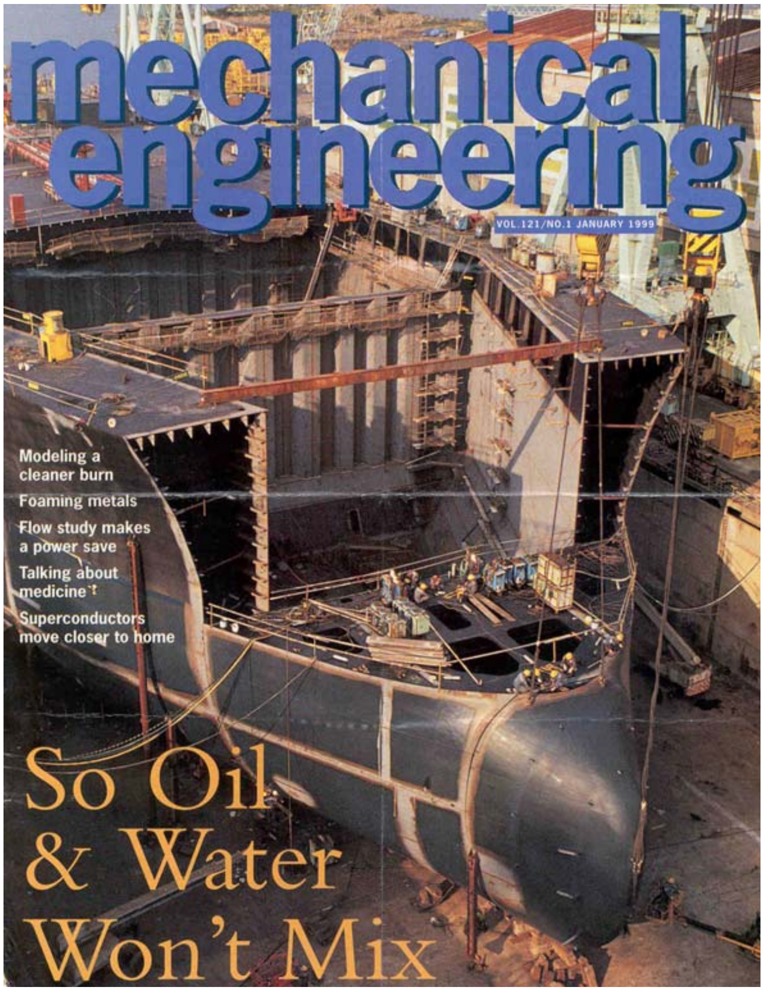
Note that a single piece of structure may resist multiple loads. Hull bottom plating for example, is a part of primary, secondary and tertiary structure!

In this course we will work our way through the important topics by starting with primary structure, then going to tertiary structure, and finishing with secondary structure. This means that we will start with beams (and *stiffeners*), then do *plating*, and finally combine the two into *panels*. We will use *three methods* of design analysis: traditional theoretical methods built on a "Mechanics of Materials" approach (often referred to as "first principles"), classification society rules (often referred to as "rules-based"), and finite element analysis or "FEA" (often referred to as a "computer-based numerical approximation method").

It is important that the student keeps up with the material as it is cumulative and rapidly presented! The student should also plan on taking a proactive approach to the design projects as they require significant time!

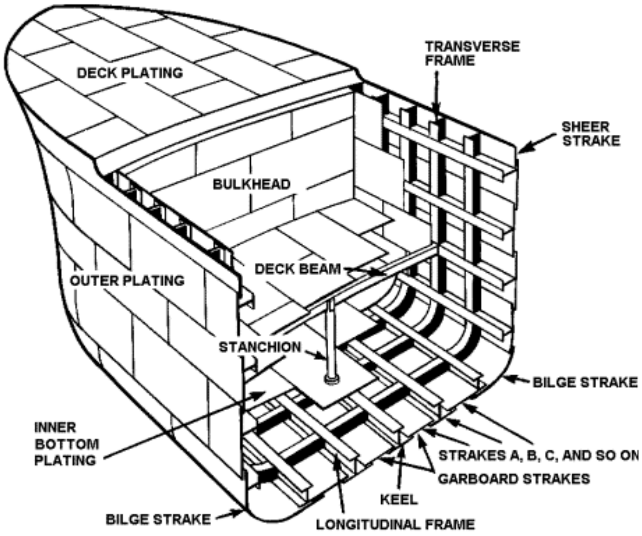
Down the road, the material in this course will help the student with the Fundamentals of Engineering exam, the Professional Engineers licensure exam, and graduate school. Currently there is a shortage of naval architects and ship structural engineers (especially U. S. citizens). This shortage is forecast to continue for the foreseeable future.

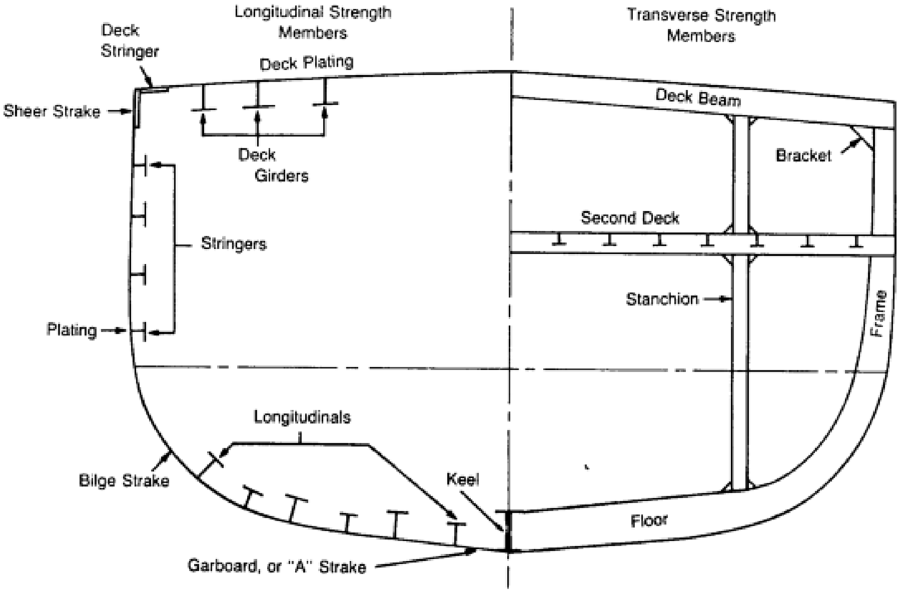
Much of the material covered in this course, and summarized in these notes, is taken from experience of the instructors as well as the very few references on ship structural design and analysis. One of the primary references is *Ship Structural Design: A Rationally-Based, Computer-Aided, Optimization Approach* (by Owen Hughes, published by SNAME, referred to as "Hughes" throughout the course notes). Another important reference is *Principles of Naval Architecture (*published by SNAME, referred to as "PNA" throughout the course notes). Other useful references are: *Ship Design and Construction* (by Thomas Lamb, published by SNAME), *The Elements of Boat Strength* (by Dave Gerr, published by McGraw-Hill), and *Ultimate Limit State Design of Steel-Plated Structures* (by Jeom Paik and Anil Thayamballi, published by Wiley).



Ship Structural Components

A ship is constructed primarily as a network of welded together cross-stiffened plates, sometimes referred to as a "grillage" (in the "old days" – prior to the 1940s – all steel ships were of riveted construction). The plates are stiffened by welded girders or "stiffeners", such as I-beams, T-beams, angles, etc. Nomenclature for ship structural components is somewhat standardized, although also somewhat confusing to the newcomer. The below figures illustrate nomenclature of some of the important ship structural components. The first illustration shows a perspective view. The second illustration shows section views, with the port side showing structural components which run longitudinally, and the starboard side showing structural components which run transversely.



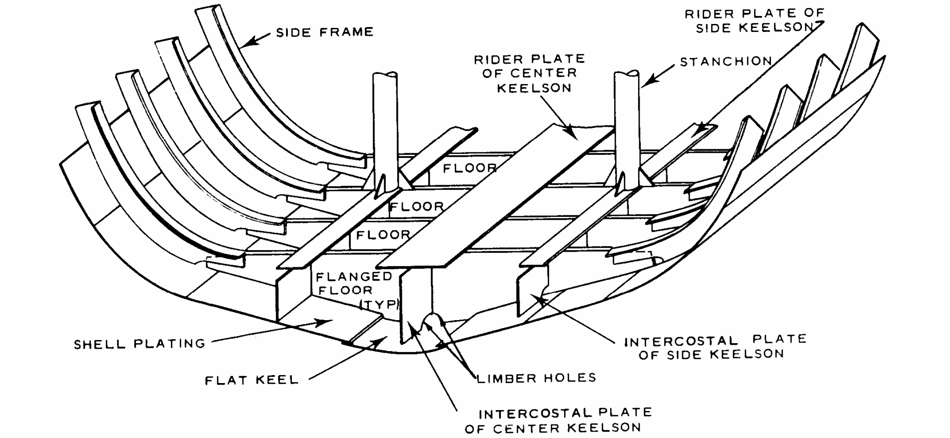


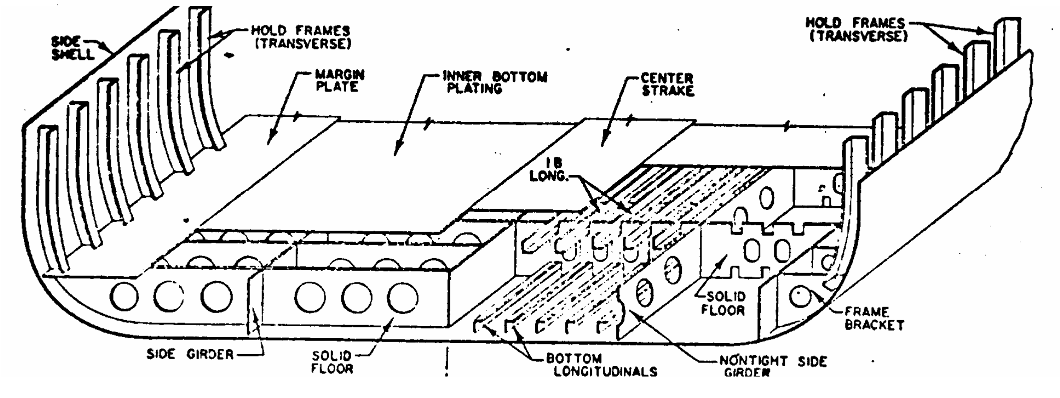
Traditionally, the term "keel" (or *center vertical keel* or *CVK*) refers to the longitudinal center plane girder which runs along the bottom of the ship (often loosely described functionally as the ship's "backbone"). The term "plating" refers to thin structure, including the outer "shell" plating (side and bottom plating), deck and inner bottom plating, and bulkhead plating. The ship's side and bottom shell plating is typically arranged in longitudinal rows referred to as "*strakes*". Strakes may be designated either by letters or by specific noun names. The *'A' strake* (also known as the *Garboard Strake*) is the strake adjacent to the keel. Strakes may be designated by successive letters outboard of the 'A' strake (see the top figure). The strake located at the turn of the bilge is referred to as the *Bilge Strake*. The upper-most strake (located at the deck edge) is referred to as the *Sheer Strake*. As you will learn in this course, the shear strake experiences high stresses, so it is often constructed from higher strength materials or of thicker plating.

The ship's plating is stiffened by welded girders or "stiffeners", which vary in name depending on their location and orientation (in the "old days" everything was riveted instead of welded). *Longitudinals* are stiffeners which run longitudinally (fore-aft) along the bottom of the ship, and stiffen the bottom plating. *Stringers* run longitudinally along the sides of the ship (nominally above the bilge strake) and stiffen the side shell plating. *Deck girders* run longitudinally and stiffen the various deck plating. *Frames* run transversely from the CVK to the main deck, and stiffen the outer plating (bottom plating and side shell plating). *Floors* are the (usually) larger portion of the frames which run from the CVK to the bilge strake. In addition to stiffening the bottom plating, the floors also function as foundations for the *inner bottom plating* and also as tank boundaries and machinery foundations. *Deck beams* run transversely and stiffen the various deck plating (note that deck beams run transversely but deck girders run longitudinally – so deck beams are fundamentally perpendicular to deck girders).

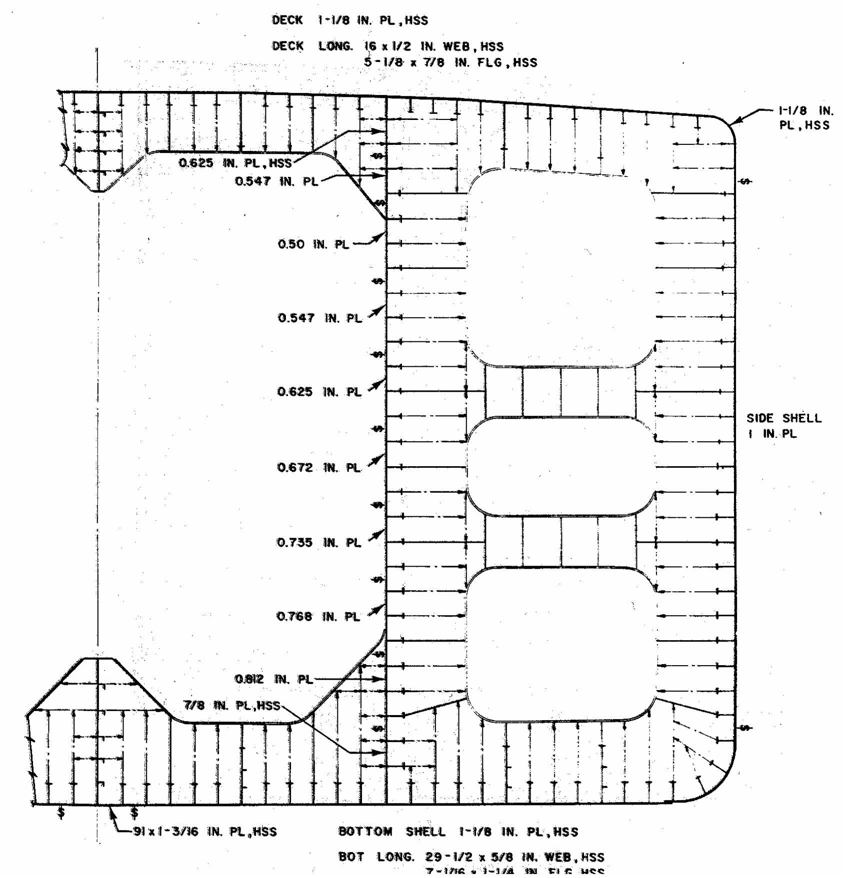
The sizes and spacing of longitudinal and transverse stiffeners vary in different types and sizes of vessels for a number of reasons. Some vessels have larger, closely-spaced frames with widely spaced longitudinals. This type of framing system is referred to as "*transverse framing*". Transverse framing systems are usually designed for ships whose primary structural loads are hydrostatic or impact-type loads, and are often found on shorter ships (such as workboats) and submarines. Some vessels have closely spaced longitudinals and more widely-spaced frames. This type of framing system is referred to as "*longitudinal framing*". Longitudinal framing systems are often chosen for ships whose primary loads are due to longitudinal bending (flexure). Most larger ocean-going ships actually have what is referred to as "combination framing" systems, which are better for dealing with combined loads (including hydrostatic, longitudinal bending, torsion, racking, etc.). Most Navy surface ships are of this type.

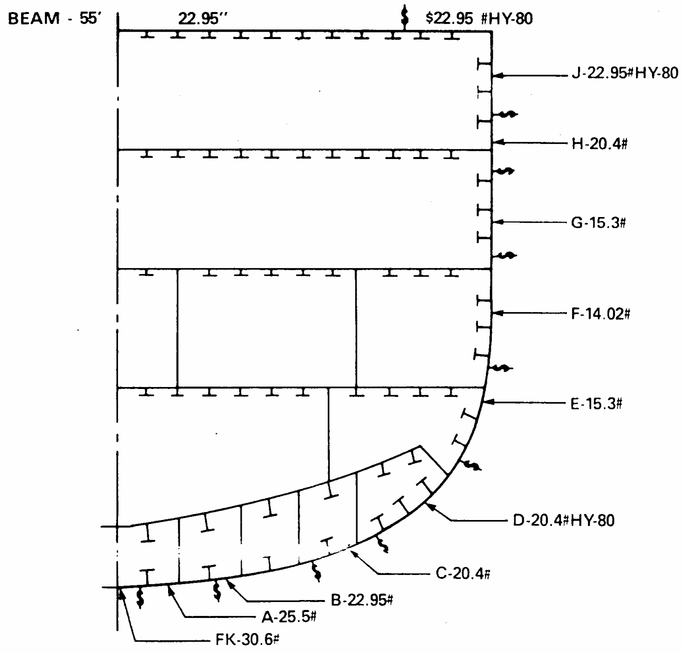
In addition to having different types of framing systems, some ships are constructed with "single hull" or "single bottom construction" (see below illustrations). This type of construction is often found on smaller ships. Most large modern ships are of "double bottom" construction (see illustration). Double bottom construction has advantages of being stronger, providing volume in which liquids can be stored without taking up valuable cargo space, and providing some damage resistance in the bottom. Double bottom construction has disadvantages of being more expensive to construct with higher maintenance costs, and it also moves the neutral axis downward (increasing bending stresses in the main deck). Also note that many oil tankers are constructed as "double hull", in which the entire bottom and side shell are enclosed with an inner skin – this having a perceived advantage of being less susceptible to oil spills, although this has been shown to be not necessarily the case.





Structural drawings provide details of structural arrangements, materials, construction methods, etc. Of the many structural drawings, the *Midships Construction Drawing* provides a section view (the midships section), providing the details of the structure amidships (sometimes structural details for other sections located fore and aft of midships are also indicated on this drawing). Shown below are several example Midships Construction Drawings. The top figure shows a midships section including a “deep frame” and other “scantlings” (structural dimensions) for a double hull oil tanker, and the bottom figure shows a midships section for a Navy destroyer (DD-963 class).





The Ship Structural Design Process

Designing a ship's structure follows the same basic process as designing any structure. It is an *iterative process*, with many compromises (trade-offs). The basic steps include:

1. Determining environmental and/or operating conditions
2. Selecting geometry
3. Determining loads (forces)
4. Estimating boundary conditions
5. Selecting analytical methods
6. Selecting materials
7. Analyzing
8. Optimizing (minimizing cost, weight and others)
9. Documenting (specifications and drawings)

An example of a flowchart method for ship structural analysis is (from Hughes):

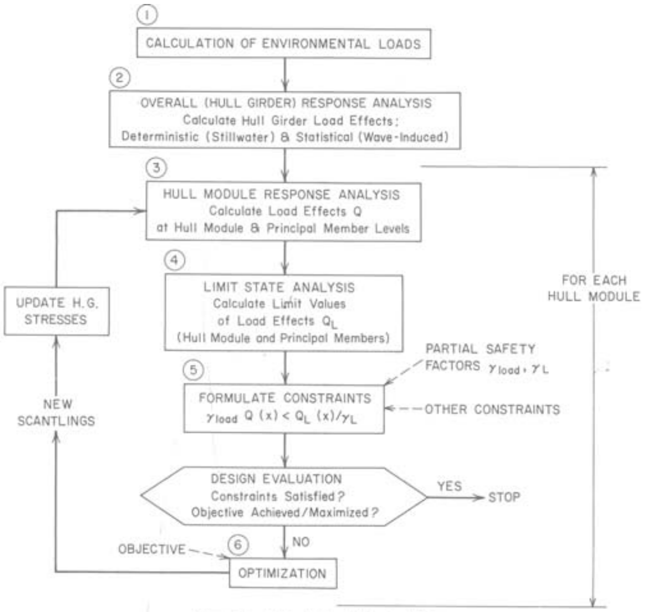


Figure 1.1 Rationally-based structural design.

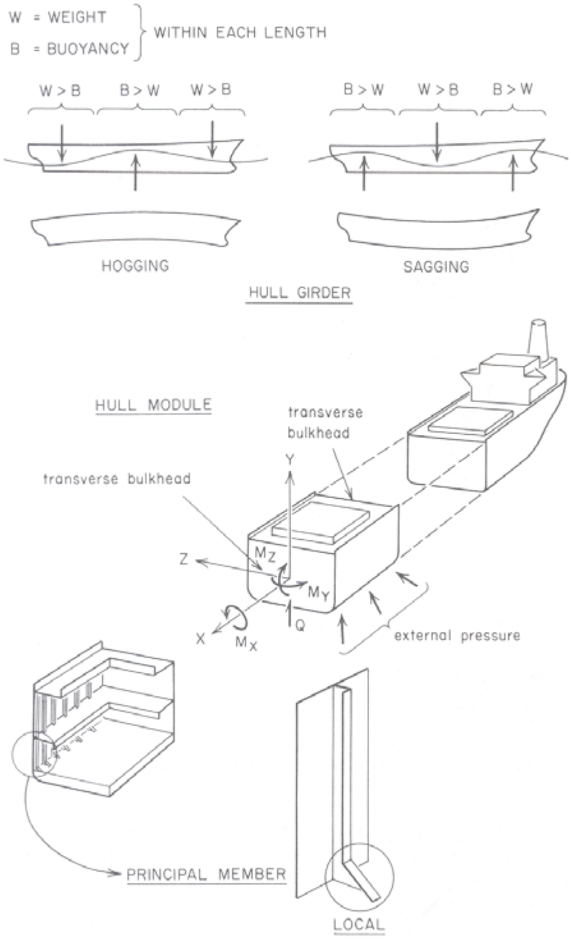


Figure 1.2 Levels of structural analysis.

Structural “Failure”

Most operators or owners would consider that something has “failed” when it can no longer perform the function for which it was intended. In the world of structural design, two criteria are generally used for considering “failure”, as each has different approaches to its analysis. The two criteria can be illustrated through a simple example – a crane boom. It is easy to visualize that the boom can fail by “breaking”. It could be that the stress level within the steel of the boom exceeds the ultimate strength of the steel and the boom fractures. The solution might be to specify a stronger material. Some polymers such as synthetic rubber are “stronger” than many steels. Does it make sense to build a crane boom out of rubber? Probably not – as the deflection would be too large. A structure that deforms beyond an acceptable or allowable limit also “fails” and would be just as useless (and possibly just as dangerous) as one that breaks. The point is that an engineer has to consider whether a particular design is “strength-driven” or “stiffness-driven”. The first (“strength-driven”) requires analysis that uses stresses and strengths of the materials. The second (“stiffness-driven”) requires an analysis that predominantly uses stiffness parameters such as the modulus of elasticity, and the “slenderness” of the structural members.

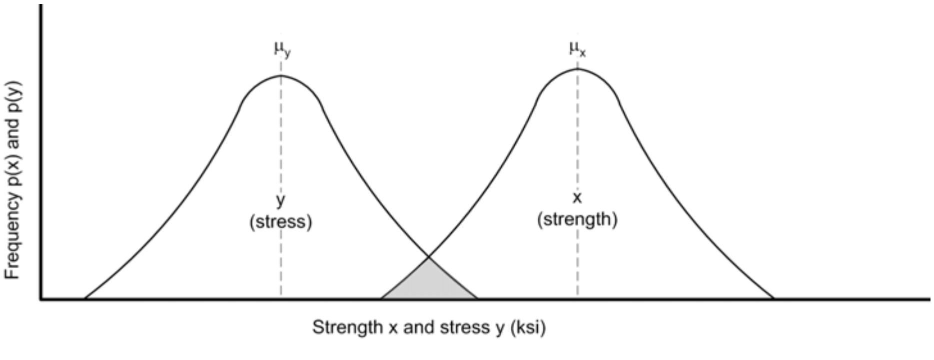
The history of naval architecture includes periods when both criteria were used. Wood is neither very stiff nor very strong, and designers using wood need to carefully consider both. Fiberglass tends to be strong (although brittle) but not very stiff. Steel is stiff, and some grades are very strong. Carbon fiber laminates are strong and stiff, but expensive. Until the last few years, ship structural design has focused on strength-driven design, as almost all construction was steel. As more and more composites (fiberglass and carbon fiber laminates) have found applications on modern marine vessels, the issues of stiffness-driven design have returned. One example was the recent problems on a 240’ fiberglass motor vessel. As the vessel deflected more than usual for a vessel of that size, the design codes (rules) did not consider the effect on other structure. As a result, a problem arose where the hull and structure deflected more than the steel propeller shaft could! This resulted in unforeseen stresses in the propeller shaft, and it bent (then it didn’t work)!

As most *catastrophic* ship structural failures are caused by the failure of strength-driven designs, this course concentrates in this area. Whether a structure will fail is typically determined (in the “Working Stress Design Method”) by calculating a *Factor of Safety*. In design, regulations (and common sense) indicate that the minimum factor of safety should be greater than one. Most marine design codes in practice today have a minimum acceptable factor of safety of at least two for most failure modes, and at least three for buckling failure modes. The American Bureau of Shipping (ABS), which provides rules for designing and building vessels in the United States, has factors of safety ranging from 1.5 to 8, depending on the application’s criticality and the load uncertainty. At the low end, a mainsail batten on a racing yacht may have a factor of safety very near one. If the batten breaks, the sail shape is compromised, but serious damage is unlikely. A batten’s low weight and stiffness are important to performance however so the batten is carefully engineered to just meet the expected worse load conditions. On the high end, failure of mission-critical (or life-critical) structural components on a naval combatant may require a higher factor of safety.

For consistency, in this class we will use a required factor of safety for “material yield” of 2 and a required factor of safety for “buckling” of 3 as constituting an adequate design.

FOS = Capacity/Load = Strength/Stress = _______________________________

A visual way to look at Factor of Safety in a real world, probabilistic method, is by comparing the probabilistic distributions of the stress and the strength of a given structural component. The loads are uncertain in most cases, so a “one hundred year event” might cause a very high stress. Similarly, a particularly weak batch of steel may give very low strengths. If the high load occurs when a low strength occurs, failure will occur. In the figure below, the difference between the two means (μ) is the safety margin, which is a function of the factor of safety. The shaded area represents where failure will occur.



Strength may be a material property such as ultimate tensile strength (σ_{ult}) or tensile yield strength (σ_Y), or it may be a buckling strength (for example column buckling capacity).

Stress within a structural component depends primarily on loading and geometry. In your previous courses (EN221, EN222, EN380), you have considered various types of stresses. Here we consider that, for strength-driven designs in the marine environment, there are 7 basic failure modes: *…(recall EN221 and EN380 and fill in this section)*

Tensile Stress (Axial)

Compressive Stress (Axial)

Shear Stress (Direct, Bending, Torsion)

Bending (or Flexural) Stress

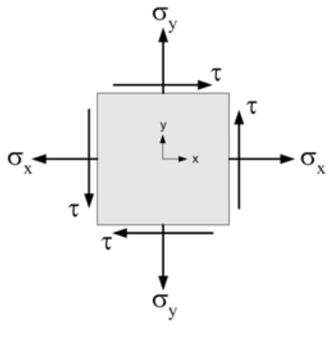
Buckling Stress (Compression or Shear)

Fatigue Damage

Corrosion

In earlier courses, for "failure" assessment, it was common to assume that only one load was applied to a structure at a time. Unfortunately for naval architects that is rarely the case! A common example is a beam in bending. The beam will have stresses caused by the bending, but will also have shear stresses. The most commonly used method to address this is to combine these stresses into an equivalent "VonMises Stress" (given formally by the Distortion-Energy Theory). This *VonMises Stress* gives an *equivalent stress* that can be compared to the tensile or compressive strengths of the material to determine a minimum factor of safety. Since most ship structures are usually built-up from thin plating (2-D plane stress), we use a *2-D version of the VonMises Stress*:

$$\sigma_{vm} = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2} \qquad \text{(2 - D "plane stress")}$$



Another common problem with ship design is that the structure often has holes or cutouts in it to reduce weight, or allow pass-through of cargo, liquids, pipes, cables or personnel. These discontinuities cause stress concentrations. As you learned in EN380, the stress concentration is addressed by a factor multiplied by the (nominal) uniform field stress. Two approaches are used: an empirical method based on tests of various configurations, and a "fracture mechanics" (theoretical) approach.

Example: $\sigma_{max} = SCF \cdot \sigma_{nom}$

where SCF is the "Stress Concentration Factor"

SCF for a circular hole is: *…(fill in from EN380 notes)*

Another issue with ships is that they don't stand still! Anyone who has walked down a passageway of a ship in a storm knows that their "weight" (as defined by weight = mass · gravity) is not constant! Each time the ship rises on a wave the person's weight "increases", which increases the load on the deck plating. A common way to deal with this in structural analysis is to use a "static-equivalent" calculation method – by modifying the basic dynamics equation and replacing the acceleration term with the standard gravitational acceleration multiplied by a factor:

$$F = m \cdot a \quad \rightarrow \quad F = m \cdot g \cdot DAF$$

where "*DAF"* is the *Dynamic Amplification Factor*. Another name for the DAF is "*g-force*". "Pulling 5 g's" means that you multiply the acceleration due to gravity by 5! Due to ship motions, most ship dynamic analyses use DAFs between 1 and 3. ISO Regulations currently in development limit the vertical acceleration forces in proposed vessels to those sustainable by humans! However, there is some debate about this as a 5 g force that lasts a millisecond does not have the same damaging influence on a human being as a 4 g force that lasts 10 seconds.

Deck Girder Exercise:

You need to check a ship design. A deck beam is loaded with 8000 pounds of vertical shear force by a stanchion. The deck beam is 8" high and has a 4" diameter lightening hole in the middle of the web. The web is ¼" plate with a tensile yield strength of 30 ksi. The DAF is 1.25. What is the factor of safety? If it is not acceptable, what can the designer do to make the design acceptable?
Hint: Use the idealized stress concentration factor.
(The answer is given on the last page of the reader)

Weekly Assignment #1: Ship Structural Design Basics

In Lab:

1. Break into groups of 2-3 and solve the following problem using the standard engineering method (and presentation format!). A lifeline stanchion on a sailboat needs to prevent someone from falling overboard. Assuming the socket base allows for a 1” diameter tube, what is the minimum wall thickness? You will use AISI Type 316L Stainless Steel (annealed plate) and you need a factor of safety of two. Use your engineering judgment to determine the missing information. Hint: the moment of inertia of a thin-walled cylinder is approximately $\pi \cdot r^3 \cdot t$

Take-home:

2. What is the difference in “failure” for a stiffness-driven design and a strength-driven design? Give an example of each (different from those in the notes).

3. What are some potential advantages and disadvantages to increasing plating thickness on a ship beyond the minimum necessary for long-term structural safety?

4. Create an Excel spreadsheet table that lists material properties (U.S. units) of the following shipbuilding materials. We will be using these materials throughout the semester on projects, quizzes and the final exam. Create your spreadsheet so the materials are on the left and properties are to the right (see below). <u>A good idea is to print an extra copy of your completed table and place it in the back cover of your binder for quick reference!</u>

   Materials: (in order)
   - AISI 1020 (annealed)
   - AISI 1020 (cold rolled)
   - ASTM A36 plate
   - HTS (High Tensile Steel, σ_Y 55 ksi)
   - 316L annealed plate
   - 17-4 ph H1150
   - Ti (ASTM Grade 2)
   - HY80
   - 6061-T6
   - 5086-H34
   - E-glass/epoxy unidirectional
   - Carbon/epoxy unidirectional
   - Douglas fir (12%)

   Properties: (in order)
   - Material name (from list)
   - Material description (ex. stainless steel, aluminum)
   - Elastic modulus (msi)
   - Tensile yield strength (ksi)
   - Tensile Ultimate Strength (ksi)
   - Shear yield strength (ksi)
   - Poisson’s ratio
   - Elongation to failure (%)
   - Weight density (lb/in^3)

   Hints: Except for HTS and the two composites, all of the materials are listed in www.matweb.com. You may need to derive one or two properties from the others. For example, for ductile materials (e.g. steel & aluminum), the shear strength is about 58% of tensile strength.

   An example (without showing the headers with units, which you should have!):

ASTM A36 plate	Low carbon steel	29.0	36.3	58.0	21.4	0.26	20	0.283

5. Develop a scaled (meaning all dimensions are to the same scale) engineering sketch (on engineering paper!) that shows the midship construction section for an 18’ flat bottom, work punt. At midships the beam at the shear is 5 feet, the depth is 2 feet and the draft is 6”. The topsides have 15 degrees of flare. The bottom plating is 3/8”, the side plating is ¼” and there is a 3” (horizontal) x 1” (vertical) keelson below the hull plating and a 1”x1” outboard rub rail at the shear. All material is 5086-H34, except for the rub rail, which is neoprene. In addition to the title block, identify on your sketch the weight per longitudinal foot of this structure. Use a scale of 1:6. Note that the boat is symmetrical about the centerline.

Note: You should start this assignment (all assignments) early so that you have time to ask questions!

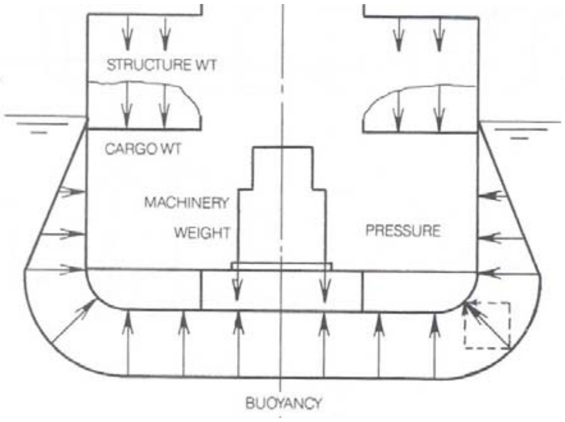
Ship Structural Loads

Usually the most difficult part of ship structural design to get right is to correctly estimate the loads! In some cases it is not too hard. For example, the foundation for the propeller shaft thrust bearing should never exceed the thrust produced by the propeller, so estimating its load is relatively easy. On the other hand, estimating the maximum wave height that will break on the deck of a ship rounding Cape Horn (and the resulting forces on the ship) is very difficult!

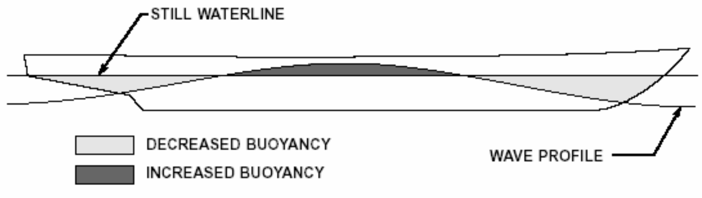
Luckily for naval architects, for mid to large size vessels, there is much historical data which can be (and is) used to produce reasonable estimates of future extreme loads. If it sounds like statistics and probability are used, you are right! The most accurate method for determining the majority of ship loads is *probabilistic load prediction*. While more accurate than the traditional, empirical deterministic methods, it is also more time consuming. In this course we will use the traditional deterministic methods, and accept a somewhat lower level of accuracy, in order to cover the key analytical methods. EN330 (Probability and Statistics with Marine Applications) and EN455 (Ship Maneuvering and Seakeeping), along with graduate school education, will cover probabilistic methods.

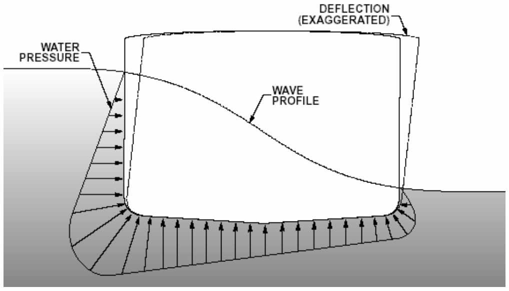
It is important for the naval architect to carefully think through what the possible loads or “load cases” are. During one of the America’s Cup design projects, the designers identified 28 potential “extreme” load cases for the analysis of the deck structure, including waves breaking over it, submergence due to capsizing, impact from the spinnaker pole, and maximum crew “live” loads. As it turned out, the first failures occurred before the vessel even went to sea, due to a load case they didn’t analyze – and hadn’t even imagined! During the commissioning ceremony, a lady walked on the deck wearing high heels, with very narrow heels. Each place she stepped, her heels punched a hole through the thin carbon fiber laminate! The (easy) solution was simply placing a thick carpet down on the deck while in port, and banning high heels while underway!

There are a number of ways of classifying loads (forces) on ships. One method is classifying the loads based upon the timeframe of the load as compared to the dynamics of the ship structure and structural components. *Static loads* are loads which are not considered to change over relatively short periods of time – or those loads which change with periods far greater than the structures’ natural frequencies of vibration or flexure. “*Stillwater loads*” include external hydrostatic pressures (without considering waves), internal tank pressures, and all onboard weights. Onboard weights include fixed or *“lightship” weight* items (including the ship’s structure, machinery and piping systems, deck gear, outfitting and furnishings, and fixed portions of weapons systems, etc.), and *variable weight* items (including cargo, fuel, water, holding and waste, provisions and stores, crew and effects, etc.). Variable weight is sometimes called “*deadweight*”. The figure below illustrates static loads including hydrostatic pressure loads and various lightship and variable weight loads on structures.

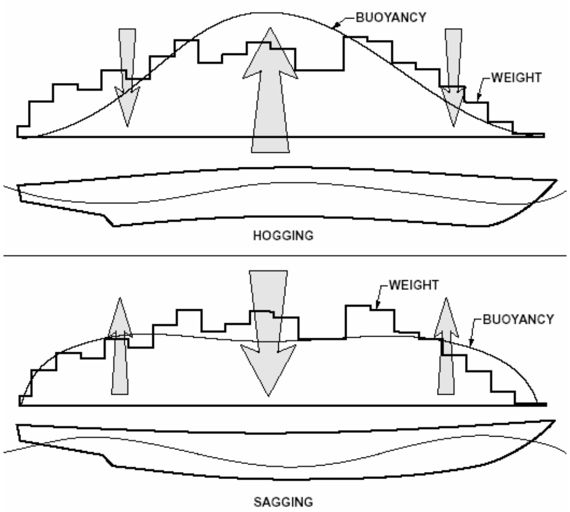


Slowly-varying loads include *wave-induced loads*, and are characterized as having a period slightly longer than the natural flexural periods of the ships primary structure. Waves cause variable hydrostatic and dynamic pressures on the hull, due to both the wave motion and the ship motion. The distribution of the pressures on the hull can be considered longitudinally (causing longitudinal bending of the ship – like a bending beam), transversely (causing transverse distortion of the hull – called “racking”) and obliquely (causing combinations of bending, racking and torsion or twisting of the hull). The figure below illustrates wave-induced pressure variations – including longitudinal and transverse effects. Wave pressures vary slowly (typically on the order of 5-15 seconds for large open ocean waves), and therefore wave-induced loads are sometimes referred to and treated as “quasi-static”.





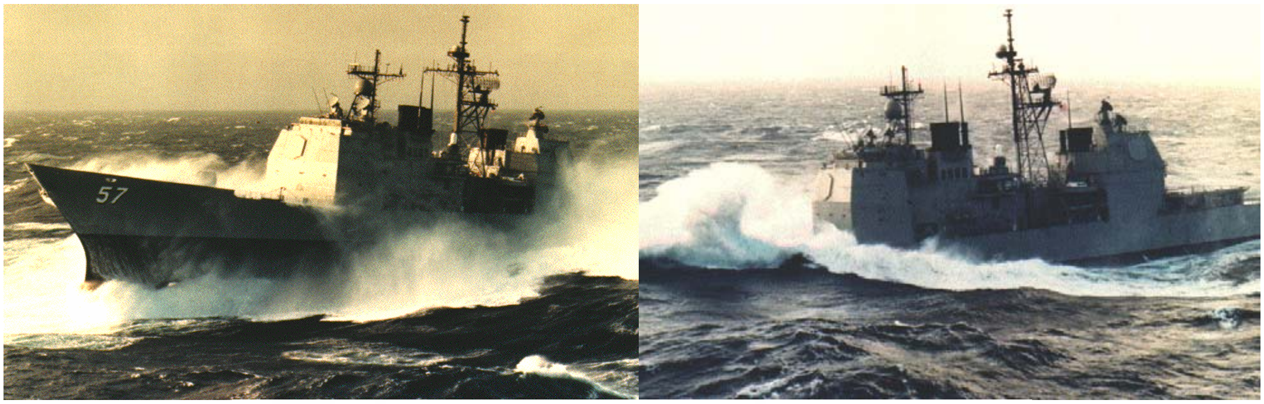
The longitudinal distribution of buoyancy applied to a ship by a passing wave (or a ship passing through a wave) creates a bending moment on the ship, which varies along the length of the ship. This is a wave-induced buoyancy distribution. The two extreme cases are given specific names. A "hogging" condition is when the crest of the wave is amidships (and trough at bow and stern – as illustrated above and below). This causes flexure of the hull which puts the main/upper decks in tension and keel and bottom plating in compression. A "sagging" condition is the reverse – with trough amidships (and crest at bow and stern). This puts the main/upper deck in compression and keel and bottom plating in tension. The net bending stress distribution, however, requires knowledge and accounting of the stillwater loading as well (including hydrostatic buoyancy distribution and weight distribution). This will be discussed in greater detail subsequently.



In addition to waved-induced buoyancy distributions, there are a number of other "slowly-varying" loads which might be important (and must be considered) when designing a ship's structure. These include: *wave slap* on sides and foredecks, sloshing of liquids in tanks, *shipping water* ("*green water"* on decks), localized inertial loads (such as masts), and launching and berthing loads.

The third basic category of loads is *"rapidly-varying loads"*, whose timescales are on the order of the natural periods of vibration or flexure of the ships structure and components (on the order of a second or less). These are basically the true *dynamic loads* on the ship structures. These types of loads include *slamming* loads, which occur because of the ships motions in waves, and result in localized buckling and shell plating damage, as well as overall flexure of the ship's hull (known as *whipping*). The effects of slamming are illustrated in the below figures. Another important dynamic load is called *springing*, which is a flexural resonance of the ships hull girder due to rapid wave encounters as the ship "drives" through the waves. Mechanical vibrations within the ships structure can also be caused by propeller and machinery rotations (although these effects are usually localized in the vicinity of machinery). Other important dynamic loads include *combat loads* (particularly important and often critical design loads for naval ships) including underwater explosions, air blasts, and even nuclear weapons.

In designing for these various loads, there are a number of methods used for analysis of stresses and deflections. Analyses can be static or dynamic, probabilistic or deterministic, and even linear or nonlinear. For most basic ship structural design, naval architects utilize "quasi-static", deterministic, linear analyses. Slowly-varying wave-induced loads are considered at a "snapshot in time" for each "load case" (e.g. hogging wave, sagging wave, etc.). For localized structural components, sometimes dynamic effects are considered using a dynamic amplification factor (DAF). Probabilistic nature of loads and strength capacities are addressed using the Factor of Safety (FOS). By requiring FOS to be above a minimum level (requiring stress levels remain well within the linear-elastic range of the material stress-strain behavior) allows analysis to be treated primarily in a linear manner.



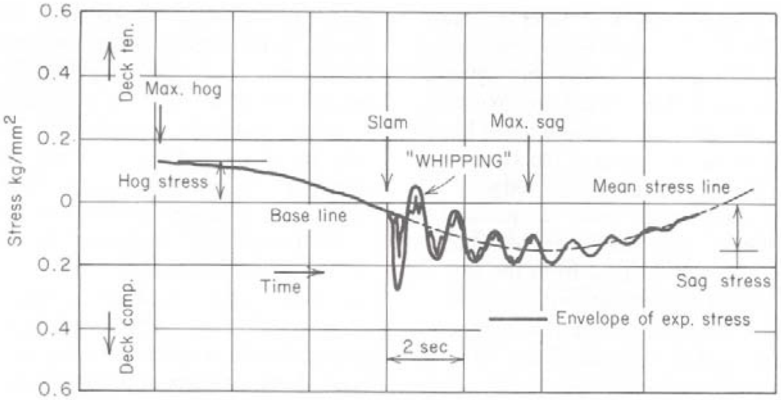
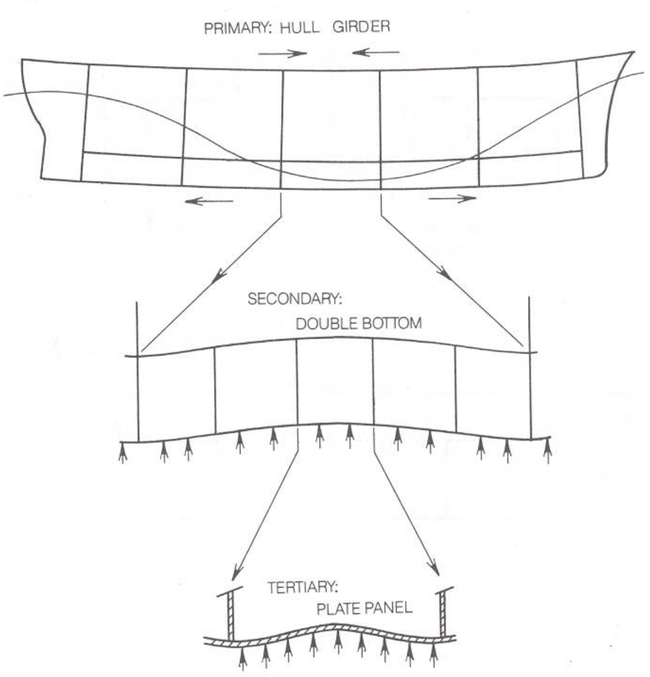


Figure 2.1 Whipping stress due to slamming.

Primary, Secondary and Tertiary Ship Structural Loads

The previous section should have pointed out one very important concept. Something a naval architect quickly learns is that any piece of ship structure is likely to see more than one load at the same time. Consider the bottom plating for example. When the vessel is "sagging", the bottom plating goes into tension. At the same time, the plating between two stiffeners is bending upward due to hydrostatic pressure. That means that the inside face of the plating has tensile stresses from *both* the hydrostatic pressure *and* from the global hull bending. The outer face has tensile stresses from the vessel sagging, but also compressive stresses from the plate bending. How much they cancel each other depends on the plate thickness, frame spacing, and longitudinal spacing. A few seconds later however the vessel goes into "hogging" and the outer face has the two tensile stresses while the inner face has a mix of tensile and compressive! In addition to global and tertiary loading going on at the time, an underwater explosion, collision with flotsam or other loads might be happening. It is easy to see why the VonMises combined stress equation is used so often.

Section 2.2 of PNA goes in to some additional depth on this topic. You should take a look at it for more insight.



Hull Longitudinal Stiffener Exercise:

A potential longitudinal design shows the structure to consist of a flat plate stiffener 6" high of 20# plate (i.e. 0.5" thickness). The bending moment on the stiffener due to hydrostatics is 16 LT/ft and the vertical shear is 2.5 LT. Due to global ship bending the stiffener will carry alternately 24,000 pounds in tension (when sagging) and 19,000 pounds in compression (when hogging). If it is made of HTS, what is (are) the factor(s) of safety for the stiffener? For this exercise only, ignore the hull plating contribution to the stiffener.
(The answer is given on the last page of the reader.)

Primary Structure: The Hull Girder Bending Concept

As discussed in the previous section, a wave crest amidships will cause a vessel to hog. Acting as a beam (or, since it runs longitudinally, we actually call it a girder), the deck of the vessel will be in tension and the hull bottom in compression. We can use the Euler beam bending equation to determine the stress in the hull bottom plating and deck plating. While the equation to find the maximum stress is simple ($M \cdot y_{max}/I$), determining the values of the bending moment (M), distance from the neutral axis to the outer fiber (y_{max}) and the moment of inertia (I) are not! While it seems like a very simple calculation for such a large structure, it is a critical calculation, and one that occasionally goes wrong.

In statics and strength of materials we used boundary conditions that included fixed, simply-supported, roller, pinned, and others. One that might not have been covered was the "elastic foundation". You can think of this as providing continuous support of a varying amount. A mattress is one example. Carrying that example further is a water bed. For ships, water is an elastic foundation. What this means is that we have to apply the basic concepts to calculate the moment as "force times distance". You also learned that if you integrate a load curve you get a shear curve, and integrating the shear curve gives the bending moment curve. For ships, the load curve is a summation of the weight curve and the buoyancy curve. We get the weights from a weight and moment sheet and the buoyancy curve is also called the "Curve of Sectional Areas" or "Section Area Curve", which is developed from the hull lines (usually the body plan if done manually).

To start this process we initially assume the vessel is in static equilibrium, which gives us the assumptions that the sum of the forces is zero and the sum of the moments is zero. This means that none of the ship's six degrees of freedom (DOF) (3 translations: surge, sway, heave and 3 rotations: pitch, yaw, roll) is out of balance, and the vessel will not accelerate in any DOF. For ships the basic case is when the weight is equal to the buoyancy and the LCG is at the same station as the LCB. The process to find the bending moments by hand is:

1. Draw a horizontal line on a piece of engineering paper that will represent the vessel's length (x-axis). Mark on it the stations, ranging from 10 on the left to 0 on the right. Include any overhangs. Sketch in the vessel profile and cargo loading as appropriate.
2. Draw another horizontal line below the first. Draw a weight curve on the line by adding up the weights at each station. More will be shown on this later. The weight curve will use the units LT/ft (or lb/ft if a small vessel). Weights will be recorded below the line (as they are downward or negative).
3. Draw another horizontal line below the weight curve and draw the buoyancy curve based on the body plan. The units will also be LT/ft. Note that forward of station 0 and aft of station 10 the values should be zero. Buoyancy is recorded above the line (as it is upward or positive).
4. Draw another horizontal line below the buoyancy curve and draw the load curve. At each station add the weight and buoyancy values to get the load curve. The units should still be LT/ft.
5. Starting at the left side, integrate the load curve to get the shear curve. Note that if there is no stern overhang, the shear at Station 10 should be zero. The units should now be LT (or lb).
6. Starting at the left side, integrate the shear curve to get the bending moment curve. Again, unless there is stern overhang, the moment at station 10 should be zero. The new units will be LT-ft (or ft-lb).

If you don't recall the basic process for load, shear, and bending moment diagrams from your Statics and Strength of Materials course (EN221), you should review your notes or text from that course.

In-class Exercise: Box-Shaped Barge Global Hull Girder Bending (Stillwater)

(Note that a full analysis would include hogging and sagging conditions as well to determine the worst case for the deck and hull bottom)

A 300’ tank barge has three equally-spaced cargo compartments. The middle compartment is full of fresh water. The barge has a beam of 25’ and a depth of 20’. The empty draft is 2’. To make this easy, we will assume the structural weight is evenly distributed, even though we would not make that assumption in a “real” analysis.

Profile ______________________________

Empty Weight Calculation:

Cargo Weight Calculation:

Weight (LT/ft) ______________________________

Total Weight and Buoyancy Calculation:

Buoyancy (LT/ft)______________________________

Load (LT/ft) ______________________________

Shear (LT) ______________________________

Moment (ft-LT) ______________________________

Maximum Shear is (value), located at (station): ___________ The bending moment there is: __________

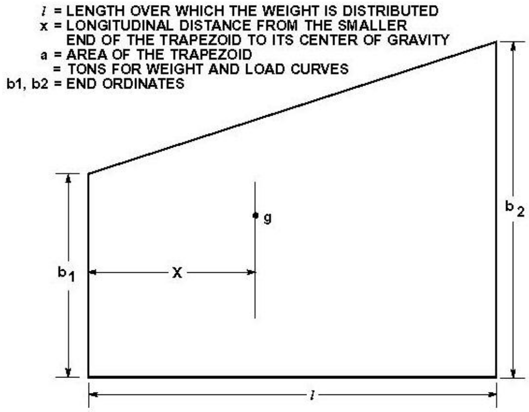
Maximum Bending Moment is (value), located at (station):_________________________________

Note: In this example we had the empty draft. If we did not know where it floated we would have had to sum up all the weights of each component to get the total weight.

Weight Curves for Ships

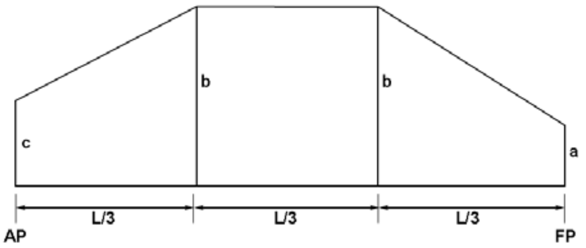
For real ships, generation of a weight curve is not as simple as illustrated in the previous example. There are several methods that may be employed (and are often employed) to generate a weight curve for a real ship. All methods utilize an accounting tool such as a spreadsheet (table) to account for the various weight items. Specific information required for each item include weight, longitudinal position of the center of gravity (lcg), and longitudinal extents (including forward and aft bounds/locations of the distributed weight). For completeness, transverse and vertical c.g. are often included in the table, even though these are not required for the weight curve calculation. Some weight items extend only over a small ship length, so they are treated as *distinct weight items* (point loads). Examples of distinct weight items include the ship's anchor, winches, masts, propellers, rudders, and even transverse bulkheads. Other weight items extend over portions of the ship's length, and therefore their distributions are important – these are *distributed weight items*. Examples of distributed weight items include the ship's hull plating and stiffeners, superstructure, cargo weights (containers, bulk cargo, cargo oil, etc.), machinery (turbines, boilers, reactors, etc.), piping and ventilation systems, to name only a few.

For distributed weight items, the weights may be uniformly distributed (in which case the lcg is located in the center of the distributed length), or they may be non-uniformly distributed (in which case the lcg is not in the center). Non-uniform weight distributions are usually modeled using a simple trapezoid, which are described by the items total weight (the total area of the trapezoid), lcg (located at the centroid of the trapezoid), forward boundary and after boundary. The heights of each end of the trapezoid are related to the total weight, total length, and center of gravity position. This is known as the *"Trapezoid Method",* and is illustrated in the figure below. Note that there is a mathematical calculation that can be made (in the spreadsheet) for calculating the heights of the ends of the trapezoid given the weight, length, and lcg.



$$b_1 = \frac{2a}{l}\left(2 - \frac{3x}{l}\right); \quad b_2 = \frac{2a}{l}\left(\frac{3x}{l} - 1\right)$$

Some distributed weight items extend over the entire length of the vessel (or nearly so), such as some piping systems, ventilation systems, and certainly the ship's hull plating. For these types of distributed weights an approximation known as *"Biles' Method"* is sometimes used to distribute these weights. This method assumes that the weight decreases near the bow and stern, and that there is an area of parallel middle body. This type of weight is therefore represented by two trapezoids (the ends) and a rectangle (parallel middle body). Like the trapezoidal method, Biles' Method provides mathematical calculations of the heights of the sides (and middle) of the distribution. This is illustrated in the figure below, for estimating the hull weight distribution of a simple vessel.



BILES METHOD ORDINATES

$$a = 0.566\,\frac{W_H}{L}$$

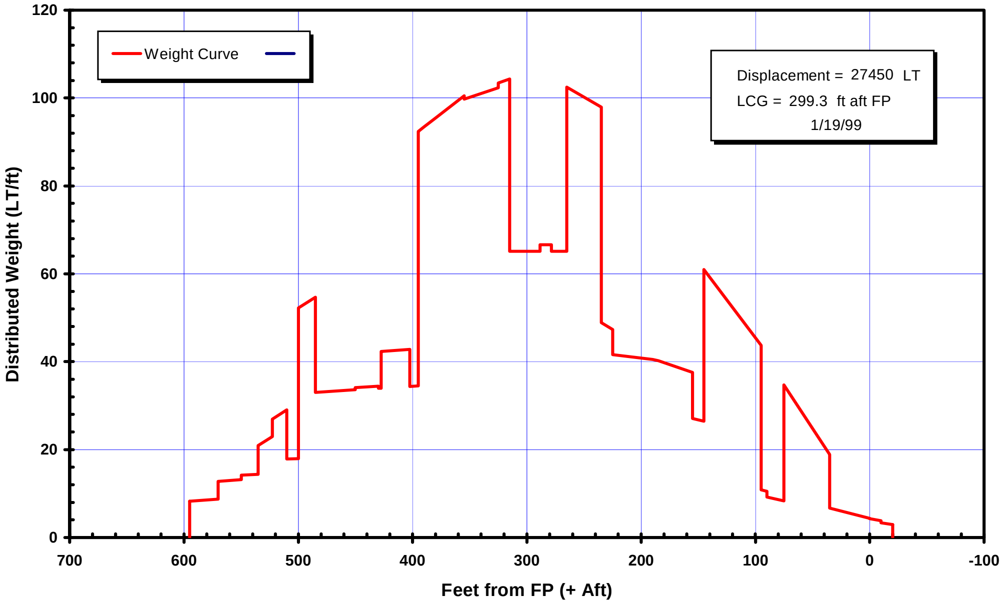
$$b = 1.195\,\frac{W_H}{L}$$

$$c = 0.653\,\frac{W_H}{L}$$

WHERE W_H = HULL WEIGHT (LESS MACHINERY)

The hull weight distribution for a more complex hull form, such as a destroyer, is only very grossly estimated using Biles' Method. A better representation for such hulls is made by estimating the distribution for a large number of sections, using the Trapezoidal Method for each section (for a particular frame spacing, for example). Another method employed in practice for the hull weight distribution is made by scaling the section area curve.

With all of the weight items accounted for (lightship and variable loads), the total weight distribution (curve) can be plotted. The figure below illustrates a weight curve for a typical small cargo ship.



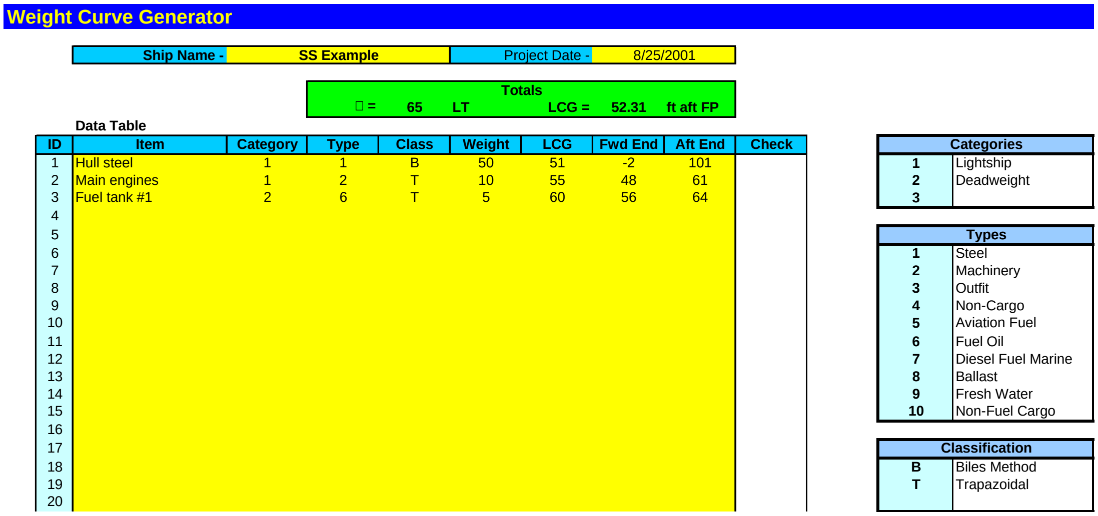
To develop an accurate weight curve we need to consider all the "major" weight items. On an aircraft carrier this can be approximately 30,000 pieces! What is a "major weight item" is up to the naval architect to determine. On a weight-critical vessel such as a racing yacht, a common rule of thumb is that everything that weighs more than 2 kg is considered as a major weight item!

Below is an example of a ledger-style spreadsheet for weight accounting and control. In modern shipbuilding each item is often weighed before installation. The first table lists the major weight items in a certain "weight group" (in this case the "machinery group"). The second table lists the total for each weight group, plus certain additional items including "margin" and "ballast".

GROUP F MACHINERY								
ITEM	**Material**	**units**	**wt/unit**	**WT**	**LCG**	**VCG**	**LMOM**	**VMOM**
ENGINE - YANMAR 6LP-DTE				837	43.00	-0.50	35991.00	-418.50
GEAR BOX				100	45.00	-1.25	4500.00	-125.00
GENERATOR- 20KW PANDA				702	48.50	-0.75	34047.00	-526.50
BOW THRUSTER				100	8.00	-1.00	800.00	-100.00
hydraulic motors				150	43.00	0.00	6450.00	0.00
SHAFT				88	51.00	-2.00	4488.00	-176.00
AQUADRIVE				70	47.00	-1.50	3290.00	-105.00
STUFFING BOX				20	48.00	-1.60	960.00	-32.00
STERN TUBE				10	49.00	-1.60	490.00	-16.00
COUPLING				30	46.00	-1.50	1380.00	-45.00
PROPLELLOR				35	56.00	-2.25	1960.00	-78.75
EXHAUST AND MUFFLER				150	60.00	0.00	9000.00	0.00
CONTROLS				20	62.00	4.00	1240.00	80.00
ALTERNATOR				25	43.00	0.00	1075.00	0.00
FUEL SUPPLY				35	43.00	0.00	1505.00	0.00
TRNASFER PUMPS		2	75.00	150	41.00	-1.00	6150.00	-150.00
OIL				30	43.00	-1.00	1290.00	-30.00
TOTAL GROUP F				**2552**	**44.91**	**-0.68**		

A. HULL STRUCTURE				12709	36.08	1.79	458522.50	22773.91
B. SUPERSTRUCTURE				954	40.33	5.87	38484.66	5598.69
C. JOINERY WORK				4621	38.57	1.51	178254.48	6978.59
D. HULL FITTINGS				3270	40.00	3.04	130813.41	9932.11
E. SPARS & RIGGING				2701	27.00	35.83	72919.41	96772.53
F. MACHINERY				2552	44.91	-0.68	114616.00	-1722.75
G. SHIP SYSTEMS				1065	43.22	0.49	46025.00	518.75
H. ELECTRICAL				1932	40.11	1.17	77492.00	2260.00
I. OUTFIT				2537	29.83	1.52	75691.00	3866.00
J. PAINT 5% OF A,B & C				914	36.93	1.93	33763.08	1767.56
K. MARGIN 5%				1663	36.88	4.47	61329.08	7437.27
--				-	-	---------	--------------------	------------------
TOTAL LESS BALLAST				34918	36.88	4.47	1287910.63	156182.66
More systems				1500	36.88	0	55326.25	0.00
BALLAST				24519	33.38	-9.79	818542.30	-239918.42
LIGHT SHIP				**60937**	**35.48**	**-1.37**	**2161779.17**	**-83735.75**

Using these tables (for this ship), a weight curve can be generated by grouping the weights by frame spacing or a given interval (such as one or five-foot blocks). The following page shows a spreadsheet method for generating a weight curve. The spreadsheet is located on the course Blackboard page. The data is entered in tabular format and a graph is generated. Additional output includes a station listing of weights that can be read in by different software programs as a comma or space delimited format. This spreadsheet tool will be utilized in Weekly Assignment #3.



Weight Curve Generator

Ship Name - **SS Example** Project Date - 8/25/2001

Totals
□ = 65 LT LCG = 52.31 ft aft FP

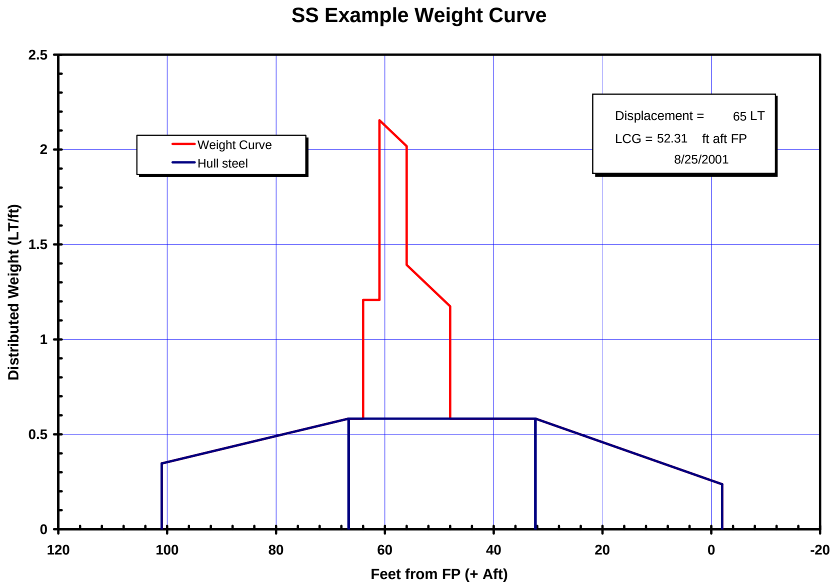
Data Table

ID	Item	Category	Type	Class	Weight	LCG	Fwd End	Aft End	Check
1	Hull steel	1	1	B	50	51	-2	101	
2	Main engines	1	2	T	10	55	48	61	
3	Fuel tank #1	2	6	T	5	60	56	64	
4									
5									
6									
7									
8									
9									
10									
11									
12									
13									
14									
15									
16									
17									
18									
19									
20									

Categories	
1	Lightship
2	Deadweight
3	

Types	
1	Steel
2	Machinery
3	Outfit
4	Non-Cargo
5	Aviation Fuel
6	Fuel Oil
7	Diesel Fuel Marine
8	Ballast
9	Fresh Water
10	Non-Fuel Cargo

Classification	
B	Biles Method
T	Trapazoidal



Buoyancy Curves for Ships

The goal of the hull girder analysis is to develop an acceptable factor of safety for the hull structure. So far we have discussed methods for generating a weight curve. This means we still need to develop the buoyancy curve. Then we can find the load, shear force and bending moment curves. We will then need to calculate the hull's section modulus and apply Euler's beam theory to find the stresses!

The previous sections showed the general approach, and for simple hull forms it is easy to extend the weight spreadsheet to calculate the bending moments. More common today the buoyancy curve is a byproduct of the hull lines development using a hydrostatics or CAD software program. For this course, we will use the HydroMax module of MaxSurf to generate a buoyancy curve.

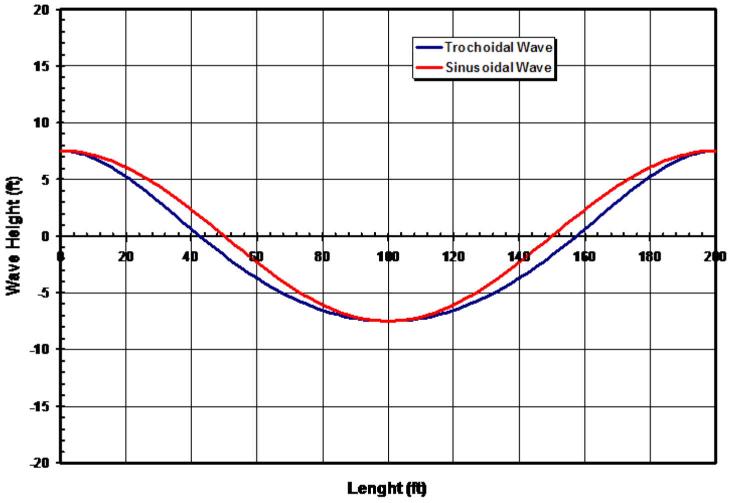
In EN342 you developed the *Stillwater* Buoyancy Curve and simply called it by another name – the "Section Area Curve". Review the appropriate sections of PNA if you have forgotten the details.

Recall from the section on loads that the longitudinal distribution of buoyancy applied to a ship by a passing wave (or a ship passing through a wave) creates a wave-induced bending moment on the ship, which varies along the length of the ship. Note that this is in addition to the stillwater hydrostatic bending moment! Recall also the two extreme cases – hogging and sagging condition. We will therefore consider these as likely "worst-case" conditions and analyze them. The approach is to consider that the ship is "statically balanced" on a wave of a given wave height and wave length.

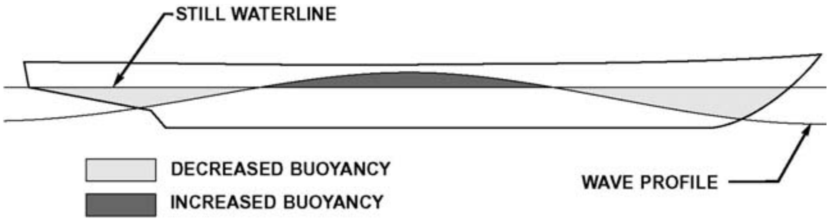
The design hogging and sagging wave profile is selected based upon classification society or design authority rules. The wave length used is usually the length between perpendiculars (L_{BP}). The wave height used depends upon the type and size of ship, and is generally given by the classification or rules authorities. For this course, we will use a design value specified for naval combatants (U.S. Navy):

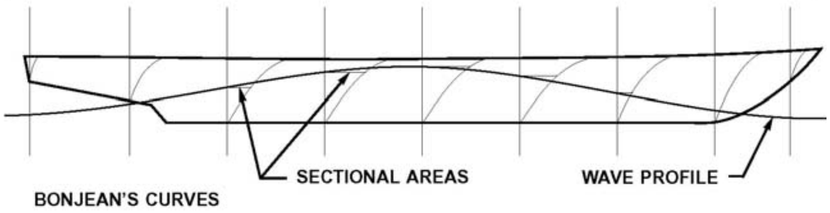
$$H_w = 1.1\sqrt{L_{BP}}$$

where H_w is the design wave height (feet), and L_{BP} is the ship's length between perpendiculars (feet). The profile or *form* of the wave is usually chosen as "trochoidal". A *trochodal wave form* has steeper crest and flatter trough than a sinusoidal wave form, and is normally used because it gives a better representation of an actual open ocean wave (see figure below). Occasionally, for shallow water, a *cnoidal wave* is used, as it has even steeper crests. We will use the troichodal wave form for this course.



For this course, we will "allow" the Hydromax software to generate our stillwater and wave-induced buoyancy distributions (and also our load, shear and moment curves!), by integrating hull offsets (which you will enter). In the "old days" (before PC computers - which most of your professors are old enough to remember), all of the integration had to be done "by hand" using hand calculators (or, in the really old days, by slide rules). One tool which came in handy for this was the "*Bonjean Curves*", which provided section areas as a function of waterline (drafts) at each station location. The Bonjean Curves could be used to calculate a modified section area curve, which incorporated the still water plus wave induced buoyancy distribution. The below figure illustrates this. It's unlikely that you would ever have to do this, but who knows…





For initial concept design studies, naval architects often use *approximate equations* to estimate the wave-induced bending moments. A quick review of the literature (PNA, etc.) will turn up dozens of approximate methods, some more accurate than others. In almost all cases, the methods are semi-empirical (meaning that they are based on fundamental theories, but are adjusted based on test results and comparisons with actual ships). They also tend to be conservative, meaning that they will usually give results (bending moments, stresses) higher than actual. This is to avoid the potential situation where a ship is under-designed due to its falling outside the typical values appropriate to the equations (the empirical parts).

In many cases the approximate equations are only applicable to a particular ship type, such as bulkers, tankers, containerships or destroyers. Understandably, the more specific the equation, the more likely it will be accurate within its limits. In PNA (Volume I, page 213), a very general added bending moment in waves equation used by ABS is given (bending moment added to the maximum stillwater bending moment). It has the form:

$$M_w = C_2\, L^2\, B\, H\, K_b$$

where, the variables are: *…(the student should complete this section)*

Alternatively, Hughes presents an approximation that is often used in Europe (by DNV):

For the vessel from Station 1 (of 10) to $C_B L/4$ forward of midships, the added bending moment in waves (bending moment added to the stillwater bending moment):

$$M_{\text{w-sagging}} = 125\, L^2\, B\,(C_B + 0.2)\, h_w \quad (N \cdot m)\,, \quad M_{\text{w-hogging}} = 165\, L^2\, B\, {C_B}^2\, h_w \quad (N \cdot m)$$

For L < 350 meters, h_w is the smallest of: $D \quad or \quad 13 - \left(\frac{250 - L}{105}\right)^3 \quad or \quad 13 - \left(\frac{L - 250}{105}\right)^3$

For L > 350 meters, h_w is $\frac{227}{\sqrt{L}}$

Note that all parameters must be given in consistent metric units (L, B, h_w in meters, M_w in N·m)

Forward of Station 1 the bending moment tapers to zero at Station 0 and aft of $C_B L/4$ forward of midships it also tapers to zero at Station 10. This gives a bending moment diagram that is trapezoidal.

It is considered "good practice" when calculating the bending moment using the buoyancy and weight curves method to compare the results to the approximate equations shown above! If there is a large discrepancy a red flag goes up and more digging is necessary.

Weekly Assignment #2: Boundary Conditions & Hull Girder Bending Analysis

In Lab:

1. Describe what the following boundary conditions imply: fixed, pinned, roller, slider, free, elastic. Include the constraints on translation and rotation and whether shear and moment can occur at that boundary location. Draw a sketch that demonstrates each condition. Draw the X, Y and Z axes and identify if there is rotation or translation in each degree of freedom. For example: "Fixed: no translations or rotations in any axis."

2. Draw the load, shear and bending moment diagrams for a horizontal cantilevered beam that is fixed on the left side. Include a distributed load w due to its own weight, and a vertical point load P, oriented down at the center. Note what the total reaction force is, and the maximum shear and bending moments. Assume a length of L.

3. Do the same exercise as #2, assuming an elastic foundation instead of fixed.

4. Sketch in profile a deck girder (with deck plating) that at the left end is welded to a bulkhead. On the right end the beam tapers to zero on a slope of 1:3 as it touches the right bulkhead. Identify the effective boundary conditions at each end of the girder (eg, does it act like fixed or simply-supported?). Draw another sketch where the left end is held by a bracket with a single large bolt and the right end is held by three bolts. Identify the boundary conditions. Does it make a difference to any of the boundary conditions if the structure is corroded?

5. Which boundary condition generates higher stresses at the ends, pinned or fixed? Why?

Take-home: Complete the following hull girder analysis

6. Draw the weight, buoyancy, load, shear and bending moment diagrams for a proposed box-shaped tank barge with five equally-spaced cargo compartments. The cargo is jet fuel. The length is 250', beam is 30', and depth is 30'. The hull bottom plate is 40#, the side shell and transoms are 30#, the bulkheads and main deck are 20#. You can assume the structural weight is evenly distributed along the length. Cargo tanks 1,3 and 5 are full, the others are empty.

7. Identify the maximum shear load and bending moment and their locations.

(Note: this last problem is very similar to one of the more difficult questions on the PE Exam. Since you will be able to do it fairly easily, realize that the PE is not an insurmountable goal!)

Moment of Inertia and Section Modulus Calculation

The previous sections were devoted to finding the maximum shear force and bending moment. The goal was to generate the information necessary to determine the maximum stress in the hull girder due to longitudinal bending. With the maximum moment determined, the next step is to calculate for the critical hull structural sections the section modulus (combines the 2nd moment of area and distance from the neutral axis to the hull bottom and deck).

Recall from EN221 (Statics and Mechanics of Materials) that the *moment of inertia* (2nd Moment of Area) of a rectangular area about its own horizontal mid-plane (i.e. its own horizontal centroidal axis) is given by:

$$I_0 =$$

The good news for naval architects is that nearly all ship structures are made up of rectangular pieces (primarily plates)!

To combine a number of rectangular components together, such as in an I-beam, we use the parallel axis theorem. Recall that the moment of inertia of a composite area about a baseline axis is given by:

$$I_{BL} = \sum I_o + \sum A_i \cdot y_i^2$$

where A_i is the cross sectional area of each part (the ith part) and y_i is the vertical distance from the centroid of that part to the baseline axis. Recall also that the height of the overall centroidal axis (which we will henceforth *loosely* refer to as the "neutral axis") above a baseline axis is found from:

$$y_{NA} = \frac{\sum A_i \cdot y_i}{\sum A_i}$$

Note that in order to find the moment of inertia about the neutral axis, we must apply the parallel axis theorem again:

$$I_{NA} = I_{BL} - A \cdot y_{NA}^2$$

Note that the total area A is given by ΣA_i

The *section modulus* (usually denoted SM, or sometimes Z) combines the moment of inertia and the maximum distance from the neutral axis (i.e. the distance to the outermost material of the beam section).

$$SM_{top} = \frac{I_{NA}}{y_{top}} \qquad SM_{bottom} = \frac{I_{NA}}{y_{bottom}}$$

The *minimum section modulus* is useful, as the maximum bending stress is found by dividing the maximum bending moment by the minimum section modulus.

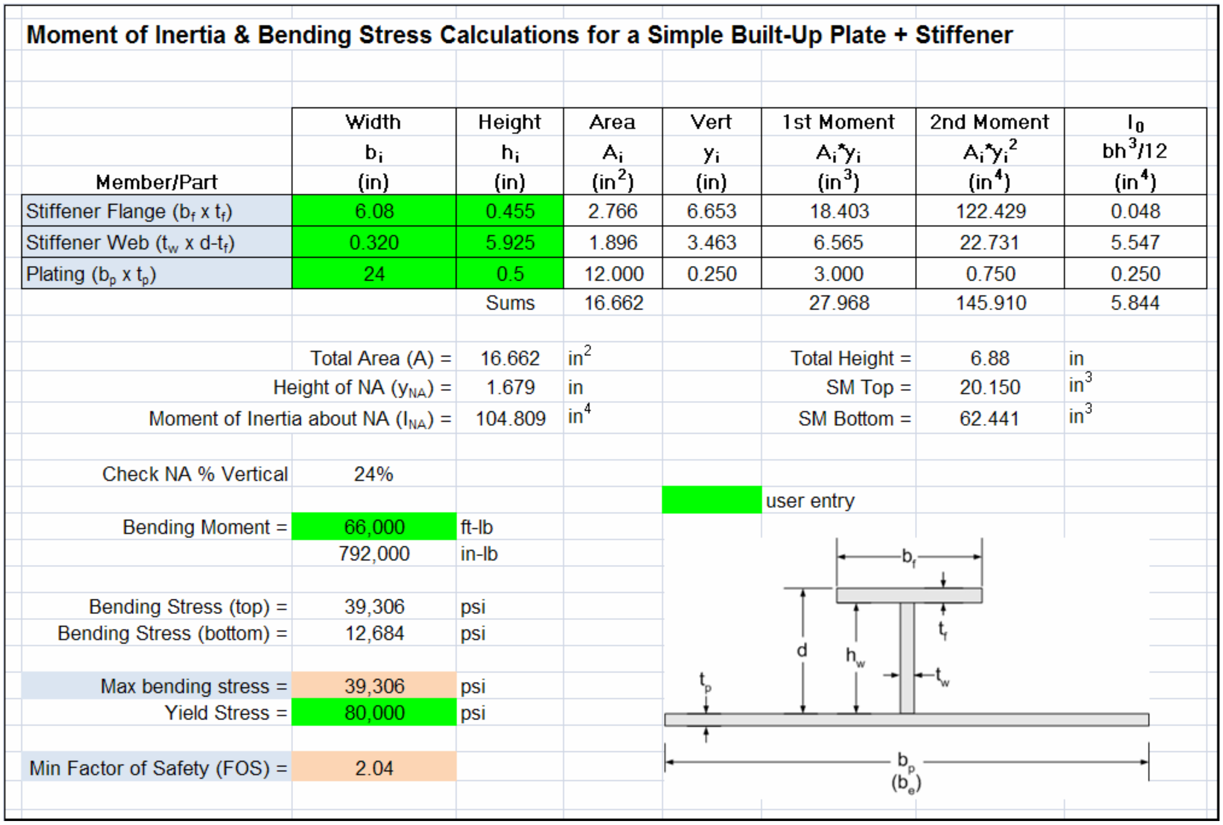
T-beam Exercise:

What is the moment of inertia and minimum section modulus of a T-beam, where the web is 3" x 0.5" and the flange is 2" x 0.25?

Where will the beam yield first when subject to a bending moment?

(The answer is given on the last page of the reader)

The following spreadsheet gives an example of an efficient way to calculate the moment of inertia, section modulus, bending stress and factor of safety for a simple beam cross-section (a simple built-up plate and stiffener). The spreadsheet is located on the course Blackboard page. Try an example (such as the above T-beam exercise) and compare a hand calculation to the spreadsheet.



Moment of Inertia & Bending Stress Calculations for a Simple Built-Up Plate + Stiffener

Member/Part	Width b_i (in)	Height h_i (in)	Area A_i (in^2)	Vert y_i (in)	1st Moment $A_i{*}y_i$ (in^3)	2nd Moment $A_i{*}y_i^2$ (in^4)	I_0 $bh^3/12$ (in^4)
Stiffener Flange (b_f x t_f)	6.08	0.455	2.766	6.653	18.403	122.429	0.048
Stiffener Web (t_w x d-t_f)	0.320	5.925	1.896	3.463	6.565	22.731	5.547
Plating (b_p x t_p)	24	0.5	12.000	0.250	3.000	0.750	0.250
		Sums	16.662		27.968	145.910	5.844

Total Area (A) = 16.662 in^2

Height of NA (y_{NA}) = 1.679 in

Moment of Inertia about NA (I_{NA}) = 104.809 in^4

Total Height = 6.88 in

SM Top = 20.150 in^3

SM Bottom = 62.441 in^3

Check NA % Vertical 24%

🟩 user entry

Bending Moment = 66,000 ft-lb

792,000 in-lb

Bending Stress (top) = 39,306 psi

Bending Stress (bottom) = 12,684 psi

Max bending stress = 39,306 psi

Yield Stress = 80,000 psi

Min Factor of Safety (FOS) = 2.04

What becomes apparent after designing a few beams is that for efficiency (minimizing weight) the beam would have the same factor of safety at the top and bottom. If the beam is made of a uniform material, that would mean the top and bottom should have the same section modulus, which means the neutral axis is in the middle. If the materials are different, you would put the higher strength material where the lower section modulus is. Of course, dissimilar materials gives you the problem of…(hint: EN380)!

When a T-beam, L-beam ("angle"), or flat bar is welded to the hull or deck plating, the question becomes "how much of the plating can I count as a flange" (i.e. what is "b_p" in the spreadsheet)? The question is a good one because of a number of factors, including the philosophical question of: "which would you want to fail first, the hull shell or the flange?" There are some rules of thumb used by the Classification Societies, as well as theoretical approaches that we will cover as part of the *effective breadth* and *effective width* concepts (shortly).

Calculating the moment of inertia and the section modulus for a complete ship's hull is almost identical to that of a simple beam as in the above spreadsheet – except that it includes many more pieces, and we may want to include the possibility of having curved and inclined plates! This is also a great use for a spreadsheet. When these calculations were done by hand, a common approximation was to ignore all the I_o contributions of the horizontal plates as they were very small. These days they are usually included as the spreadsheet calculates them automatically, but you may see both methods used as the difference is very small, except for large stiffeners such as a CVK. Another common practice is to calculate the moment of inertia of just the starboard half of the ship's cross-section, and then multiply it by two to get the full moment of inertia. Don't try that if the vessel is asymmetrical (such as a CVN)!

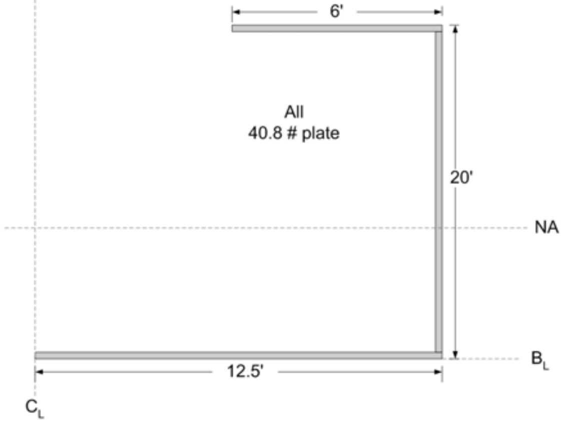
The basic process for calculating the Section Modulus for both “deck” (main deck) and “keel” (bottom plating) for a ship is as follows:

1. Calculate the area of each component (A_i)
2. Calculate the height of each component area (centroid) above the baseline (y_i)
3. Calculate the 1st moment of each component area about the baseline (A_iy_i)
4. Calculate the moment of inertia (2nd moment of area) of each component about the baseline ($A_iy_i^2$)
5. Calculate the moment of inertia of each component about its own horizontal centroidal axis (I_0)

   Note for vertical or horizontal plates (with breadth b, height h): $I_0 = bh^3/12$
6. Calculate the height of the NA above the baseline ($y_{NA} = \sum A_iy_i / \sum A_i$)
7. Calculate the moment of inertia of the total section about the baseline ($I_{BL} = \sum A_iy_i^2 + \sum I_0$)
8. Calculate the moment of inertia of the total section about the NA ($I_{NA} = I_{BL} - Ay_{NA}^2$)
9. Calculate the section modulus for deck and keel

Example:

Consider a simple box-shaped barge:



Create a table

Item	Scantlings (b x h) (in)	Area A_i (in^2)	Height above BL y_i (in)	1st moment about BL $A_i y_i$ (in^3)	2nd moment about BL $A_i y_i^2$ (in^4)	2nd moment about own centroid I_0 (in^4)
Deck	72 x 1	72	239.5	17,208	4,129,938	6
Side pl	1 x 238	238	120	28,560	3,427,200	1,123,439
Bottom pl	150 x 1	150	0.5	75	37.5	12.5
Total (½ sction)		$\sum A_i =$ 460		$\sum A_i y_i =$ 45,843	$\sum A_i y_i^2 =$ 7,557,175.5	$\sum I_0 =$ 1,123,457.5

Height of NA above BL: $y_{NA} = \dfrac{\sum A_i y_i}{\sum A_i} = \dfrac{45{,}843}{460} = 99.66\,\text{in} \approx 100\,\text{in} \approx 8.3\,\text{ft}$

Total Moment of inertia of section about NA:

$$I_{BL} = I_{NA} + A \cdot y_{NA}^2 \rightarrow$$

$$I_{NA} = I_{BL} - A \cdot y_{NA}^2 = \underbrace{\sum I_0 + \sum A_i y_i^2}_{I_{BL}} - \left(\sum A_i\right) \cdot y_{NA}^2 = 1{,}123{,}457.5 + 7{,}557{,}175.5 - (460) \cdot (99.66)^2 = 4{,}111{,}860\,\text{in}^4$$

…but this is only ½ the section, so the full $I_{NA} = 8{,}223{,}720\,\text{in}^4$

What about the maximum bending stress and FOS ?

$y_{max} = y_{deck} = 140$ in so $SM_{deck} = \dfrac{I_{NA}}{y_{deck}} = \dfrac{8{,}223{,}720\ \text{in}^4}{140\ \text{in}} = 58{,}741\ \text{in}^3$

From the previous loading example:

$$M_{max} = 34{,}800\,\text{ft} \cdot \text{LT} \rightarrow \sigma_{deck} = \frac{M_{max}}{SM_{deck}} = \frac{34{,}800\,\text{ft - LT}}{58{,}741\,\text{in}^3}\left(12\frac{\text{in}}{\text{ft}}\right)\left(2240\frac{\text{lb}}{\text{LT}}\right) = 15{,}925\frac{\text{lb}}{\text{in}^2}$$

$$\text{FOS} = \frac{\sigma_Y}{\sigma_{deck}} = \frac{36\,\text{ksi}}{15.9\,\text{ksi}} \approx 2.3$$

Below is an example of a full hull section modulus calculation spreadsheet, which is also located on the course Blackboard page. See PNA Chapter 4, Section 3.3 (and especially Table 2) for an additional hand calculation example.

Global Hull Girder Section Modulus and Bending Stress Spreadsheet

Units (USCS/SI):	USCS	(USCS, SI)
Height of Deck:	24	in

Green boxes are for data entry!

Plating (Major components) (one-half of ship)

Component	# of items	Plate wt #	Width	Thickness	Angle	Total Area (A_i)	Height wrt BL (y_i)	1st Moment (A_iy_i)	2nd Moment ($A_iy_i^2$)	Local Moment (I_0)
			in	in	deg	in^2	in	in^3	in^4	in^4
hull bottom plate	1.0	10.2	36.00	0.2500	0.00	9.0000	0.13	1	0	0
side plate	1.0	10.2	24.00	0.2500	90.00	6.0000	12.00	72	864	288
gunwale	1.0	40.8	2.00	1.0000	90.00	2.0000	23.00	46	1058	1
deck	1.0	10.2	6.00	0.2500	0.00	1.5000	24.00	36	864	0
				0.0000		0.0000		0	0	0
				0.0000		0.0000		0	0	0
				0.0000		0.0000		0	0	0
				0.0000		0.0000		0	0	0
				0.0000		0.0000		0	0	0
				0.0000		0.0000		0	0	0
				0.0000		0.0000		0	0	0
					Sum	18.5		155.1	2786.1	288.7

Longitudinals (Small components) (one-half of ship)

Component	# of items	Area	Total Area (Ai)	Height wrt BL (y_i)	1st Moment (A_iy_i)	2nd Moment ($A_iy_i^2$)
		in^2	in^2	in	in^3	in^4
longitudinal	1.0	0.75	0.75	1.5	1	2
					0	0
					0	0
					0	0
					0	0
					0	0
					0	0
					0	0
					0	0
					0	0
					0	0
	Sum		0.75		1.1	1.7

Example: (Baseline at the bottom)
gunwale strake 1" x 2"
side plating 0.25" x 24"
bottom plate 0.25 x 72"
longitudinal 0.25" x 3"
chine radius 1" for 90 degrees
deck 0.25" x 6"

Curved Plating (one-half of ship)

Component	# of items	wt #	Curve Radius	Circle Fraction	Thickness	Total Area (A_i)	Height wrt BL (y_i)	1st Moment (A_iy_i)	2nd Moment ($A_iy_i^2$)	Local Moment (I_0)
			in	%	in	in^2	in	in^3	in^4	in^4
chine radius	1.0	10.2	1.0	25.0	0.2500	0.3927	0.3	0	0	0
					Sum	0.4		0.1	0.0	0.0
					TOTAL SUM	**19.6**		**156.4**	**2787.9**	**288.8**

Half-Ship Values

y_{NA}	8.0	in	
1/2 I_{BL}	3076.6	in^4	
A x y^2_{NA}	1244.8	in^4	
1/2 I_{NA}	1831.8	in^4	
% ht of NA abv BL	33%		Note: The goal is as close to 50% as is practical.

Full Ship Values

Area	37.0	in^2
I_{NA}	3663.7	in^4
SM_{deck}	228.4	in^3
SM_{keel}	460.2	in^3

Stillwater Moment	100	LT-ft	Notes: Identify whether from ABS, DnV, GHS or hand calcs
Hogging Moment	150	LT-ft	
Sagging Moment	170	LT-ft	
DAF	1		Include dynamic amplification factor, if appropriate

Material yield	34000	psi
Allowable FOS	2.3	
Material allowable	14783	psi

			Actual FOS	OK?
Stillwater deck stress	11768	psi	2.9	OK
Stillwater bottom stress	5841	psi	5.8	OK
Hogging deck stress	17652	psi	1.9	FAIL
Hogging bottom stress	8761	psi	3.9	OK
Sagging deck stress	20005	psi	1.7	FAIL
Sagging bottom stress	9929	psi	3.4	OK

A few additional notes:

Not all structure can be considered “structurally effective” for longitudinal bending. It makes sense that only *continuous longitudinal structure* is counted, and it must be *rigidly connected*. Hatch covers and stanchions, for example, are not counted as contributing to global (longitudinal) hull strength. A general “rule-of-thumb” is that structural components must be continuous for at least 40% of the L_{BP} to be counted. Additionally, there are “shadow zones” near hatch openings and discontinuities, so that even longitudinally-continuous members might not be counted in these locations. See PNA for more information on “shadow zones”.

For an inclined plate (width w, thickness t, angle to horizontal θ), the local horizontal moment of inertia is given by (this is incorporated into the spreadsheet):

$$I_0 = \frac{wt(w^2 \sin^2 \theta + t^2 \cos^2 \theta)}{12}$$

For curved plate (radius r, area a) approximately thickness times arc length)), the horizontal local moment of inertia is given by (also incorporated into the spreadsheet):

$$I_0 = \left(\frac{1}{2} - \frac{4}{\pi^2}\right) a\, r^2 \quad h = \frac{(\pi - 2)}{\pi} r$$

Weekly Assignment #3: Weight Curve & Hull Section Modulus Calculation for Mariner Class

In Lab:

Using the data on the following pages and the Weight Curve Excel spreadsheet, complete a weight curve for the Mariner class cargo ship. Present your data table (A1:K70) and the weight curve graph.

1. You must review the material provided about the ship to determine the length over which the weight item is distributed. Inputs to the spreadsheet include the item's weight, the LCG, the forward extent of the weight, and the aft extent of the item. You also choose whether it is a trapezoidal or Biles taper. Note that the LCG must be in the middle third of the extents or the "check" will fail. The distances on the profile view are by frame location rather than feet, and the frame spacing is not uniform! The easiest thing to do is to add to the spreadsheet a sheet that does a frame location-to-feet table.

2. The liquid cargo is usually determined from the tank volume, the percent filled and the fluid density. For future reference, JP5 sg = 0.755, DFM = Fuel Oil = 0.795, SW=1.025, FW = 1. In this case the weights are already tabulated for you.

3. You must check that your calculated displacement is within 2 LT of the given displacement (21100 LT) and the LCG is within six inches (266.5 ft).

Take-home:

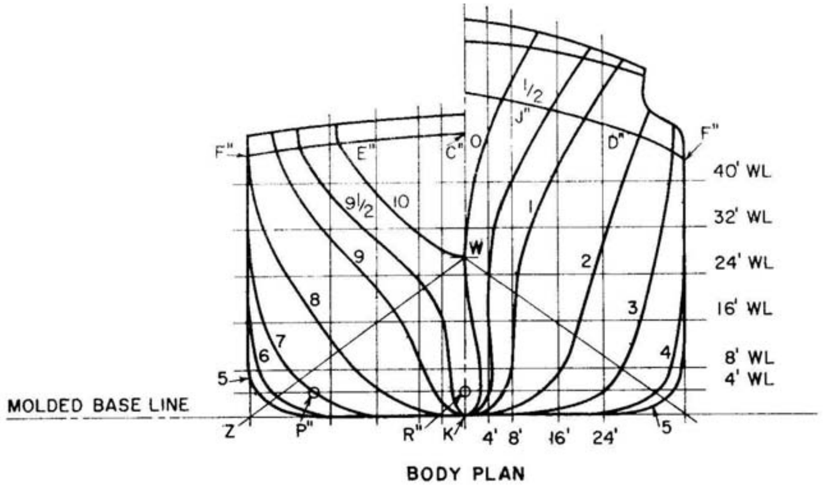
4. Using the section modulus spreadsheet, calculate the midship section modulus (deck and hull) for the Mariner (see below). Assume all the material is A36 equivalent. You will find that the dimensions are not exactly the same as the molded lines. That is common as each shipyard will often have slight differences. Ignore the lap as a structural member. Attach a copy of your spreadsheet.

5. You now have the weight curve and section modulus. What more do you need to calculate to determine if the vessel will fail by global hull girder bending?

Information for the Mariner Class:

The Mariner Class is a general cargo ship designed in the late 1940s as the C4-class and was built by over a dozen U.S. shipyards. It was the last large production run of American-made cargo ships. A number are still listed in the USNS Ready Reserve Fleet register and are in "moth-balls" at the James River (VA) or Suison Bay (CA) storage areas. They were used in the Korean War, Vietnam War and Operation Desert Shield/Storm. Many were converted to container ships, hospital ships, transports, ammo ships, passenger liners and other types. Some are still in service around the world.

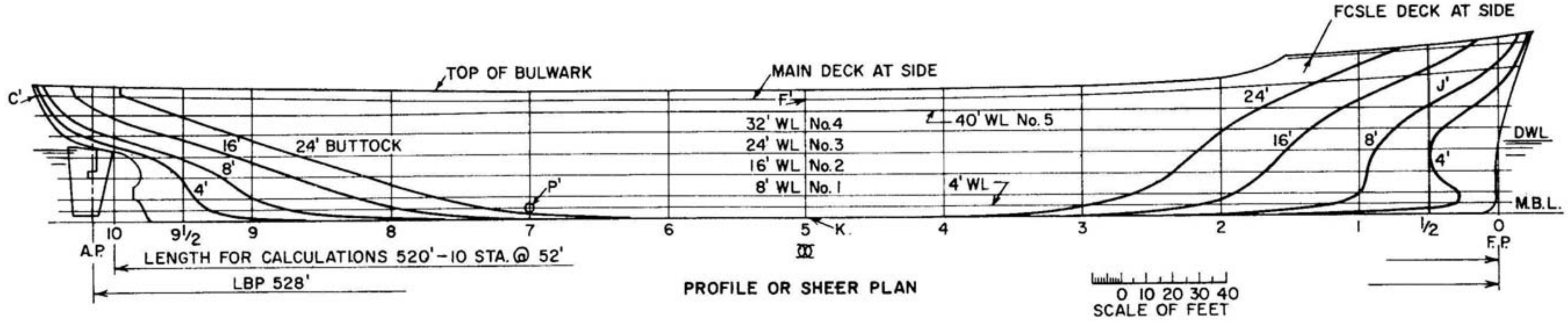
The following pages provide information necessary for completion of this weekly assignment. The Lines Drawing, Inboard Profile Drawing, and Midships Construction Drawing are all provided in high resolution (.jpg images) posted on the course Blackboard page. You should print these out as necessary to facilitate completion of this assignment. You can print the Lines Drawing and Inboard Profile Drawing on 11" x 17" paper in Rickover Room 126.



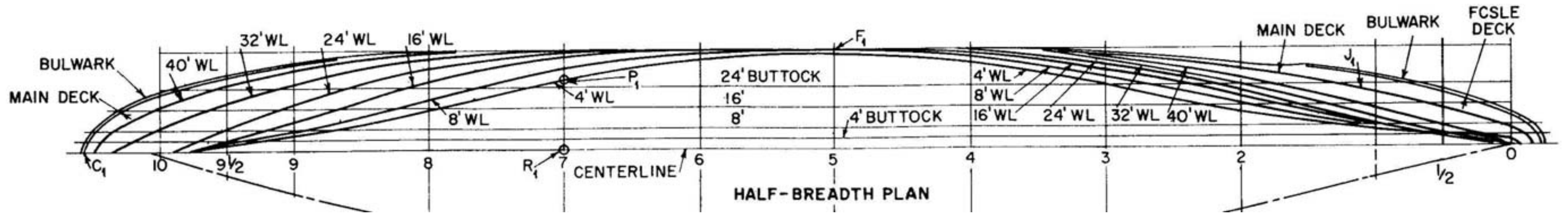
BODY PLAN

PRINCIPAL DIMENSIONS

LENGTH OVER ALL	563'-7 3/4"
LENGTH BETWEEN PERPENDICULARS	528'-0"
LENGTH FOR CALCULATIONS	520'-0"
BREADTH MOLDED	76'-0"
DEPTH MOLDED, TO MAIN DECK AT SIDE (STA 5)	44'-6"
DRAFT, MOLDED, TO DESIGNED WATERLINE (DWL)	27'-0"
DRAFT, TO LOAD WATERLINE (LWL)	29'-10"
DISPLACEMENT, MOLDED, SALT WATER-27' DWL	18,674 TONS
DISPLACEMENT, LOADED, SALT WATER-29'-10" LWL	21,093 TONS



PROFILE OR SHEER PLAN



HALF-BREADTH PLAN

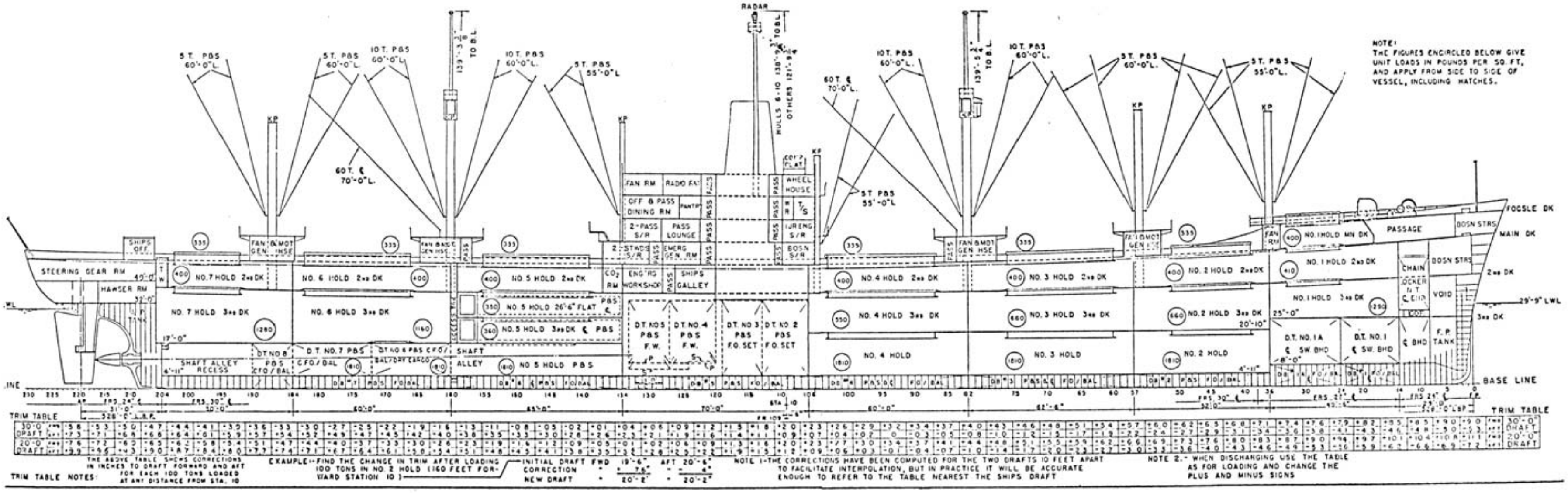
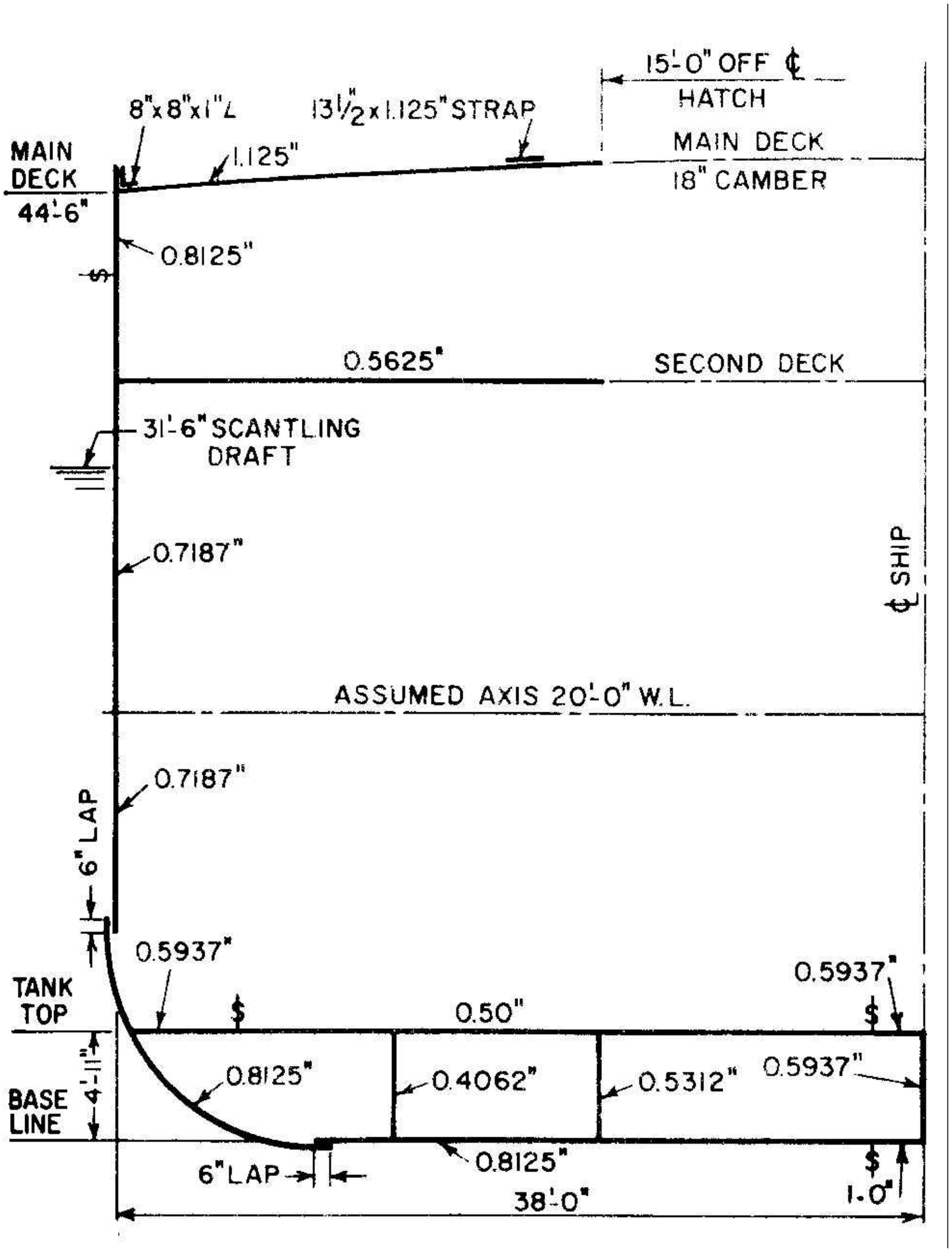


Fig 7.—Inboard Profile

Weight Summary
Mariner Class

Item	Weight	LCG	Start	End
	LT	ft aft of FP	Frame No.	Frame No.
	Note: Frame location is not the same as station location			
Lightship				
Hull Steel	3997	273.4	-7	230
Forecastle	95	0	-5	5
Deckhouse	479.6	275	105	134
Aft house	120	521	213	220
Total Steel	4691.6	274.4		
Boilers	394.2	279.3	106	127
Main Engines	499.2	314.3	127	134
Shafting	115	421.5	134	215
Total Machinery	1008.4	312.8		
Hull Engineering	683.7	280	0	219.75
Wood and Outfit	1289	265.2	0	219.75
Total Outfit	1972.7	270.33		
Deadweight - Full Load				
Crew and Stores	63	293.3	105	134
Fresh Water				
No. 4 Deep Tank	123.7	296.8	120	127
No. 5 Deep Tank	108.4	313	127	133
Distilled Water	24.9	256.8	106	109
Total FW	257	299.7		
Fuel Oil and SW Ballast				
#1 Deep Bottom Tank	131.4	55.6	14	36
#2 DBT	143.9	106.6	36	57
#3 DBT	342.3	163.8	57	82
#4 DBT	485.4	223	82	106
#5 DBT	560	284.8	106	134
#6 DBT	420.7	351.8	134	160
#7 DBT	191.1	412.4	160	184
#1 Deep Tank	386.9	57	14	36
#2 DT	203.5	260.8	106	113
#3 DT	173.9	277	113	119
#6 DT	406.6	401.2	160	172
#7 DT	260.2	430.7	172	184
#8 DT	102.1	454	184	190
Fore Peak Tank	110.8	17.1	0	14
Aft Peak Tank	93	506.8	204	218
Total Fuel and Ballast	4011.8	266		
Dry Cargo				
Hold #1	573.3	56.8	14	36
Hold #2	1102.8	105.3	36	57
Hold #3	1870.7	161.9	57	82
Hold #4	1992.8	222.2	82	106
Hold #5	1386.7	353.8	134	160
Hold #6	1436.9	414.5	160	184
Hold #7	732.2	469.5	184	203
Total Dry Cargo	9095.4	255.6		
Total Deadweight	13427.2	259.7		
Total Displacement	21099.9	266.5		


15'-0" OFF ¢
HATCH
8"x8"x1"L
13 1/2"x1.125" STRAP
MAIN DECK
44'-6"
1.125"
MAIN DECK
18" CAMBER
0.8125"
0.5625"
SECOND DECK
31'-6" SCANTLING
DRAFT
0.7187"
¢ SHIP
ASSUMED AXIS 20'-0" W.L.
0.7187"
6" LAP
TANK
TOP
0.5937"
0.50"
0.5937"
BASE
LINE
4'-11"
0.8125"
0.4062"
0.5312"
0.5937"
6" LAP
0.8125"
1.0"
38'-0"

Composite Beam Approach (Dissimilar Materials)

At this point we have calculated the bending moment and the section modulus of beams and hull girders. That gives us the bending stresses due to waves and hull weight distributions. We used Euler's beam equation as the numerical model for the "simple beam" analysis. The problem is that Euler's equation is somewhat limited as it assumes that all the material in the moment of inertia calculation has the same stiffness (Modulus of Elasticity). It is easy to see the issue if we replaced the steel main deck with one made of neoprene! It would be just as watertight, but it sure would not carry the same amount of load!

There is a relatively simple way to take into account different materials in a bending beam. It is not as accurate as using a higher-order method available in finite element analysis, but it works well enough for preliminary design if the factors of safety are adequate. In a nutshell the method "replaces" one of the materials with an amount of the other material that would give it an equivalent *axial stiffness* (EA). In other words, you create a flange of the base material that is the same thickness but may be wider or narrower than the actual flange. The method is called the "composite beam" method by naval architects, or the "equivalent area" method by civil engineers.

The method works especially well for simple composite beams such as steel reinforced concrete (civil engineers use this all the time in buildings and bridges), but is adapted and used also to some extent for simple analyses of thin walled beams such as ships. It is also quite often used in "composite construction". In the 19th century, wood frames capped with a bronze or steel strap and metal frames with wood planking, were frequently used in ship construction. Today (going into the 21st century), it means fiberglass frames capped with carbon fiber, or similar "composite" configurations.

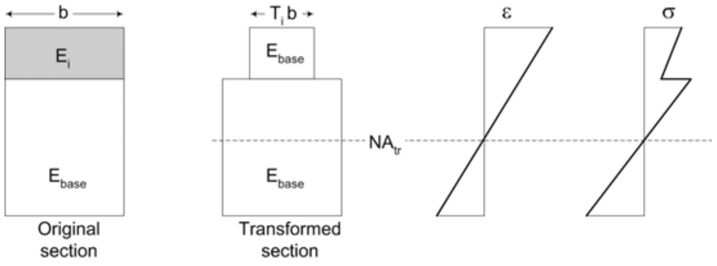
The most common use in ships is for decks or superstructures of different materials (e.g. an aluminum superstructure with a steel hull). For the method to be most accurate it ideally must be a *full-width* superstructure, uniform over its length, and rigidly connected. The method is also applied to other structures such as stiffened panels.

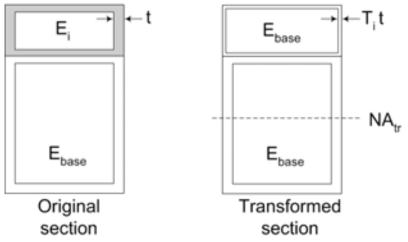
The basic steps of the composite beam approach are:

1. Define the *transformation factor*: $T_i = \frac{E_i}{E_{base}}$ (for example: material *i* = aluminum, base material = steel)
2. *Transform* the *breadth* of all structural components of material *i* by multiplying the current breadths by the transformation factor T_i (this can be done in a table). Note: The net *height* of the transformed section must remain the same (we transform only the breadth of the components).
3. Using the transformed breadth for components of material *i*, calculate the section properties for the transformed section:
    - (i) Neutral axis of the *transformed* section (NA_{tr})
    - (ii) Moment of inertia of the *transformed* section (I_{tr})
    - (iii) Section modulus for the extreme fibers of each material (using NA_{tr} and I_{tr})
4. Calculate the stresses *within each material*:
    - (a) Within the base material $\sigma_{base} = \frac{M\,y}{I_{tr}}$ (y is distance from NA_{tr})
    - (b) Within material *i* use $\sigma_i = T_i \frac{M\,y}{I_{tr}}$

Note that because of this transformation, a material located closer to the neutral axis might have a lower factor of safety!

The figures on the next page illustrate this transformation.



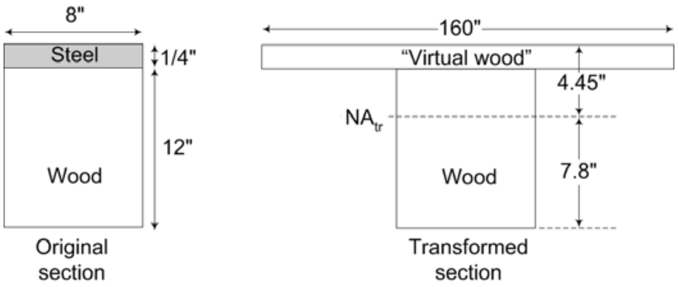


Example:

Consider a steel-capped wood beam.

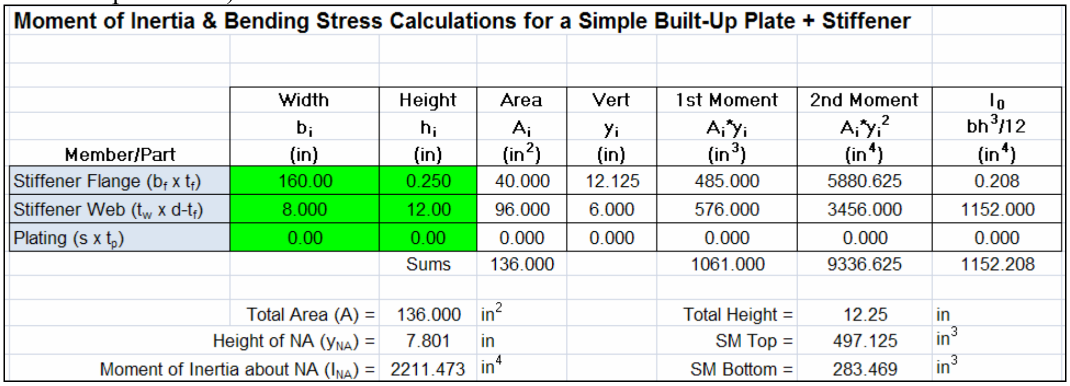
What is the minimum FOS for a mahogany & 316L (annealed) SS composite beam w/ a moment of 40,000 ft-lb?

Use $E_{wood} = 1.5$ msi and $E_{steel} = 30$ msi



Steps:

1. Calculate the transformation factor (arbitrarily choosing the wood as base): $T_{steel} = \dfrac{E_{steel}}{E_{wood}} = \dfrac{30\text{ msi}}{1.5\text{ msi}} = 20$
2. Transform the *breadth* of the steel component by multiplying current breadth by transformation factor T
$$b_{steel} = 20 \cdot 8\text{in} = 160\text{in}$$
3. Using the transformed breadth of steel, calculate the transformed section properties (using a simple beam calculation spreadsheet):



Moment of Inertia & Bending Stress Calculations for a Simple Built-Up Plate + Stiffener

Member/Part	Width b_i (in)	Height h_i (in)	Area A_i (in^2)	Vert y_i (in)	1st Moment $A_i{*}y_i$ (in^3)	2nd Moment $A_i{*}y_i^2$ (in^4)	I_0 $bh^3/12$ (in^4)
Stiffener Flange (b_f x t_f)	160.00	0.250	40.000	12.125	485.000	5880.625	0.208
Stiffener Web (t_w x d-t_f)	8.000	12.00	96.000	6.000	576.000	3456.000	1152.000
Plating (s x t_p)	0.00	0.00	0.000	0.000	0.000	0.000	0.000
		Sums	136.000		1061.000	9336.625	1152.208
	Total Area (A) =	136.000	in^2		Total Height =	12.25	in
	Height of NA (y_{NA}) =	7.801	in		SM Top =	497.125	in^3
	Moment of Inertia about NA (I_{NA}) =	2211.473	in^4		SM Bottom =	283.469	in^3

(i) Neutral axis of the *transformed* section $NA_{tr} = 7.8\text{ in}$

(ii) Moment of inertia of the *transformed* section $I_{tr} = 2211\text{in}^4$

4. Calculate the maximum stresses and minimum FOS for each material:
   (a) Wood

$$\sigma_{\text{wood}} = \frac{My}{I_{tr}} = \frac{(40{,}000\,\text{ft - lb})\left(12\,\frac{\text{in}}{\text{ft}}\right)(7.8\,\text{in})}{(2211\,\text{in}^4)} = 1693\,\text{psi}$$

$$\text{FOS}_{\text{wood}} = \frac{\sigma_{\text{Ywood}}}{\sigma_{\text{wood}}} = \frac{6630\ \text{psi (ABS)}}{1692\text{psi}} = 3.9$$

   (b) Steel

$$\sigma_{\text{steel}} = T\frac{My}{I_{tr}} = 20\frac{(40{,}000\,\text{ft - lb})\left(12\,\frac{\text{in}}{\text{ft}}\right)(12.25 - 7.8\,\text{in})}{(2211\,\text{in}^4)} = 19{,}321\,\text{psi}$$

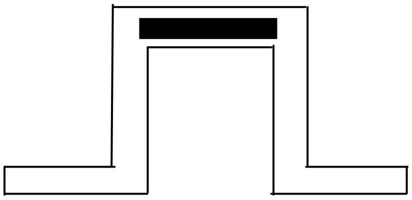
$$\text{FOS}_{\text{steel}} = \frac{\sigma_{\text{Ysteel}}}{\sigma_{\text{steel}}} = \frac{30{,}000\ \text{psi}}{19{,}321\ \text{psi}} = 1.55$$

Since the factor of safety for the steel is below the acceptable value of 2, what do we do?
Solutions include adding material to the cap and/or web. You might make it wider for instance. Another option is to change the material. To get a factor of safety of 2 we would need a yield strength of 38.6 ksi. If we assume that the 316L has a linear plastic range, with a yield of 30 ksi and an ultimate of 81, by cold-working it 20% we will raise the yield to 40.2 ksi! The downside is that the stainless will be less-ductile, but we can live with that.

Composite Beam Exercise:

A 3/16” thick fiberglass hat section has a web cap that is 1/4” thick unidirectional carbon. What is the maximum moment that the stiffener can handle?

E of unidirectional carbon in epoxy = 17 msi
E of fiberglass cloth in epoxy = 2 msi
Compressive strength of the carbon cap = 86 ksi
Tensile strength of the carbon cap = 120 ksi
Compressive strength of the fiberglass web and cap = 36 ksi
Tensile strength of the fiberglass web and cap = 40 ksi
The cap width is 3” and the stiffener height is 4”.
The bottom flanges are each 2” wide.



(The answer is given on the last page of the reader)

Weekly Assignment #4: Hull Girder Bending Analysis for Mariner Class

In Lab:

1. Use Maxsurf and Hydromax to generate weight, buoyancy, load, shear, and bending moment curves for the Mariner class in the full load condition (for which you generated a weight curve using Excel in the previous assignment). To do this, you will need to digitize the body plan and generate a "tri-mesh" hull surface in Maxsurf, save the Maxsurf design, then import the design into Hydromax to run the longitudinal strength analysis. This should be done following the steps described in class. Generate and plot these curves for the stillwater condition, as well as the hogging and sagging wave conditions (use the appropriate design wave height, with wave length, using a trochoidal wave).

Take-home:

2. Using your calculated maximum bending moments (for each condition), and the section modulus calculation for the midships section (which you calculated in the previous assignment), determine the minimum FOS for the ship at the hull bottom and main deck vs. material yield. (Later we will look at buckling).

3. Determine the FOS for hogging and sagging using the *approximate* methods used by ABS and DNV. For ABS, use the handouts. For DNV, use the equations presented in the notes on page 21 (given in Hughes). Watch your units!

4. From the maximum bending stress calculated above, determine the maximum longitudinal tensile and compressive force in the hull bottom and main deck (for each condition).

5. Present and discuss your results for 1-4 above. Include a hard copy of each plot, and a screen capture of your digitized body plan (use the section view), and a perspective view (render it to visualize the 3-D hull surface). Provide *all of your calculations* for 2-4. Be sure to compare your Maxsurf/Hydromax results with ABS and DNV.

6. You have been asked to help restore the classic yacht Coronet. She has white oak wood frames capped with silicon bronze. Her frames are 3" wide and 4" tall white oak and the proposed silicon bronze caps are ¼" thick. The bending moment is 2,000 ft-lb. Is the design acceptable? Show all of your work to get the answer, and explain why or why not.

Shear Stress in Small Open Beam Sections

Back when we were looking at hull girder loading we went through the process of calculating the weight and buoyancy curves. Having these gave us the load curve, then the shear force curve, and finally the bending moment curve. The shear force curve also helps us to find the shear stress in the hull. In earlier courses you solved shear stress problems, mostly due to “direct shear” and torsion, but you did not spend a lot of time looking at more global shear stress issues. The reason was that in long (“slender”) beam-like components, the shear stresses are often small and are overshadowed by bending stresses. Because ship hull girders cannot be considered long “slender” members, and because of unusual geometry and loading, we cannot just ignore (or overly simplify) shear stresses for ship structures – we can not ignore the shear stress contribution to VonMises!

In the “Box-Shaped Barge Global Hull Girder Bending” in-class exercise we did earlier, we noted that the shear force was zero at Stations 0, 5 and 10. Shear force at the ends was zero because the vessel had no overhanging bow or stern, so no weight was unsupported. The midships had zero shear force because the loading was symmetric. While it makes for a nice quick example, unfortunately “in the real world” these conditions rarely occur. The good news is that the shear force is often a small part of the combined stress (remember VonMises!). The bad news is that sometimes it is very difficult to calculate.

The challenge for the naval architect is to design to the minimum allowable factor of safety, without “over-designing” (which makes the ship “over-weight”). It would be nice to say that the bending stress gives the highest stress, and it is located at the location farthest from the ship’s neutral axis, and where the bending moment is the highest. While global bending stresses are often high, it is the *combined stress* we have to worry about, and that comes from adding the bending stress from the global bending to the shear stress from the global bending, and then additional stresses due to local bending due to hydrostatic pressure or cargo loads.

Shear stresses can be due to “direct shear” (such as in a bolt in direct shear), torsional loading (such as in propeller shafts), or when a structural member is bending (such as beams - and ships!). The torsion equation for shear stress (as a function of the radial location) for a circular shaft is the very familiar:

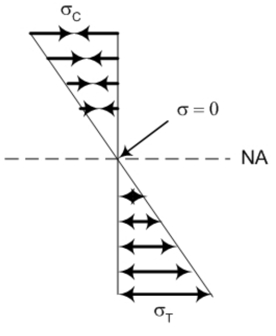
$$\tau(r) = \frac{T \cdot r}{J}$$

Where T is the internal torque, r is the radial distance, and J is the polar moment of inertia of the shaft cross-section.

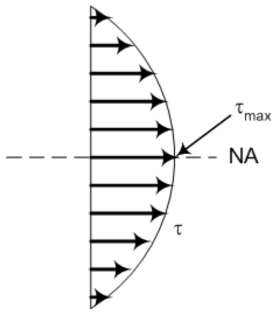
The equation for shear stress due to bending (as a function of the distance from the neutral axis) is the familiar:

$$\tau(y) = \frac{V \cdot Q(y)}{I \cdot t(y)}$$

where V is the internal shear force, y is the distance from the neutral axis, I is the moment of interia (2^{nd} moment of area) of the beam cross-section, Q is the 1^{st} moment of area of the area above y, and t is the thickness of the cross-section at y. Recall that, for a rectangular cross section, the shear stress distribution is parabolic, with maximum at the middle (neutral axis), and minimum of zero at the top and bottom. This is of course, different than the bending stress (normal stress) distribution, which is linear from zero at the neutral axis to maximum at the top and bottom. This comparison is illustrated in the below figure.



Bending Stress Distribution



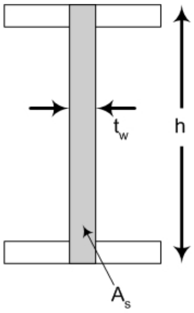
Shear Stress Distribution

Recall also that the maximum shear stress (τ_{max}) occurring at the neutral axis (y = 0) for the rectangular cross section is:

$$\tau_{max} = 1.5\frac{V}{A} \quad \text{(rectangular cross - section)}$$

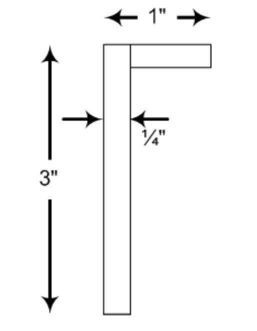
which is precisely 1.5 times the *average shear stress* $\tau_{avg} = V/A$.

A common approximation often used in structural design manuals for *small open sections*, such as channels (C-beams), I-beams, angles (L-beams), etc., is to only count a *vertical shear area (A_s)*, and then use the simple shear equation $\tau = V/A$ to *estimate* the maximum shear stress (see below figure). This obviously has some accuracy issues, but due to shear lag (which we will study later), and relatively low shear loads in many civil engineering structures, it has a reasonable track record. As an example, for a I-beam with web thickness t_{web} and overall height h, then $A_s = t_{web} \cdot h$



Example:

Consider an open "L" (angle) beam with a shear load of 10,000 lb and a tensile load of 15,000 lb. What is the FOS if it is HY-80?



$$A_s = 3\text{in} \cdot 0.25\text{in} = 0.75\text{in}^2 \qquad \tau_{max} \approx \frac{V}{A_s} = \frac{(10{,}000\text{lb})}{(0.75\text{in}^2)} = 13{,}300\text{psi}$$

$$FOS_{shear} = \frac{\tau_Y}{\tau_{max}} = \frac{0.58\sigma_Y}{\tau_{max}} = \frac{0.58(80{,}000\ \text{lb/in}^2)}{13{,}300\ \text{lb/in}^2} = 3.49$$

But, this is not the whole story! The "L" also has a tensile load, so we need to use the VonMises (equivalent) stress:

$$\sigma = \frac{P}{A} = \frac{15{,}000\text{lb}}{(0.25\text{in})(3\text{in} + 0.75\text{in})} = 16{,}000\text{psi}$$

$$\sigma_{vm} = \sqrt{\sigma_x^2 + \sigma_y^2 - 2\sigma_x\sigma_y + 3\tau^2} = \sqrt{16{,}000^2 + 0 - 0 + 3(13{,}300)^2} = 28{,}050\ \text{psi}$$

$$FOS_{vm} = \frac{\sigma_Y}{\sigma_{vm}} = \frac{80{,}000\ \text{psi}}{28{,}050\ \text{psi}} = 2.85$$

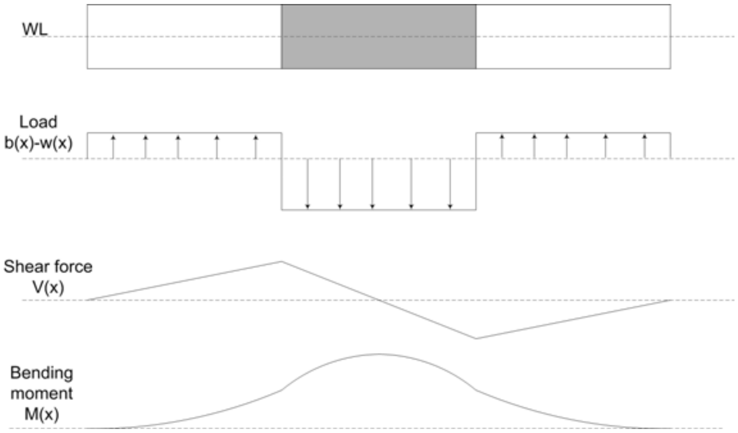
Beam Shear Exercise:

You have designed a flat bar stiffener as a deck beam and someone wants to connect a chain hoist to it. You need to calculate the shear stress prior to looking at the stress concentration factor. You have a rectangular steel beam of 20# plate that is 4" high. What is the average shear stress, maximum shear stress and the shear stress at a point 3" from the top, if the vertical shear load is 8 short tons?
(The answer is given on the last page of the reader)

Shear Stress and Shear Flow in Closed and Large Beam Sections

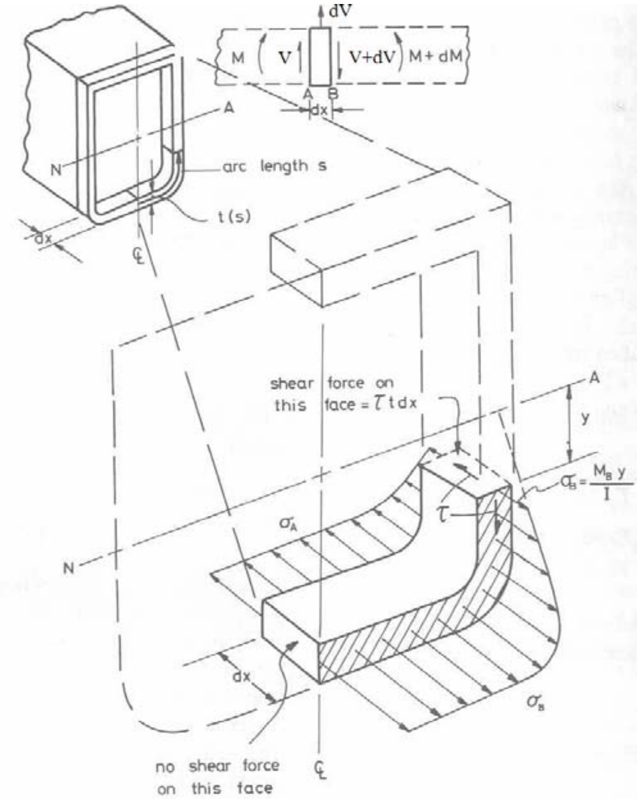
The simple shear stress equations shown above work fine for small, open structural sections, such as those often used in civil engineering construction (such as bridges and buildings). However, ship sections are neither small nor open, so a more detailed analysis is needed. Large sections, and those like box beams that are closed (an aircraft wing spar is another example) experience a characteristic called *shear flow*. This concept, while somewhat challenging, is almost unique to naval architecture and ignoring it has caused a number of very expensive ship structural failures. The most recent example was the 123’ Coast Guard Cutters of 2005!

We will start with a review of how we found the shear force. Recall the box-shaped barge example.



So how is shear force transmitted through the ship section and into shear stress? Like a solid rectangular beam section, it is not constant across the section! But, a ship structure is in fact a complex thin-walled structure (so it is a little more complicated than a solid beam section).

Consider a general thin-walled symmetric (about a centerline plane) box beam, subjected to a vertical shear force V. As we saw above, when developing the shear force and bending moment curves, V(x) varies along the length of the ship and causes a corresponding variation in the bending moment M(x).



For a small cross-section of the ship having a length dx (as the x-axis is the principal axis in the ship), the variation in V and M at the fore and aft ends of dx cause an inequality in bending stress across the element (σ_A and σ_B in the figure). Here is where it gets interesting. If we isolate a portion of the differential segment by making two cuts, one cut at the centerline (CL) and the other at an arc length "s" running counterclockwise along the bottom plating and then topside from CL (see the figure), the imbalance of longitudinal stress within the bottom portion of the beam must be balanced by a shear force across the cut sections. Because of symmetry, there is no shear stress at the CL, so all shear stress must be at the other cut (at arc length s).

Longitudinal <u>equilibrium</u> (in the x-direction) requires

$$\tau(s)\cdot t(s)\cdot dx = \int_0^s \sigma_B(s)\cdot t(s)\cdot ds - \int_0^s \sigma_A(s)\cdot t(s)\cdot ds$$

Substituting our bending stress equation (σ = My/I) on both faces (A and B) gives

$$\tau(s)\cdot t(s)\cdot dx = \frac{M_B - M_A}{I}\int_0^s y(s)\cdot t(s)\cdot ds = \frac{dM}{I}\int_0^s y(s)\cdot t(s)\cdot ds$$

Substituting dM = Vdx (from simple beam theory) gives

$$\tau(s)\cdot t(s) = \frac{V}{I}\int_0^s y(s)\cdot t(s)\cdot ds$$

Thus the shear stress is a function of "s" (the arc length from the "open" end), and is:

$$\boxed{\tau(s) = \frac{V\cdot Q(s)}{I\cdot t(s)} \qquad Q(s) \equiv \int_0^s y(s)\cdot t(s)\cdot ds}$$

Note the similarity to the basic shear stress equation for small open sections: $\tau(y) = \dfrac{V\cdot Q(y)}{I\cdot t(y)}$

Since naval architects try to avoid changing the plating thickness too often (as the tapers add cost), if the thickness is constant, then

$$\tau(s) = \frac{V\cdot \int_0^s y(s)\cdot ds}{I} \qquad (t = \text{constant})$$

Note that we have <u>defined</u> the integral *Q(s)* as the *1st moment about the NA of the cumulative section area starting from the "open" (shear stress-free) end of the section*. Note that the integral Q(s) is maximum at the NA (and 0 at all "open" ends of the section – including the CL plane!), and thus *shear stress is maximum at the neutral axis NA*, just like it would be for a solid section.

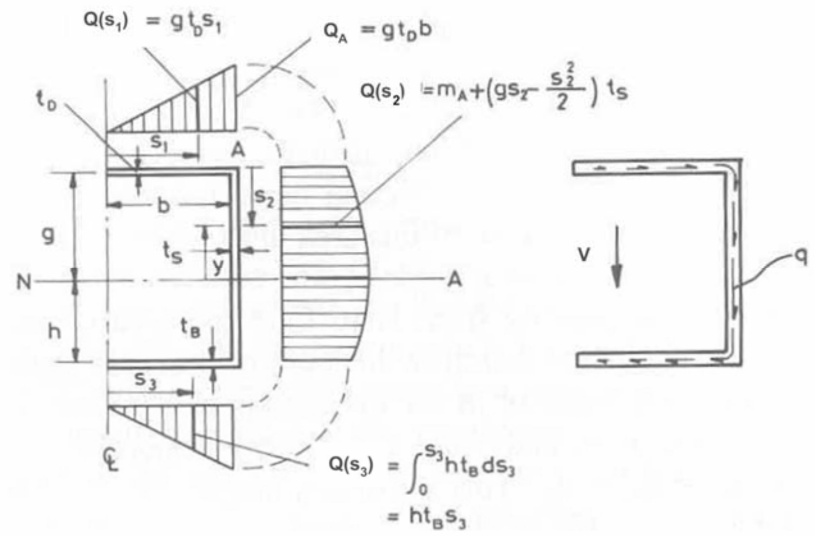
Note also that V and I are for the entire section (i.e. V(x) and I(x)), while Q (and maybe t) is a function of the arc length s from the "open" end (i.e. Q(s) and t(s)).

The product $\tau\cdot t$ (shear stress x thickness = lb/in^2 x in = lb/in) is appropriately named the ***<u>shear flow</u>*** (in analogy to pipe flow) and is often denoted by symbol $q(s) = \tau(s)\cdot t(s) = \frac{V}{I}Q(s)$

Note that the ratio V/I is a constant at each cross-section (x). Thus, this ratio simply scales the integral Q(s) to get the shear flow q(s).

**We can think of shear stress (more precisely shear flow q) as <u>flowing around</u> the thin-walled section of the box beam, being zero at the "open" ends and maximum at the NA. In fact, the laws of conservation apply to shear flow (examples later).

As a specific case, consider a box-beam with *constant plating thickness t* as shown below*:*



For horizontal portions, y is constant, and therefore Q (and therefore q and τ) increases linearly with arc length s (the horizontal distance from CL). "g" is the distance from the NA to the deck.

In the deck: $Q(s_1) = \int_0^{s_1} y \cdot t \cdot ds_1 = g\, t\, s_1 \quad \rightarrow \quad \tau(s_1) = \frac{V \cdot g \cdot t \cdot s_1}{t \cdot I} = \frac{V \cdot g}{I} s_1$ (linear in s) (similar for bottom)

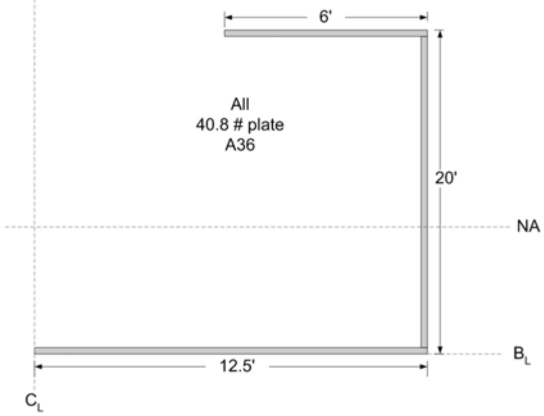
In the side shell, since y(s) = g – s2:

$$Q(s_2) = Q_A + \int_0^{s_2} y \cdot t \cdot ds_2 = Q_A + tg\int_0^{s_2} ds - t\int_0^{s_2} s_2 ds = \left(g \cdot s_2 - \frac{1}{2} s_2^2 \right) t$$

$$\rightarrow \quad \tau(s_2) = \tau_A + \frac{Q}{I}\left(g \cdot s_2 - \frac{1}{2} s_2^2 \right) \quad \text{(parabolic in s) (max. at NA)}$$

Example:

Consider our box-beam barge example from previous classes:



From the previous class: y_{NA} = 100 in = 8.3 ft I = 8,233,720 in^4 V = V_{max} = 464 LT = 1,039,360 lb

$$\tau(\text{deck edge}) = \frac{(1{,}039{,}360\text{lb}) \cdot (240\text{in} - 100\text{in})}{(8{,}233{,}720\text{in}^4)}(72\text{in}) = 1{,}272 \text{ psi}$$

$$\tau(\text{bilge chine}) = \frac{(1{,}039{,}360\text{lb}) \cdot (100\text{in})}{(8{,}233{,}720\text{in}^4)}(150\text{in}) = 1{,}893 \text{ psi}$$

$$\tau(\text{NA}) = 1{,}272\text{psi} + \frac{(1{,}039{,}360\text{lb})}{(8{,}233{,}720\text{in}^4)}\left((140\text{in})(140\text{in}) - \frac{1}{2}(140\text{in})^2 \right) = 2{,}509 \text{ psi}$$

**Note that if the plating thickness t varies around the cross-section, then this variation must be included in the integration of Q(s) (i.e. we must use the full equation for τ(s)).

Multi-Cell Shear Flow

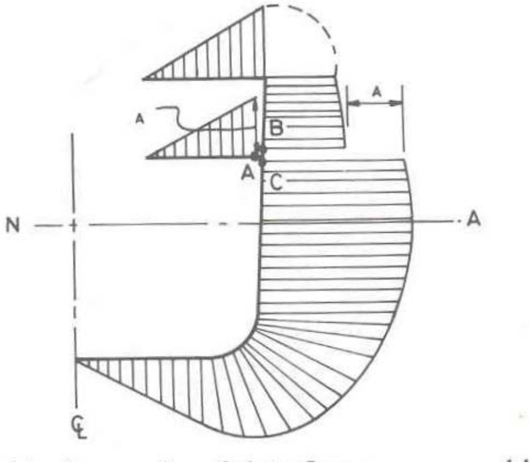
For more complex ship structural cross-sections where there are multiple decks or longitudinal bulkheads, and where thickness t varies around the cross-section, a numerical technique is explained in PNA and Hughes.

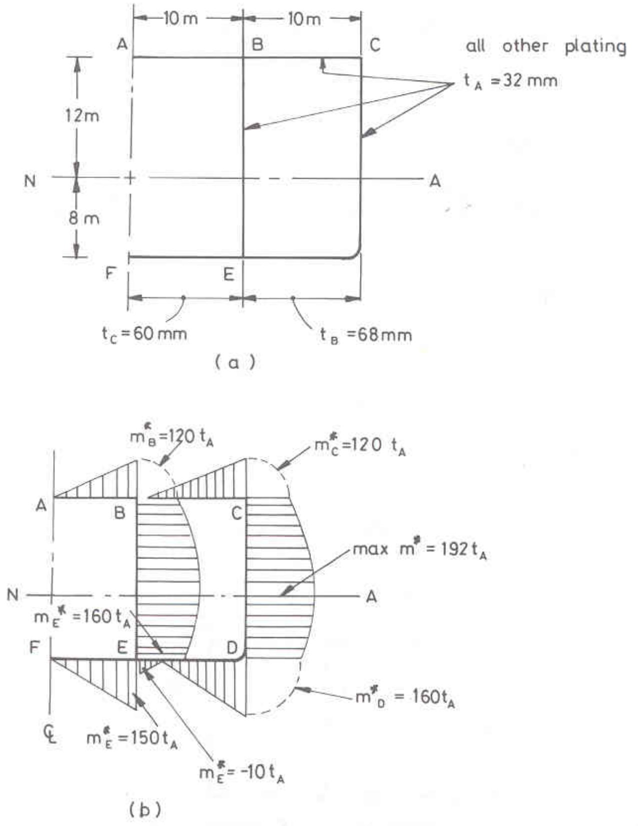
This technique capitalizes on the conservation of shear flow q(s) around the open and closed sections of the cross-section.

Several examples from Hughes (see below figures):

1. Multiple open decks, with large bilge radius (left figure)
2. Internal longitudinal bulkheads (right figure)

Note that the maximum shear stress is at the neutral axis (NA), but the shear stress at the deck edge is not negligible! Note *conservation of shear flow* at corners and "branch points" (shear flows add or subtract).





Apart from some barges and canoes, it is rare to find a completely open," single-cell" hull! In reality, hulls are divided into multiple "cells" by decks, longitudinal bulkheads and tank tops. How we calculate shear flow in these cases is an extension of how we calculate it in the single cell case, with one twist. First though, let's think about a two cell hull divided by a "tween" deck (left figure above). By definition, on centerline or where the deck terminates at the inboard end, the shear stress is zero. The shear flow then increases linearly outboard. It reaches a peak corresponding to the equations:

$$\tau(s) = \frac{V \cdot Q(s)}{I \cdot t(s)} \qquad Q(s) \equiv \int_0^s y(s) \cdot t(s) \cdot ds$$

Now, from the equations above, what is the equation of shear stress of a deck located on the neutral axis? Well, since y is the distance from the neutral axis to the deck, and in this case is zero, the shear flow and also the shear stress is *zero* on that deck! This makes sense when we realize that shear flow is a response to load carried in bending, and material located at the neutral axis has no bending stress and is therefore not carrying any bending load.

If the deck is located anywhere but the neutral axis, it is going to carry some bending load, and therefore will have some shear flow. As that flow grows as the distance from the deck or centerline increases, the deck is carrying more load. When the deck intersects the topsides (shell plating), that load must be transferred into the topsides. At this intersection the shear flow from the decks above is added in, creating a *jump in shear flow* at each intersection, as illustrated in the figure above.

A bigger challenge is the case where longitudinal bulkheads such as those used in wing tank structures are used. The figure above (right figure) shows the general situation. Using the pipe flow analogy and starting at the centerline on the bottom, we know that shear flow builds linearly until it reaches the intersection of the bottom with the longitudinal bulkhead. The question is: how much goes through the bulkhead and how much goes through the shell plating (topsides)? The challenge is that this is a structurally *statically indeterminate* problem, and we must use a statically determinate approach to get an answer. The solution is to temporarily impose "imaginary cuts" in the structure that will stop the shear flow and create a determinate solution. When this happens it is clear that the "cuts" will distort due to the uneven loading (try bending a rolled piece of paper with a gap in the top). If we then "close the gaps", and find the force required to restore equilibrium, we will get the shear flow "jump" across the "cut".

It should be clear that some type of numerical technique should be used to do this calculation. PNA (Section 3.5 of Chapter 4) discusses this method (note that PNA uses "N" for shear flow instead of "q"). A better presentation is given in Hughes (Section 3.7, starting on page 117). For this course it is sufficient for the student to understand just the basic concept of how we solve longitudinal multi-cell shear flow problems, but should be able to calculate shear stress anywhere in a single cell or multi-decked vessel (such as illustrated in the left figure above).

Weekly Assignment #5: 4-Point Beam Bending Lab & Cable Guide T-Stiffener Analysis

In-lab: *4-Point Beam Bending Lab*

Objectives: To have a "hands-on" lab in beam bending and strain measurement. To be able to correlate field readings from strain gages to theory and design practice.

Deliverables:

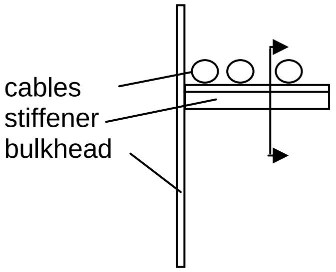
1. A description of what you did in "Executive Summary" format. This must include a brief discussion of the procedure, your results and conclusions. Include all the following:
2. An engineering sketch (scale = 1:5) of the beam test setup. Show enough views to describe the experiment (consider someone who did not see the setup, but for whom you must describe what you've done). If you do the sketch using AutoCAD, you will get an extra 10% bonus!
3. Load, shear and bending moment diagrams (scale = 1:10).
4. Example calculations page (or pages) showing at least one example of every calculation required.
5. Plots illustrating important results of the experiment. Include as a minimum:
    i. Load vs. *experimental* and *theoretical* stress for y_{max} gages (1, 2, 3, 4, and 5). Compare experiment vs. theory for each gage location. Discuss Euler and Schade approaches. Be careful of which gage you apply Schade's approach to, only one is correct!
    ii. Load vs stress for "neutral axis" gages (8 and 9). Are they correct? If not, why? Can you justify it with numbers?
    iii. Load vs stress for hatch corner gages 6, 7, 14, and 15. See figure 56 in PNA for one possible stress concentration factor. Does this SCF approach give a close answer? If not, discuss why?
6. A calculation showing how close the beam was to yield! (i.e. minimum factor of safety and where it was located.) Hint: check the gage with the highest strain!

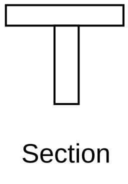
Instructions:

Fairly often engineers are tasked with determining "how close to failure" a piece of structure is in the field. This lab combines that experimental determination with a reinforcement of the beam and girder calculations we have been practicing. During the lab period you will load a 6061-T6 box beam in a four-point bending condition with loads ranging from 0 to 3000 pounds at 500 pound increments. The beam has a number of strain gages applied at strategic spots. The strain readings will be saved in a text file and will be e-mailed to you. Before you leave you should record the dimensional information you need to complete your calculations! If however, for some reason you don't record all the information you need, you can return to the lab at a later time. Before you leave though, you will need to run a quick calculation to determine if the readings "make sense".

Take-home:

A cable guide on a large ship is to be supported by T-stiffeners made of 6061-T6. Each stiffeners can each be modeled as a cantilever beam. Assume each stiffener supports 500 pounds of cables, and the weight is centered 6" from the bulkhead (see below). For the stiffener, the top piece is 1" wide and 1/8" thick. The vertical piece is 1" high and 1/8" thick. Use a length of 12" for the T-stiffener.





Section

a. Show your assumed load-shear-moment diagram. Pick an appropriate DAF. What is the maximum shear load on the stiffener?
b. What is the maximum bending moment on the stiffener?
c. What is the maximum shear stress on the stiffener and where is it precisely located (show on both views)?
d. What is the maximum bending stress on the stiffener and where is it precisely located (show on both of the above sketches)?
e. Using an equivalent stress analysis, what is the FOS of the stiffener? Do you feel it is acceptable? If not, what design change can you make that will make it acceptable?

Asymmetric Bending: Asymmetric Loading and Asymmetric Sections

So far we have only checked the stresses and FOS in the ship when it is upright. In this condition, we have essentially only considered *vertical bending of symmetric sections*.

However, ships do *roll* in a seaway, and/or may have a permanent list due to off-center flooding damage (such as battle damage or collision), or due to off-center loading (such as improper cargo loading or grounding). Most of the time, a vessel is "weakest" when it is upright, but not always! Because of this we need to know how to calculate for a rotated hull (or rotated load). Asymmetric loading also occurs due to quartering seas. The process for evaluating asymmetric loading or asymmetric sections is relatively straight forward as it is based on combining force components.

Most modern naval architecture software incorporates asymmetric loading (and in some cases asymmetric sections due to structural damage), so it is unlikely that you will need to do these calculations "by hand". However, it is necessary that you understand the basic process (and implications) for dealing with such situations.

Consider a ship inclined to an angle θ as shown in the figure below, with a y-z coordinate frame as shown (y axis is the centerline and z axis is the upright neutral axis of the cross-section). The *vertical* bending moment has components M_y and M_z about the CL and NA respectively, given by:

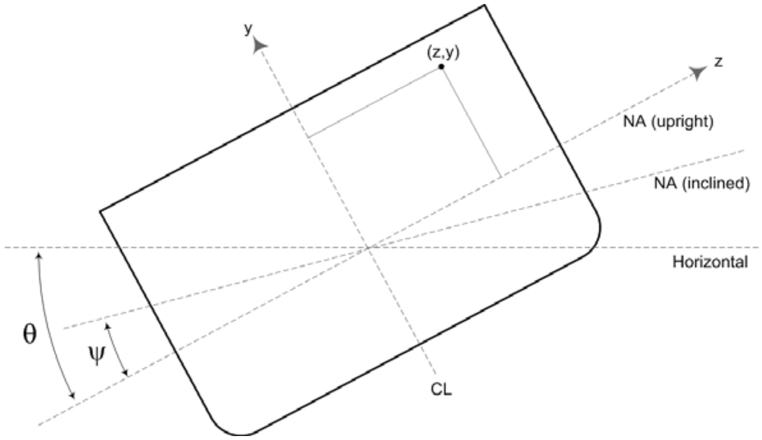
$$M_y = M\sin\theta \qquad M_z = M\cos\theta$$

where M = the vertical bending moment (due to weight and buoyancy distributions). Defining moments of inertia I_{NA} as about the upright NA and I_{CL} as about the CL (note symmetry), we can write the bending stress at a point (y,z) by superposition:

$$\sigma(y,z) = \frac{M\cos\theta\cdot y}{I_{NA}} + \frac{M\sin\theta\cdot z}{I_{CL}} \qquad (1)$$

The maximum values of bending stress will occur at the upper and lower corners, where both y and z are maximum (the deck edge at the top and bilge strake at the bottom).

$$\sigma_{max} = \frac{M\cos\theta\cdot y_{max}}{I_{NA}} + \frac{M\sin\theta\cdot z_{max}}{I_{CL}} \quad \leftrightarrow \quad \sigma_{max} = \frac{M\cos\theta}{SM_{NA-deckedge}} + \frac{M\sin\theta}{SM_{CL-deckedge}}$$



The maximum stress will occur at an angle of inclination θ when

$$\frac{d\sigma_{max}}{d\theta} = 0 \quad \rightarrow \quad \frac{-M\sin\theta}{SM_{NA-deckedge}} + \frac{M\cos\theta}{SM_{CL-deckedge}} = 0 \quad \rightarrow \quad \tan\theta = \frac{SM_{NA-deckedge}}{SM_{CL-deckedge}} \quad \rightarrow \quad \theta(\sigma_{max}) = \tan^{-1}\left(\frac{SM_{NA-deckedge}}{SM_{CL-deckedge}}\right)$$

For typical large ocean-going ships, $\theta(\sigma_{max})$ are approximately 30 degrees (± 10 degrees).

The bending stress is zero at the neutral axis of the *inclined* condition. From (1):

$$\frac{M\cos\theta\cdot y}{I_{NA}} + \frac{M\sin\theta\cdot z}{I_{CL}} = 0 \quad \rightarrow \quad y = -\left[\frac{I_{NA}}{I_{CL}}\tan\theta\right]z$$

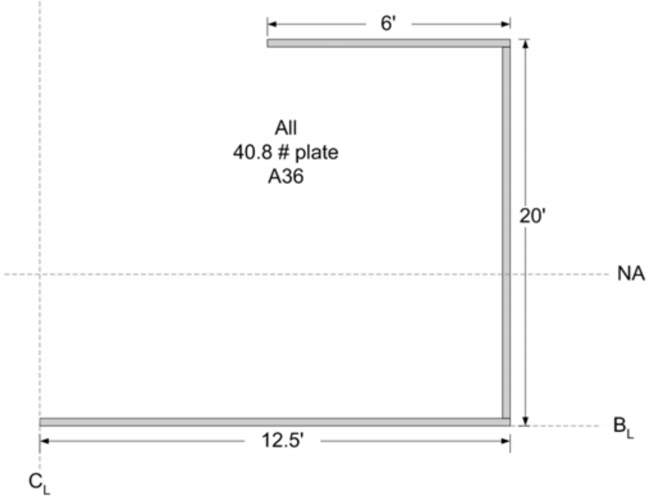
This in an equation for a line (in y and z) defining the neutral axis of the inclined condition. Thus we can find the angle of the neutral axis of the *inclined* condition with respect to the original upright neutral axis:

$$\tan\psi = \frac{y}{z} = \left[\frac{I_{NA}}{I_{CL}}\tan\theta\right] \quad \rightarrow \quad \psi = \tan^{-1}\left[\frac{I_{NA}}{I_{CL}}\tan\theta\right]$$

Note that if $I_{NA} = I_{CL}$ then neutral axis remains horizontal! (this is the case for a square cross-section only).

Example:

Continuation of the barge example from before



Recall: $I_{NA} = 8{,}223{,}720\ \text{in}^4$ (full section) $SM_{NA\text{-deckedge}} = 58{,}741\ \text{in}^3$ M = 34,800 ft-LT

Now: $I_{CL} = 14{,}893{,}672\ \text{in}^4$ (full section) $SM_{CL-\text{deckedge}} = \dfrac{I_{CL}}{z_{max}} = \dfrac{14{,}893{,}672\ \text{in}^4}{150\ \text{in}} = 99{,}291\ \text{in}^3$

At what angle of inclination does maximum bending stress occur?

$$\theta(\sigma_{max}) = \tan^{-1}\left(\frac{SM_{NA}}{SM_{CL}}\right) = \tan^{-1}\left(\frac{58{,}741}{99{,}291}\right) = 30.6^0$$

What is the maximum bending stress at the deck edge?

$$\sigma_{max} = (34{,}800\text{ft}\cdot\text{LT})\left(\frac{2240\text{lb}}{\text{LT}}\right)\left(12\frac{\text{in}}{\text{ft}}\right)\left[\frac{\cos 30.6^0}{58{,}741\text{in}^3} + \frac{\sin 30.6^0}{99{,}291\text{in}^3}\right] = 18{,}503\ \text{psi}$$

$$\text{FOS} = \frac{\sigma_Y}{\sigma_{max}} = \frac{36\ \text{ksi}}{18.5\ \text{ksi}} \approx 1.9$$

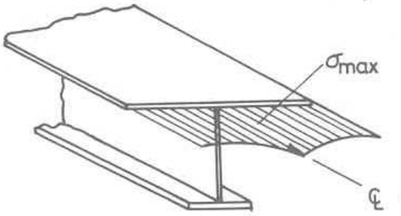
Compare to FOS = 2.3 for vertical-only bending from previous class. In this case the stress went up when heeled!

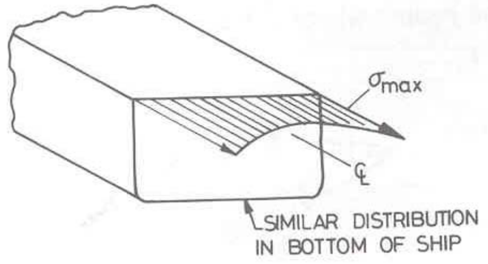
What factors, such as B/D ratio, might influence this trend?

We can think of this as *bounding* our maximum bending stress as the ship is heeled, or as the ship rolls in the seaway!

Shear Lag and "Effective Breadth"

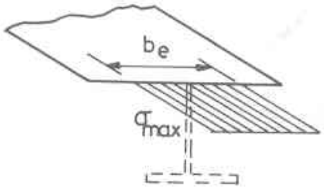
Simple beam theory assumes that "plane section remain plane", and therefore that bending stress is directly proportional to distance from the NA (linear distribution), and thus in any web & flange type beam (I-beam, Box beam, etc.) the stress should be constant across the flange. However, the strain in a bending beam comes primarily from the curvature of the web, and only reaches the flanges through shear which is transmitted across the web-flange connection. The elongated (or shortened) web "pulls" the flange along, through shear forces, setting up *shear distortion* (and stresses) in the flange. The shear distortion of the flange is such that the portion of the flange away from the web undergoes less distortion (and therefore less stress). The bending stress in the flange away from the web is said to "lag behind", and therefore the term *shear lag* has been used to describe this effect.

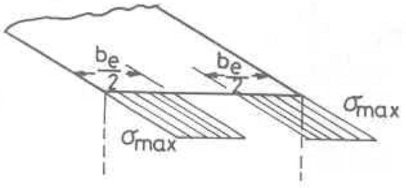




The exact distribution of bending stress across the flange can be calculated through theory, but it is too complex for basic design work (and certainly for this introductory course). For typical ship hulls, it has been found to be only a few percent different. However, for non-slender or shallow beams, with wide flanges and/or short webs (such as ship's double bottoms and tank tops), it can be significant. Because it really only becomes important for "non-slender" beams, shear lag effects are typically not considered for "slender" beams used in civil engineering construction (buildings and bridges), but it is important to consider shear lag effects in ship structural design. The problem with shear lag is that using the simple Euler beam equation would predict stresses less than the maximum stresses occurring at the junction of the web and flange (as shown above).

One solution to this problem was proposed by Commodore Henry Shade (USNA Class of 1923) [1]. Shade proposed defining an "*effective breadth*" of the flange plating which, when used in calculating the moment of inertia (and section modulus) of the beam section, would give the correct maximum stress at the junction of the web and flange using simple beam theory (σ = My/I). The concept is illustrated in the below figures. Shade's *effective breadth* is normally denoted with b_e (but in some literature with the Greek letter λ).





In other words, since simple beam theory (Euler's equation) calculates the *average stress*, and it is the *maximum stress* that will yield first, the concept of effective breadth is used to help us calculate the actual maximum stress in the part, specifically at the web-flange junction. The way we do this is to calculate the moment of inertia (and section modulus) based on the *effective breadth* rather than the actual breadth of the flange. Since bending stress σ = My/I, by making I smaller, the stress goes up.

Schade, in his original papers (published in the SNAME Transactions in 1951 and 1953), produced dozens of graphs showing effective breadth factors for a wide variety of structural shapes (including I-beams, T-beams, Box-beams, multiple-web girders/panels, etc.) and a variety of loading conditions (including uniformly distributed loads, non-uniform loads, and point loads). Additional details can be found in Hughes and PNA, as well as Shade's original papers in SNAME Transactions.

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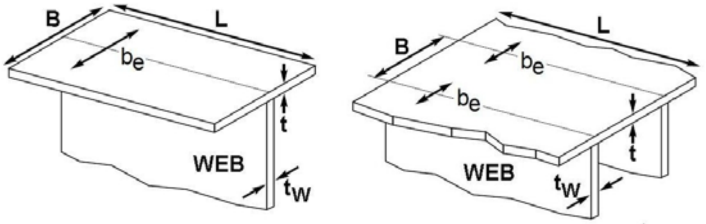
[1] After graduation from the Naval Academy, Commodore Schade served on destroyers and then attended MIT as training for the Ship Constructor Corps (the predecessor to the Engineering Duty community). From 1935-1937 he was in Germany earning a Doctor of Engineering degree and studying their naval ship designs. After working on the Iowa-class battleship design he became a senior engineer on the Essex class carriers and then led the Midway-class carrier program. He achieved flag rank as Commodore (O-7) in 1947 and headed the Department of Naval Research. After retiring from the Navy in 1949 he became a professor at UC Berkeley.

For basic ship structural design (and for this course), we are concerned especially with shear lag and effective breadth in the design of stiffened plating (panels), specifically the selection/design of stiffener scantlings (dimensions) necessary to support hydrostatic and other distributed loadings. Thus, we are concerned primarily with uniform loading (hydrostatic pressure at depth) and with single and multi-web girders. These special cases from the many dozens of Shade Curves is plotted in the below figure. Note that one curve is for a single web girder flange (like an I-beam or T-beam), and the other for multiple-web girder flanges (like stiffened plating panels).

The Shade Curve is used by first calculating the ratio L_0/B (note the effect of the beam end conditions), then extracting the ratio b_e/B using the curve for the correct type of girder flange (single or multi-web), then finally calculating the effective breadth b_e by multiplying by the flange width or stiffener spacing B.



By studying the above Shade Curves, you should notice that for longer (or "slender") beams (as $L_0/B \rightarrow 10$), there is very little shear lag effect ($b_e/B \rightarrow 1.0$). Conversely, for shorter (or "non-slender") beams (especially for $L_0/B < 3$), shear lag effect is important. This is why civil engineers are not normally concerned with shear lag effects, as most civil engineering structures are built using longer (or "slender") structural beams, but naval architects are concerned with shear lag effects. You might notice that a multiple-web girder is essentially stronger than an equivalent single web girder for the same breadth (B) (note that b_e/B is greater than 1 for larger L_0/B for the multiple-web girder, and is always greater than the value for the single web girder). This is essentially because of the Poisson effect, which we will revisit later when we study plate bending.

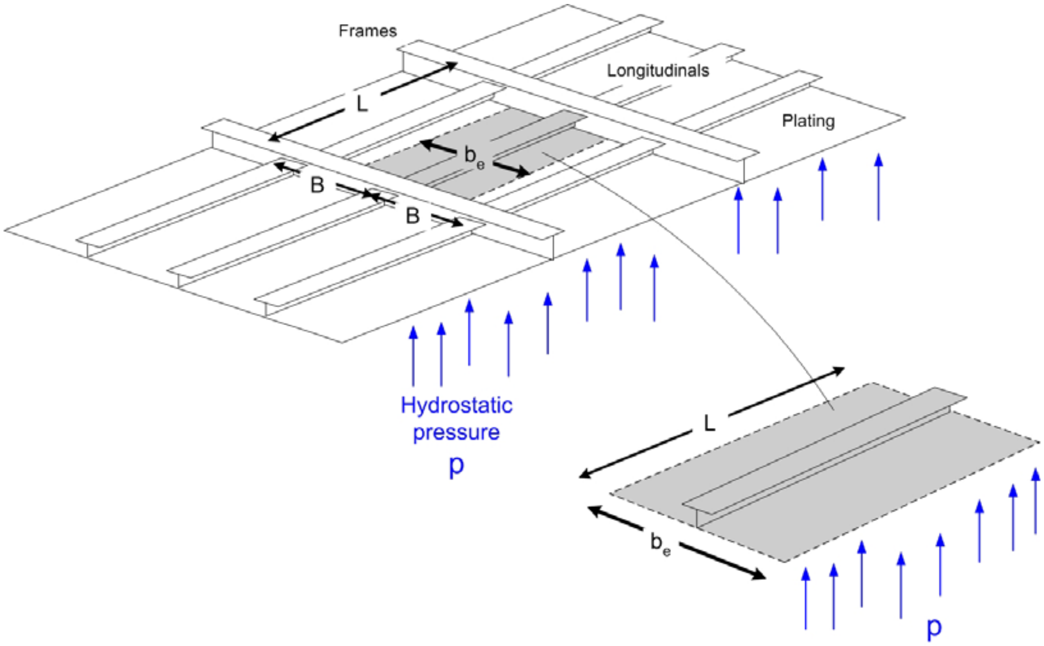
A simplifying design rule used by ABS and other classification societies, is to use an *effective breadth* that is either *the stiffener spacing (B) or 1/3 of the length between frames or supports (L), whichever is less*:

$$b_e = B \quad \text{or} \quad b_e = \frac{L}{3} \quad \text{(whichever is less)} \quad \textit{(ABS Rules)}$$

Can you see why this rule is used, based on the above Shade Curves?

Application of Shear Lag and Effective Breadth – Stiffener Design:

An important application of shear lag and effective breadth in naval architecture is in the design of plate stiffeners, such as longitudinals, stringers, etc. For this application, we consider each stiffener (between frames), with an *effective breadth of attached plating*, to act as a bending beam, with a uniformly distributed load (hydrostatic pressure on the plating) (we will include additional stresses due to global hull girder bending *later*). This is illustrated in the figure below. In design practice, the beam end conditions (at the frames) are taken as simply-supported, since this gives a "worst-case" load (with maximum bending moment at the center of the beam span).



Example:

Consider longitudinals supporting bottom plating on a ship (assume A-36 steel).
What is the maximum bending stress and the minimum FOS (vs. material yield)?

Consider only hydrostatic pressure (we will ignore global hull girder bending stress – for now!).
The following information is provided:

$$\text{H (hydrostatic head)} = 32\text{ ft} \quad \text{L (frame spacing)} = 4\text{ ft} \quad \text{B (stiffener spacing)} = 3\text{ ft}$$

$$t_p\text{ (plate thickness)} = 0.5\text{ in} \quad t_f\text{ (flange thickness)} = 0.5\text{ in} \quad t_w\text{ (web thickness)} = 3/8\text{ in} = 0.375\text{ in}$$

$$h_w\text{ (web height)} = 6.0\text{ in} \quad b_f\text{ (flange breadth)} = 8.0\text{ in}$$

For a simply-supported beam with a uniform load: $M_{max} = \frac{wL^2}{8}$ where w is the uniform load (force/length).

In this case w is due to the hydrostatic pressure on the plate:

$$w = (\text{density})(\text{depth})(\text{stiffner spacing}) = (\rho g H)(B) = \left(64\frac{\text{lb}}{\text{ft}^3}\right)(32\text{ft})(3\text{ft}) = 6144\frac{\text{lb}_f}{\text{ft}}$$

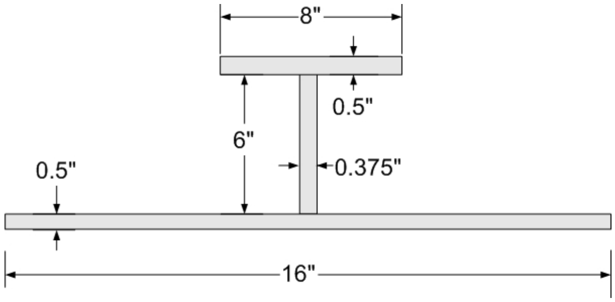
Since the stiffener spacing B = 3 ft, and the ends are simply-supported (L_0 = L = 4 ft), then L_0/B = 4/3 = 1.33.

As an illustration of using the Shade curve (uniform load, multi-web girder), we have $b_e/B \approx 0.5$, so the effective breadth of attached plating is $b_e \approx 1.5$ ft.

Or, using the ABS Rules, b_e = L/3 = 1.33 ft (which less than B = 3 ft).

Note that we can use the full breadth of the flange since $b_f < B$ and $b_f < L/3$

Thus, we have the below effective beam section (stiffener with effective breadth of attached plating).



Using a section modulus calculation (spreadsheet) for these scantlings gives:

$$\mathrm{SM_{min} = SM_{top} = 27.625\ in^3}$$

Maximum bending stress and minimum FOS:

$$\mathrm{M_{max}} = \frac{\mathrm{wL^2}}{8} = \frac{(6144\ \mathrm{lb/ft})(4\mathrm{ft})^2}{8} = 12{,}288\ \mathrm{ftlb}$$

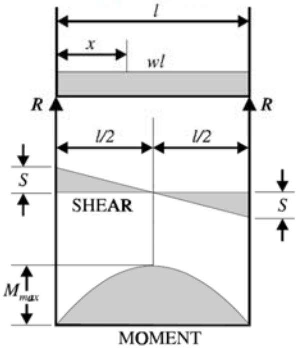
$$\sigma_{\mathrm{max}} = \frac{\mathrm{M_{max}}}{\mathrm{SM_{min}}} = \frac{12{,}288\ \mathrm{ftlb}}{27.625\ \mathrm{in}^3}\left(12\frac{\mathrm{in}}{\mathrm{ft}}\right) = 5{,}338\frac{\mathrm{lb}}{\mathrm{in}^2} = 5.34\ \mathrm{ksi}$$

$$\mathrm{FOS_{min}} = \frac{\sigma_{\mathrm{Y}}}{\sigma_{\mathrm{max}}} = \frac{36\ \mathrm{ksi}}{5.34\ \mathrm{ksi}} = 6.7 \quad > \ 2 \quad (\mathrm{OK})$$

Additional notes about maximum bending moments and deflections for beams:

In the previous example, we used a maximum bending moment at the center of the beam ($M_{max} = wL^2/8$). This is used in many ship structural design applications, since it is the "worst-case" moment for a uniformly-loaded beam (note that hydrostatic pressure provides a uniform load on a horizontally-oriented beam/stiffener). This maximum moment can be derived for a simply-supported uniformly loaded beam using the Principle of Static Equilibrium (recall Statics and Mechanics of Materials – EN221). Maximum bending moment (and also maximum shear force and maximum deflection) can likewise be determined in a similar manner for any combination of end condition and load type. Rather than do this "from scratch" for each possible case, it is often useful to make use of design manuals, which usually contain listings of maximum bending moments (M_{max}), shear forces (V_{max}), and deflections (y_{max}) for a wide range of such conditions. Some cases used often in ship structural design (and which we will use in this course) are shown below.

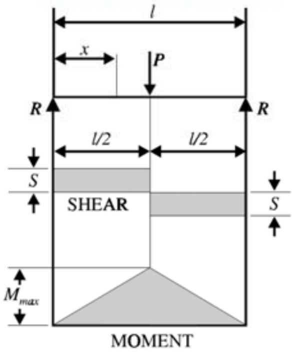
Simply-Supported Ends
Uniform Load



$$M_{max} = \frac{wl^2}{8} \quad \text{(at center)}$$

$$y_{max} = \frac{5}{384\mathrm{EI}}\,wl^4 \quad \text{(at center)}$$

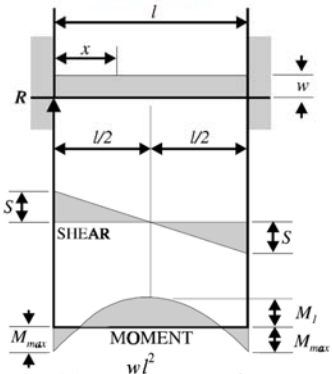
Simply-Supported Ends
Concentrated Load (Center)



$$M_{max} = \frac{Pl}{4} \quad \text{(at } P\text{)}$$

$$y_{max} = \frac{Pl^3}{48\mathrm{EI}} \quad \text{(at } P\text{)}$$

Fixed Ends
Uniform Load

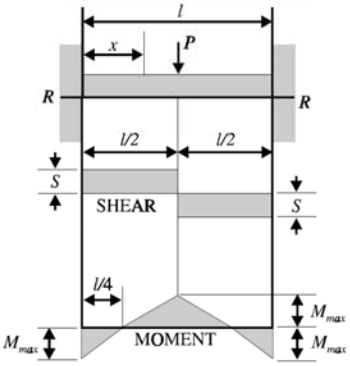


$$M_{max} = \frac{wl^2}{12} \quad \text{(at ends)}$$

$$M_1 = \frac{wl^2}{24} \quad \text{(at center)}$$

$$y_{max} = \frac{wl^4}{384\mathrm{EI}} \quad \text{(at center)}$$

Fixed Ends
Concentrated Load (Center)



$$M_{max} = \frac{Pl}{8} \quad \text{(at center and ends)}$$

$$y_{max} = \frac{Pl^3}{192\mathrm{EI}} \quad \text{(at center)}$$

Weekly Assignment #6: Balsa Beam Design Project

In this design project you will work in *3-person teams* to design, build, and test a beam made of balsa wood.

Design requirements:

The span between supports is 40 inches (you should assume the beam to be simply-supported).

Your beam must be at least 43 inches in length.

The material is balsa wood, which will be provided by the instructor. The balsa is available in sheets of length 44 inches, width 3 inches, and available thicknesses are 1/16, 1/8 and 3/16 inches. You may cut and glue the balsa pieces together as desired. When you are ready to build the beam, swing by the instructor's office for the supplies. He will first make a quick review of your beam to see if it is buildable (be prepared to explain how). Realize it will take at least 24 hours for the glue to dry.

The beam must support a 25 pound weight centered between the two supports. The weight will be hung below the beam using a piece of 2 inch webbing (consider how this will load the beam).

There are no constraints on the beam geometry. However, you should consider the loading and beam stability!

Design a beam such that the maximum deflection of the beam is 1/8 inch (0.125 inch) while supporting the weight.

Report:

Document your design by submitting a three-part report.

The first part will be a description of the project, including your proposed solution (in memo format).

The second part is the supporting calculations (hand and spreadsheet).

The third part is the construction drawing. The drawing may be hand drawn or AutoCAD (If AutoCAD is used, a 10% bonus is possible, with award based on quality and completeness of the drawing).

The *initial report* is due Monday morning of Week #8.

Test:

We will test the beams as part of the lab period of Week #8.

The *final report* (including additional description of the testing results, and any changes you would then make to your design) will be due Monday morning of Week #9.

Bonus:

The team with the *lightest weight* beam which also meets <u>all</u> of the design requirements (including the deflection limit), will receive an extra 10% on their score!

Hint: In previous classes, the number 1 problem was that the as-built beams were too stiff! What were the bad assumptions those students made?

Hull–Superstructure Interaction

Up to now we have considered the hull to be a single, continuous beam that we can analyze using Euler and Schade methods, plus shear flow. A good question is, "how does the superstructure interact structurally with the rest of the hull?"

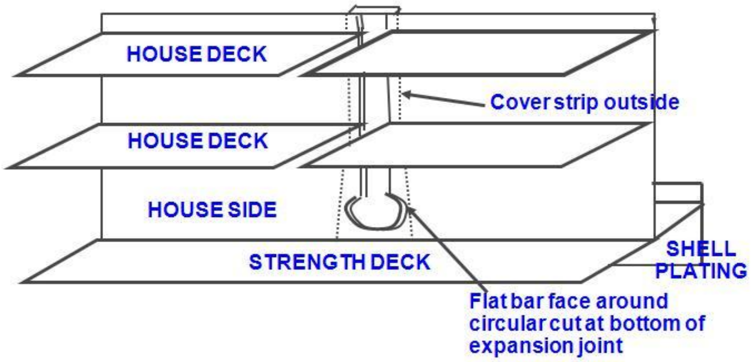
Like most naval architecture answers, this one starts with, "it depends!" Follow-on questions might include: "How long and wide is the deckhouse?", "How is it structurally connected to the deck?" and "What material is it made of?". The basic approach is to determine whether the deckhouse is structurally "stiff enough" to influence the bending characteristics of the hull. Similarly, an equally important question is "does the hull's deflection influence the deckhouse?"

Here are some thought provoking questions about hull/superstructure interactions. Imagine a hull subject to a hogging moment. Does that put the deck in tension or compression? Hopefully you immediately answered, tension! The deck is getting longer due to the hogging. Think of a short deckhouse (about 20% of the vessel's length) that is just sitting on the deck at midships and is not attached. The deck will just expand and contract underneath the deckhouse and no loads (apart from friction and gravity) will transmit to the deckhouse, except that the deckhouse is being pushed up in the middle by the deck and is unsupported at the forward and aft ends as the deck has dropped due to hogging deflection! That is a similar case to the hull only getting hydrostatic support in the middle! In other words, the deckhouse would hog as well as the hull! In this case, while the deck is in tension, the bottom of the deckhouse will be in compression!

Now imagine that while the vessel is hogged, the rectangular deckhouse is bolted to the deck at only the four corners. How do you get the four corners to cinch down to the deck? You screw the bolts down tight, which puts a large tensile stress on the bolts. When the vessel hogs, the two forward bolts will also move forward while the two aft bolts will move aft. That will put the bottom of the deckhouse in tension. At the same time, the forward and aft bolts move down relative to the middle. That push up in the middle may cause the deckhouse to bend, which increases the deckhouse hog. On the other hand, since the bottom of the deckhouse now moves with the deck, the bottom of the deckhouse is now in tension!

What is the goal? Well, structures last longer and can be built lighter if they don't carry loads, so naval architects will often use methods that have some boundary condition fixity so that the deckhouse is neither in tension nor compression. Since not attaching it would give a compressive load, and making it rigidly attached would give a tensile load, the solution is to use a compliant joint. The most common is an *expansion joint* as shown in the below figure. Where the joints go is important, and from the discussion above it should be clear that the highest loads will be at the forward and aft ends of the deckhouse. These are common stress "hot spots" on ships, with cracking seen on many ships, including the DDG963, CG47, PC and others.

This is a good time to remind the student of the old engineering proverb, "the load goes to the stiffest structure". It is often hard to determine how the superstructure will react unless its *relative stiffness* compared to the hull is known. This is therefore a very good application for finite element analysis, which will be our next topic. You may want to read section 3.13 of PNA for more information.



Advanced Computer Methods: Introduction to Finite Element Analysis (FEA)

The single biggest development in ship structural design and analysis over the last decade has been the introduction and acceptance of finite element analysis (FEA). This tool has significantly changed the way we design ships and its use will continue to grow. It offers both faster and more accurate solution to complex structural problems. For the structural design engineer, it is necessary to know both the basic theory that it uses (so you know its limitations) and how the particular computer code functions. FEA is not “reality”, and FEA computer programs are not always correct, so sometimes it can provide some incredibly bad results (although it is usually human “operator” error which produces the incredibly bad results with FEA computer programs)!

The FEA method (also called FEM for “finite element method”), goes back to ancient mathematicians. Archimedes used the basic technique with a 96-side polygon to approximate the value of π to 30 decimal places (recall that the circumference of a circle = 2πr)! However, modern FEA really came about along with the electronic computer, being initiated in the 1960’s at the University of California Berkeley. It was first utilized in the marine design industry in the 1970’s, but did not begin wide-spread use until the mid-1990’s (a little over 10 years ago)!

FEA is used in structural design to solve complex structural problems. Applications include linear, nonlinear (due to nonlinearities in material, geometry (contact, etc.), boundary conditions, etc.), dynamic (ballistic, impact, transient, harmonics/vibration), heat transfer, and fluid dynamics. For this course, we will focus on linear applications, but will discuss some of the others in class.

In FEA, a structure is divided into a large number of discrete sub-structures called “*elements*”. Each element is “connected” to adjacent elements at points called “*nodes*”, where the *“loads”* (forces and moments) are applied, and “*displacements”* (translations and rotations) are determined. For the total structure, including the many elements which make it up, the laws of solid mechanics are applied (within each element and between elements): equilibrium of forces and moments (statics and dynamics), compatibility of displacements (translations and rotations), laws of material behavior (stress-strain or “Hooke’s” laws), to name only a few.

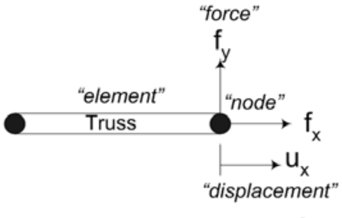
The mechanical behavior of each element is modeled using equations from solid mechanics. The basic *types of elements* include (plus other variations):

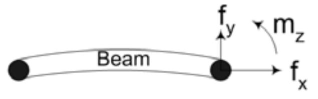
Truss elements: 2-node elements which allow only *translation* at the nodes (1-3 DOF per node). Used for bars and rods.

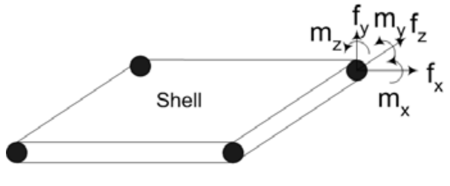
Beam elements: 2-node elements which allow *translation and rotation* at each node (2-6 DOF per node).

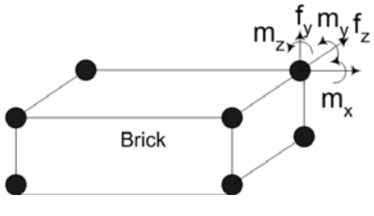
Shell elements: 3-node (triangular shell) or 4-node (quad shell) elements. Used for “thin” plates (flat or curved).

Solid elements: 4-node (tetrahedral) or 8-node (brick) elements. Used for “thick” plates or solid parts.





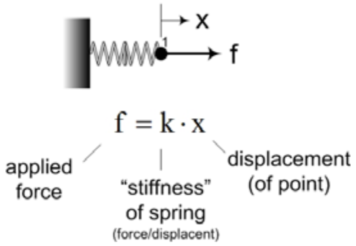




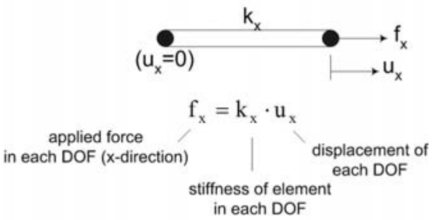
Once the structure is *modeled* using these discrete elements, then the elements must be interconnected at the nodes (compatibility), then the loads (or displacements) must be applied (at the nodes), then the displacements and stresses can be determined (at the nodes) using methods of <u>matrix math</u> (known as “linear algebra” or “matrix algebra”).

It is important to realize that FEA is a numerical approximation of a complex structure (it is not "reality"). It is made by breaking the structure up into small (discrete) parts whose mechanical behavior can be *better* modeled and predicted than the entire structure as a whole. Note that the model and results can usually be made more accurate by making the element sizes smaller – but it is still only as good as the simplified mechanics equations used (solid or "continuum" mechanics)! Why a model with really small elements (called "fine mesh") is not always more accurate is a function of how stress is calculated. A simple example is that of tensile stress P/A. As the area (A) gets smaller, the stress goes up. If the force (P) cannot be easily resolved at the nodes, then the FEA code will over-predict the stresses at certain nodes. You will see examples of this in class.

Let's take a better look at the basic idea of FEA. Structural elements (truss, beam, shell, solid) are modeled as spring-like objects. Consider a simple 1-D truss element (a "bar") attached at one end with a force applied at the other to act as a spring as shown in the figure below. The force (f) is equal to the displacement of the end point (x) multiplied by a spring constant (we call "stiffness" k). Or, alternatively, the displacement (x) can be determined by *dividing* the applied force (f) by the stiffness (k).



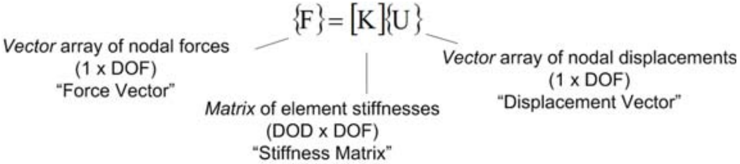
The equivalent "finite element" is a 1-D truss element (bar) as shown below. Note that, in FEA we use "*u*" to denote the displacement in each degree of freedom (DOF). For the 1-D bar element (similar to the spring) we have



Note that the "stiffness" for a 1-D truss element (bar) can be derived as you did in EN221 (take a look at your notes):

$$k = \frac{\text{force}}{\text{displacement}} = \frac{F}{\frac{F \cdot L}{A \cdot E}} = \frac{A \cdot E}{L} \quad \text{(1-D bar element)}$$

For many elements (combining together to make the structure), we create arrays (vectors and matrices), which are denoted with capital letters F, K and U.



This can be solved mathematically using a matrix inversion method (this is analogous to simple division for the 1-D case):

$$\{F\}=[K]\{U\} \quad \rightarrow \quad \{U\}=[K]^{-1}\{F\}$$

Once the displacements $\{U\}$ are known, then the strains and stresses can be determined:

$$\varepsilon = \frac{\Delta u}{L} \quad \text{(strain for the 1-D bar …more complicated for 2-D and 3-D)}$$

$$\sigma = E \cdot \varepsilon \quad \text{(stress for the 1-D bar …more complicated for 2-D and 3-D)}$$

*** For more information, and some example marine applications of FEA, read the Professional Boatbuilder magazine articles (handouts) "N[E]Ex = Not Easily Explained" by Scott R. Sutherland, and "FEA" by Geoff Green. Also view the Powerpoint "Intro to Marine Finite Element Analysis" by Prof. Paul Miller, located on the course Blackboard page.*

The Stiffness Matrix Method

As shown above, the FEA solution for the displacement array $\{U\}$is found by multiplying the inverse of the stiffness matrix [K] and the force array $\{F\}$. While the fundamental idea is somewhat straight forward, the mathematics to implement this (especially calculating the inverse of the stiffness matrix) is not, and requires a detailed understanding of matrix algebra (linear algebra).

To give a better understanding of some of the fundamental mathematics (without delving too deeply into the theory of matrix algebra), we will discuss several simple applications of FEA which illustrate the general process. This simplified version of FEA is often called the "Stiffness Matrix Method".

Consider a simple 1-D spring as previously discussed. For a spring with movement in the x-direction only:

$$f = k \cdot x$$

f = force (lb)
x = axial displacement (in)
k = stiffness or spring constant (lb/in)

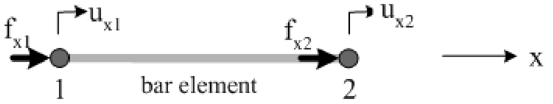
Note: whether a variable is lower or upper case does make a difference! The lower case variables refer to a single element, while the upper case refers to multiple elements joined together in a structure. For instance, [K] refers to the global stiffness matrix.

For each *node* in a structure, we have 6 possible *displacements* – in 6 possible degrees of freedom (DOF): 3 (linear) translations (u_x, u_y, u_z) and 3 (angular) rotations (r_x, r_y, r_z). Displacements may be specified at certain nodes (such as boundary conditions (BCs) or calculated. For each node, we also have 6 possible forces: 3 linear forces (f_x, f_y, f_z) and 3 moments (m_x, m_y, m_z).

The stiffness matrix for each element depends upon geometries and material properties.

Stiffness Matrix Method – Bar Elements:

We start with a simple 1-D bar element (a 1-D version of the "truss" element) (*axial displacement only along the x-axis*). Note that a "truss" element has "pinned" ends, and therefore can carry no end moments. The bar element has 2 nodes (one at each end, and 2 total DOF (1 translation at each node).



What does the stiffness matrix look like for this bar element (called the "element stiffness matrix")?

Applying a force at node 1 f_{x1} (with no force f_{x2} at node 2), we have the following (from static equilibrium):

$$f_{x1} = (u_{x1} - u_{x2})k$$

Apply a force at node 2 f_{x2} (with no force f_{x1} at node 1), we have (from static equilibrium):

$$f_{x2} = (u_{x2} - u_{x1})k$$

If we have forces at both nodes 1 and 2, then we have *simultaneous* equations:

$$f_{x1} = ku_{x1} - ku_{x2}$$
$$f_{x2} = ku_{x2} - ku_{x1}$$

which can be written in "matrix form":

$$\begin{Bmatrix} f_{x1} \\ f_{x2} \end{Bmatrix} = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \cdot \begin{Bmatrix} u_{x1} \\ u_{x2} \end{Bmatrix}$$

We see that the "element stiffness matrix" is a "square matrix" and is of size DOF x DOF (here 2 x 2). We also see that the element displacement array is 1 x DOF (here 1 x 2) and is therefore a vector, and the force array is also 1 x DOF (here 1 x 2) and is also a vector.

Also note that the axial stiffness of a bar is $k = \frac{AE}{L} \left(\frac{lb}{in}\right)$

We can then re-write the element stiffness matrix as:

$$\left[k^e\right] = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} = \begin{bmatrix} \frac{AE}{L} & -\frac{AE}{L} \\ -\frac{AE}{L} & \frac{AE}{L} \end{bmatrix} = \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

Recall that we desire the displacements, which we can get by "inverting" the stiffness matrix: $\{u\} = [k]^{-1} \cdot \{f\}$

From "matrix algebra", for a 2 x 2 matrix $[A] = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $[A]^{-1} = \frac{\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}}{ad - bc}$

But for our element stiffness matrix $\left[k^e\right] = \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$, inversion gives $\left[k^e\right]^{-1} = \frac{AE}{L} \frac{\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}}{(1)(1) - (-1)(-1)}$

This inverted matrix is singular! (the inversion gives a 0 on the bottom, or *infinite* values!)

Thus the displacements are unbounded (infinite), which means our structure would have "rigid body motion" (RBM).

This means that we need to *prescribe* some type of *constraints* on our displacements (we call these constraints "boundary conditions").

For our bar element, in order to avoid RBM, we need at least 1 of the 2 nodes to be constrained. Then this effectively "reduces" the stiffness matrix to one that is <u>not</u> singular!

As an example, let's fix node 2 ($u_{x2} = 0$). Then

$$\begin{aligned} f_{x1} &= k_{11}u_{x1} \qquad & f_{x1} &= \text{the applied force} \\ f_{x2} &= k_{21}u_{x1} \qquad & f_{x2} &= \text{the reaction force} \end{aligned}$$

Thus we can solve for our unknown displacement at node 1, and also our unknown reaction force at node 2.

Once we have displacements at nodes 1 and 2, we can find the average axial strain in the bar element:

$$\varepsilon = \frac{u_{x1} - u_{x2}}{L}$$

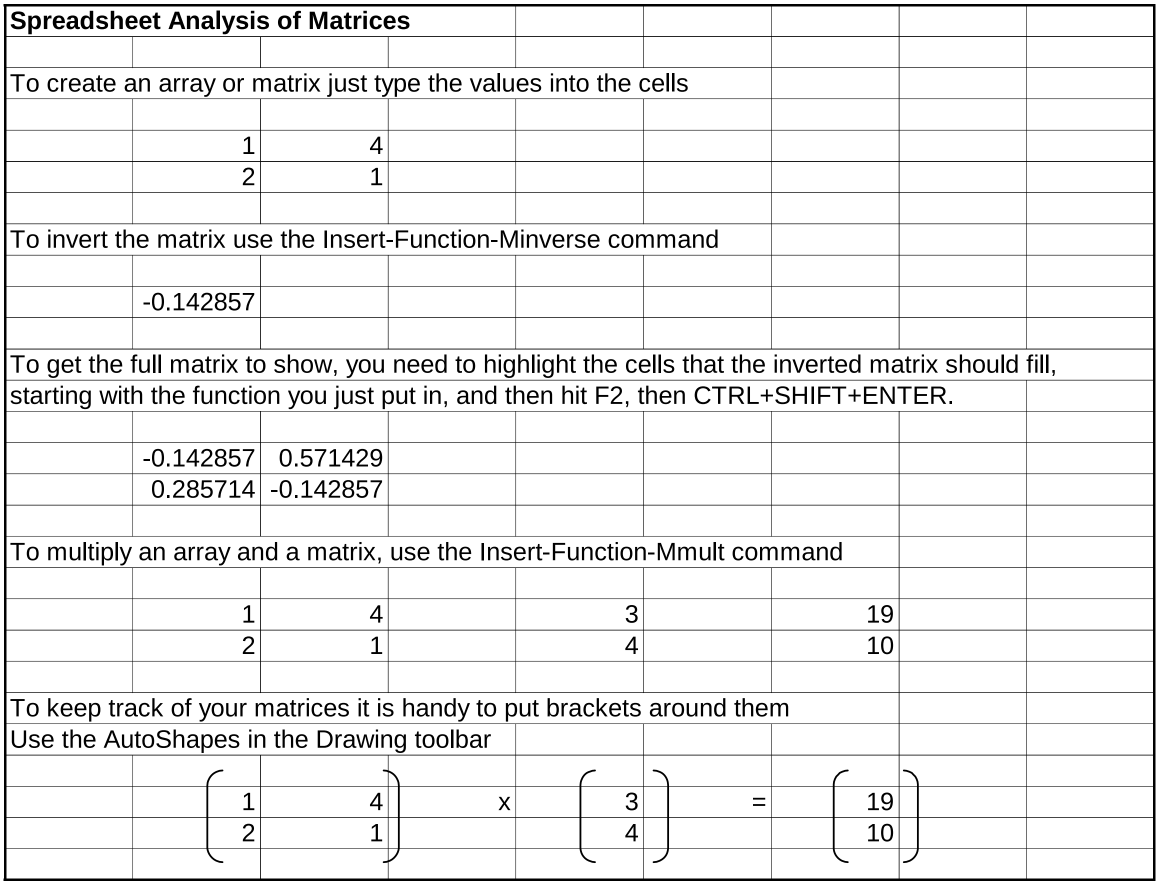
and the average axial stress in the bar element (using Hooke's Law):

$$\sigma = \varepsilon E$$

This is the solution to a FEA problem!

There are a couple things to note as a check that you are setting up the problem correctly. First, list all of your DOF (translations and rotations at each node – the maximum will be 6 x the number of nodes). The size of the force vector must be the same as the DOF. To get from one to the other, the Stiffness matrix must be of the size DOF x DOF.

This is a good time to introduce spreadsheet matrix operations. Spreadsheets are very useful for solving simple matrix problems. Most students have not learned the techniques however, so here is a primer on using Excel to solve matrix problems.



Spreadsheet Analysis of Matrices

To create an array or matrix just type the values into the cells

	1	4						
	2	1						

To invert the matrix use the Insert-Function-Minverse command

	-0.142857							

To get the full matrix to show, you need to highlight the cells that the inverted matrix should fill,
starting with the function you just put in, and then hit F2, then CTRL+SHIFT+ENTER.

	-0.142857	0.571429						
	0.285714	-0.142857						

To multiply an array and a matrix, use the Insert-Function-Mmult command

	1	4		3		19		
	2	1		4		10		

To keep track of your matrices it is handy to put brackets around them
Use the AutoShapes in the Drawing toolbar

$$\begin{pmatrix} 1 & 4 \\ 2 & 1 \end{pmatrix} \quad x \quad \begin{pmatrix} 3 \\ 4 \end{pmatrix} \quad = \quad \begin{pmatrix} 19 \\ 10 \end{pmatrix}$$

OK, so now we have solved a simple bar element problem. But what about larger structures (made up of numerous bar elements)?

To add elements together (elements that are connected at nodes), we use lower case to denote elements and upper case to denote multiple elements (complete structures). Thus:

$$\{f\}=\left[k^e\right]\{u\} \;\rightarrow\; \{F\}=[K]\{U\}$$

For clarity, we denote elements by small case letters (a, b,…), element nodes by numbers (1,2,3,4,5,6,…), and structure nodes (“global”) by capital letters (A, B, C,…). See the below figure for a 2-element axial bar.

For the 2-element axial bar, the forces at the nodes can be related:

$$F_A = f_{a1} \quad F_B = f_{a2} + f_{b1} \quad F_C = f_{b2}$$

The displacements of the nodes can also be related:

$$U_A = u_{a1} \quad U_B = u_{a2} = u_{b1} \quad U_C = u_{b2}$$

Combining these relations with the basic results for the single element bar from the previous pages, we have:

$$\begin{aligned} F_A &= k_a U_A - k_a U_B \\ F_B &= -k_a U_A + k_a U_B + k_b U_B - k_b U_C \\ F_C &= \qquad\qquad\quad - k_b U_B + k_b U_C \end{aligned} \qquad \text{where } k_a = \frac{A_a E_a}{L_a} \quad k_b = \frac{A_b E_b}{L_b}$$

This can be written in “matrix form” as:

$$\begin{Bmatrix} F_A \\ F_B \\ F_C \end{Bmatrix} = \begin{bmatrix} k_a & -k_a & 0 \\ -k_a & k_a + k_b & -k_b \\ 0 & -k_b & k_b \end{bmatrix} \begin{Bmatrix} U_A \\ U_B \\ U_C \end{Bmatrix}$$

$[k^e{}_a]$ $[k^e{}_b]$

Note that the stiffness matrix [K] can actually be “<u>assembled</u>” from the element stiffness matrices [k^e] along the diagonal. Note also that [K] is still singular, and constraints are required to eliminate RBM.

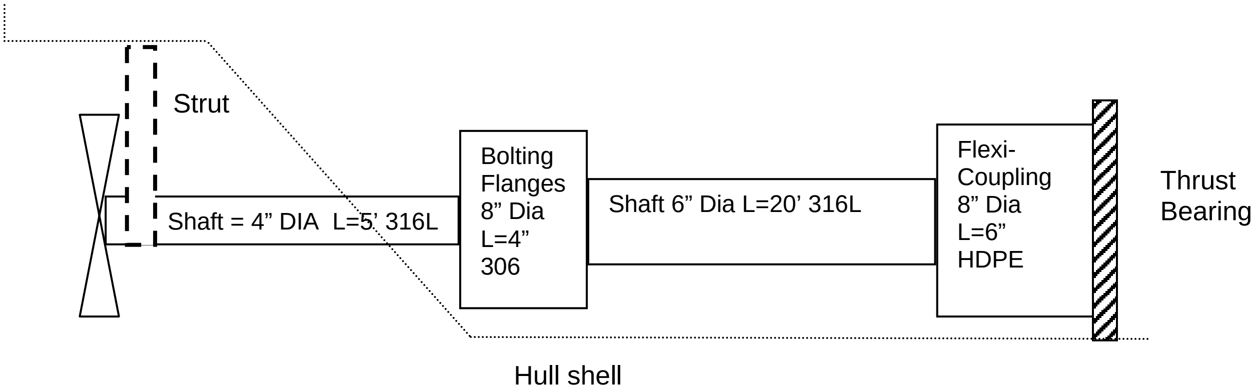
Consider for the 2-element bar a constraint (boundary condition) at node A so that $U_A = 0$, with applied forces F_A and F_B:

> This 3 x 3 matrix problem can be solved by “partitioning” matrix [K] (and then inverting partitioned sub-matrix). We partition around <u>known</u> displacements (BCs) as illustrated below.
>
> $$\begin{Bmatrix} F_A \\ F_B \\ F_C \end{Bmatrix} = \begin{bmatrix} k_a & -k_a & 0 \\ -k_a & k_a + k_b & -k_b \\ 0 & -k_b & k_b \end{bmatrix} \begin{Bmatrix} U_A = 0 \\ U_B \\ U_C \end{Bmatrix}$$
>
> Note that $U_A = 0$ is the known BC, but it could also be non-zero (but known).
>
> *Partitioning* around the known U_A gives the following relations, which can be used to solve for the unknown displacements (U_B and U_C) and unknown support reaction at the constraint (F_A):
>
> For the lower partition:
>
> $$\underbrace{\begin{Bmatrix} F_B \\ F_C \end{Bmatrix} = \begin{bmatrix} k_a + k_b & -k_b \\ -k_b & k_b \end{bmatrix} \begin{Bmatrix} U_B \\ U_C \end{Bmatrix}}_{\text{Solve for } U_B \text{ and } U_C} + \underbrace{\begin{bmatrix} -k_a \\ 0 \end{bmatrix} \{U_A = 0\}}_{=0}$$
>
> <u>plus</u> for the upper partition:
>
> $$\underbrace{F_A = k_a\{U_A = 0\} + \begin{bmatrix} -k_a & 0 \end{bmatrix} \begin{Bmatrix} U_B \\ U_C \end{Bmatrix}}_{\text{Solve for unknown support reaction } F_A \text{ } (U_B \text{ and } U_C \text{ known})}$$
>
> Finally, solving for strain and stress: $\varepsilon_a = \dfrac{U_B - U_A}{L_a}$ $\quad \varepsilon_b = \dfrac{U_C - U_B}{L_b}$ $\quad \sigma_a = \varepsilon_a E_a$ $\quad \sigma_b = \varepsilon_b E_b$

In-Class Exercise: Propulsion Shaft Analysis Using the Stiffness Matrix Method

You need to analyze the propeller drive train, shown schematically below. The goal is to figure out how far the propeller will move forward under load, thereby giving you a minimum spacing between the propeller and strut. Determine the displacements at each node, and the strains, stresses and factors of safety for each element of the structural component described below. Work in teams of two. Show [K], [F], and [U] and their inverses, as required. Identify the component that fails first, and by what mode. Is this an acceptable design? What is the required clearance between the propeller and the strut?

The propeller thrust is 150 kips. The ultra-high molecular grade HDPE flexi-coupling ("Drive-Saver") is a vibration and noise damper. As a thermoplastic, the compressive strength is about three times the tensile strength.



The next page shows a basic set up for this problem. You can use this one as a guide or you can develop your own. The values have been changed in a few spots (so you just can't copy and paste)!

We haven't covered buckling in detail yet, but as a refresher, here is a summary of the key information from the column buckling material (to be covered in a few weeks):

For "practical" critical buckling stress, we often use the radius of gyration defined by $I = \rho^2 A \quad \rho = \sqrt{\frac{I}{A}}$

Note that Roark's and many "Strength of Materials" have a section on the properties of areas.
The critical buckling stress for an "ideal" (Euler) column is:

$$\sigma_{cr} = c\frac{\pi^2 E}{\left(\frac{L}{\rho}\right)^2}$$

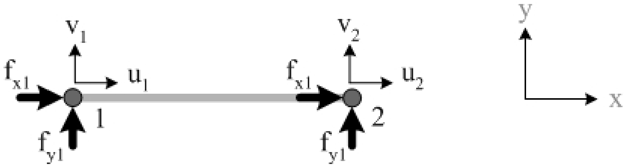
where $\frac{L}{\rho}$ is a "slenderness ratio"

c is a coefficient for end conditions
(c = 1 pinned, c = 4 clamped, c ≈ 2 for welded ship stanchions)

Spreadsheet Analysis For Matrix Stiffness Method									
E 316/306	10000000	psi	E HDPE	72500	psi				
yield	55100	psi	yield	8700	psi				
Element	Diameter	Area	Length	AE/L					
		sq in	in	lb/in					
a	4	12.56636	60	2094393					
b	8	50.26544	4	125663600					
c	5	19.63494	240	818122					
d	8	50.26544	6	607374					
	F	=	K	U					
	150000		2094393	-2094393	0	0	0		UA
	0	=	-2094393	127757993	-125663600	0	0		UB
	0		0	-125663600	126481722	-818122	0		UC
	0		0	0	-818122	1425496	-607374		UD
	Fe		0	0	0	-607374	607374		0
	Partitioned to solve for displacements								
	150000		2094393	-2094393	0	0		UA	
	0	=	-2094393	127757993	-125663600	0		UB	
	0		0	-125663600	126481722	-818122		UC	
	0		0	0	-818122	1425496		UD	
		U	=	K^{-1}	F				
	UA		3.354E-06	2.877E-06	2.869E-06	1.64643E-06	150000		
	UB	=	2.877E-06	2.877E-06	2.869E-06	1.64643E-06	0		
	UC		2.869E-06	2.869E-06	2.869E-06	1.64643E-06	0		
	UD		1.64643E-06	1.64643E-06	1.64643E-06	1.64643E-06	0		
	UA		0.5031						
	UB	=	0.4315	in					
	UC		0.4303						
	UD		0.2470						
	UE		0						
		a	-0.0012						
	Strains	b	-0.0003						
		c	-0.0008						
		d	-0.0412						
		a	-11937						
	Stress	b	-2984	psi					
		c	-7639						
		d	-2984						
		a	-4.6						
	FOS	b	-18.5						
		c	-7.2						
		d	-2.9						
Buckling Calculation									
element	a	b	c	d					
C	1	1	1	1	end fixity				
E	10000000	10000000	10000000	72500	psi				
L	60	4	240	6	in				
gyradius	1	2	1.25	2	in				
sigma crit	27416	24674126	2677	79506	psi				
FOS	-2.3	-8268.4	-0.4	-26.6					
Note: To find the reaction force, FD, we can just plug back in and solve.									
	150000		2094393	-2094393	0	0	0		0.5031
	0	=	-2094393	127757993	-125663600	0	0		0.4315
	0		0	-125663600	126481722	-818122	0		0.4303
	0	lb	0	0	-818122	1425496	-607374		0.2470
Reaction =	-150000	lb	0	0	0	-607374	607374		0.0000

Stiffness Matrix Method – Truss Elements & Coordinate Transformations:

Addition of another translation DOF (y-displacement) at each node to a bar element gives us a "truss" element.



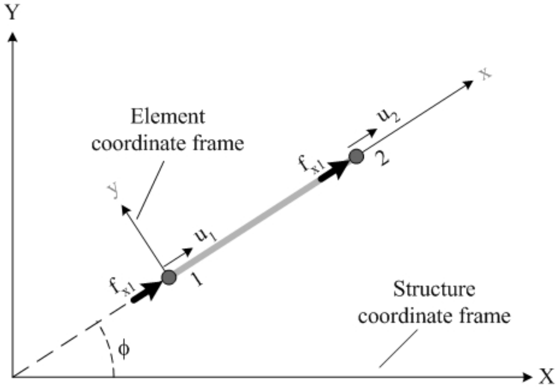
The element matrix equation can be written in a similar manner to the bar element, except we add the additional DOF at each node. Also note that the y-displacements do not produce a force directly to the truss element (since it is pin-ended).

$$\{f\} = \left[k^e\right]\{u\} \;\rightarrow\; \begin{Bmatrix} f_{x1} \\ f_{y1} \\ f_{x2} \\ f_{y2} \end{Bmatrix} = \frac{AE}{L}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{Bmatrix}$$

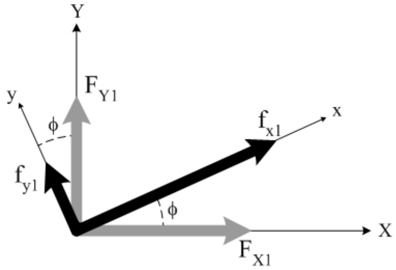
The "global" structure stiffness matrix can be built as before (by "assembly" using the element stiffness matrices).

As you should recall from EN221, the members of a truss structure are oriented at angles to one another (forming triangular sub-structures which provide the rigidity to the truss). Therefore, in order to combine different truss elements, each with different element coordinates (x and y), a transformation must be used. The way this is done is to establish a set of global coordinates (X and Y), into which the local element coordinates (x and y) are transformed. As you can imagine, simple trigonometry is used for this transformation.

Consider a truss element oriented at an angle ϕ from the horizontal as shown in the figure below.



Consider specifically node 1, as shown in the figure below.



Forces applied in the *global* X-Y coordinate frame can be *transformed* into the element x-y frame as follows (this is a standard vector coordinate transformation):

$$\begin{aligned} f_{x1} &= F_{X1}\cos\phi + F_{Y1}\sin\phi \qquad & f_{x2} &= F_{X2}\cos\phi + F_{Y2}\sin\phi \\ f_{y1} &= -F_{X1}\sin\phi + F_{Y1}\cos\phi \qquad & f_{y2} &= -F_{X2}\sin\phi + F_{Y2}\cos\phi \end{aligned}$$

Thus, the forces in the *global* coordinate frame {F} can be transformed to forces in the *element* coordinate frame {f} with a matrix multiplication:

$$\begin{Bmatrix} f_{x1} \\ f_{y1} \\ f_{x2} \\ f_{y2} \end{Bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 & 0 \\ \sin\phi & \cos\phi & 0 & 0 \\ 0 & 0 & \cos\phi & \sin\phi \\ 0 & 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{Bmatrix} F_{X1} \\ F_{Y1} \\ F_{X2} \\ F_{Y2} \end{Bmatrix}$$

The matrix is called the "*transformation matrix*", and is denoted [T], thus

$$\{f\} = [T]\{F\} \quad \ldots\text{where } [T] = \begin{bmatrix} \cos\phi & \sin\phi & 0 & 0 \\ \sin\phi & \cos\phi & 0 & 0 \\ 0 & 0 & \cos\phi & \sin\phi \\ 0 & 0 & -\sin\phi & \cos\phi \end{bmatrix} \quad \textit{“Transformation matrix” (2-D)}$$

Just as the force vector can be transformed using the transformation matrix, so too can the displacement vector be transformed using the (same) transformation matrix! Thus:

$$\{f\} = [T]\{F\} \text{ and } \{u\} = [T]\{U\}$$

Because of its unique structure, $[T]^{-1} = [T]^{T}$ (inverse = transpose). This is a very useful property for the transformation matrix.

Using a little "matrix algebra":

$$\{f\} = [k^e]\{u\} \rightarrow [T]\{F\} = [k^e][T]\{U\} \rightarrow \{F\} = [T]^{-1}[k^e][T]\{U\} = [T]^{T}[k^e][T]\{U\}$$

$$\{F\} = \underbrace{[T]^{T}[k^e][T]}_{\equiv[K^e]}\{U\} = [K^e]\{U\}$$

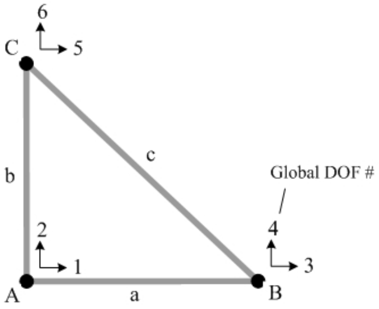
Note that we have defined a "*global element stiffness matrix*": $[K^e] \equiv [T]^{T}[k^e][T]$

In terms of the global coordinate system (X, Y), the global element stiffness matrix for a truss element is:

$$[K^e] = \frac{AE}{L}\begin{bmatrix} \cos^2\phi & \sin\phi\cos\phi & -\cos^2\phi & -\sin\phi\cos\phi \\ \sin\phi\cos\phi & \sin^2\phi & -\sin\phi\cos\phi & -\sin^2\phi \\ -\cos^2\phi & -\sin\phi\cos\phi & \cos^2\phi & \sin\phi\cos\phi \\ -\sin\phi\cos\phi & -\sin^2\phi & \sin\phi\cos\phi & \sin^2\phi \end{bmatrix} = \frac{AE}{L}\begin{bmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{bmatrix} \quad (c \equiv \cos\phi,\ s \equiv \sin\phi)$$

Use this "global element stiffness matrix" [K^e] to assemble your global structure stiffness matrix [K].

Example: Consider a simple truss frame structure (pinned joints).



Note the numbering of the nodes and that each node has 2 DOFs (total 6 DOF).
Assume for this example that the truss is supported (pinned) at A and C (so that $U_1 = 0$, $V_2 = 0$, $U_3 = 0$, $V_4 = 0$) and that a force is applied at B (so that F_3 and F_4 are known). Assume also that all truss elements have same E (elastic modulus) and same A (cross-sectional area).

It’s useful to make a “Transformation Table”:

Element	ϕ (deg)	$\cos\phi$	$\sin\phi$	$\cos^2\phi$	$\sin^2\phi$	$\cos\phi\sin\phi$
a	0	1	0	1	0	0
b	90	0	1	0	1	0
c	135	-0.707	0.707	0.5	0.5	-0.5

Assemble the global element stiffness matrices (1 for each element). Note the annotation for global DOFs (1-6).

$$\left[K^a\right]=\frac{AE}{L_a}\begin{array}{cll} \text{global DOFs for U} & & \\ \begin{array}{cccc}1&2&3&4\end{array} & & \\ \left[\begin{array}{cccc}1&0&-1&0\\0&0&0&0\\-1&0&1&0\\0&0&0&0\end{array}\right] & \begin{array}{c}1\\2\\3\\4\end{array} & \text{global DOFs for F}\end{array}$$

$$\left[K^b\right]=\frac{AE}{L_b}\begin{array}{cll} \text{global DOFs for U} & & \\ \begin{array}{cccc}1&2&5&6\end{array} & & \\ \left[\begin{array}{cccc}0&0&0&0\\0&1&0&-1\\0&0&0&0\\0&-1&0&1\end{array}\right] & \begin{array}{c}1\\2\\5\\6\end{array} & \text{global DOFs for F}\end{array}$$

$$\left[K^c\right]=\frac{AE}{L_c}\begin{array}{cll} \text{global DOFs for U} & & \\ \begin{array}{cccc}3&4&5&6\end{array} & & \\ \left[\begin{array}{cccc}0.5&-0.5&-0.5&0.5\\-0.5&0.5&0.5&-0.5\\-0.5&0.5&0.5&-0.5\\0.5&-0.5&-0.5&0.5\end{array}\right] & \begin{array}{c}3\\4\\5\\6\end{array} & \text{global DOFs for F}\end{array}$$

Note for this problem: $L_a = L_b$ and $L_c = \sqrt{2}\,L_a = \sqrt{2}\,L_b$ thus:

$$\left[K^c\right]=\frac{AE}{L_a}\begin{array}{cll} \text{global DOFs for U} & & \\ \begin{array}{cccc}3&4&5&6\end{array} & & \\ \left[\begin{array}{cccc}0.35&-0.35&-0.35&0.35\\-0.35&0.35&0.35&-0.35\\-0.35&0.35&0.35&-0.35\\0.35&-0.35&-0.35&0.35\end{array}\right] & \begin{array}{c}3\\4\\5\\6\end{array} & \text{global DOFs for F}\end{array}$$

Assemble the global structure stiffness matrix based on each global element stiffness matrix (note that we simply add along the diagonal):

$$\left[K\right]=\frac{AE}{L}\begin{array}{cll} \text{global DOFs for U} & & \\ \begin{array}{cccccc}1&2&3&4&5&6\end{array} & & \\ \left[\begin{array}{cccccc}1+0&0+0&-1&0&0&0\\0+0&0+1&0&0&0&-1\\-1&0&1+0.35&0-0.35&-0.35&0.35\\0&0&0-0.35&0.35&0.35&-0.35\\0&0&-0.35&0.35&0+0.35&0-0.35\\0&-1&0.35&-0.35&0-0.35&1+0.35\end{array}\right] & \begin{array}{c}1\\2\\3\\4\\5\\6\end{array} & \text{global DOFs for F}\end{array}$$

$$\begin{array}{cc}
 & \text{global DOFs for U} \\
 & \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\
[K]=\dfrac{AE}{L} & \left[\begin{array}{cccccc}
1 & 0 & -1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & -1 \\
-1 & 0 & 1.35 & -0.35 & -0.35 & 0.35 \\
0 & 0 & -0.35 & 0.35 & 0.35 & -0.35 \\
0 & 0 & -0.35 & 0.35 & 0.35 & -0.35 \\
0 & -1 & 0.35 & -0.35 & -0.35 & 1.35
\end{array}\right]
\begin{array}{l} 1 \\ 2 \\ 3\ \ \text{global DOFs for F} \\ 4 \\ 5 \\ 6 \end{array}
\end{array}$$

Assemble the stiffness matrix equations, and apply the BCs:

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{Bmatrix} = \frac{AE}{L}\begin{bmatrix}
1 & 0 & -1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & -1 \\
-1 & 0 & 1.35 & -0.35 & -0.35 & 0.35 \\
0 & 0 & -0.35 & 0.35 & 0.35 & -0.35 \\
0 & 0 & -0.35 & 0.35 & 0.35 & -0.35 \\
0 & -1 & 0.35 & -0.35 & -0.35 & 1.35
\end{bmatrix}\begin{Bmatrix} U_1=0 \\ V_2=0 \\ U_3 \\ V_4 \\ U_5=0 \\ V_6=0 \end{Bmatrix}$$

Partition around known BC's:

$$\left\{\begin{array}{c} F_1 \\ F_2 \\ \hline \boxed{F_3} \\ \boxed{F_4} \\ \hline F_5 \\ F_6 \end{array}\right\} = \frac{AE}{L}\left[\begin{array}{cc|cc|cc}
1 & 0 & -1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & -1 \\
\hline
-1 & 0 & \boxed{1.35} & \boxed{-0.35} & -0.35 & 0.35 \\
0 & 0 & \boxed{-0.35} & \boxed{0.35} & 0.35 & -0.35 \\
\hline
0 & 0 & -0.35 & 0.35 & 0.35 & -0.35 \\
0 & -1 & 0.35 & -0.35 & -0.35 & 1.35
\end{array}\right]\left\{\begin{array}{c} U_1=0 \\ V_2=0 \\ \hline \boxed{U_3} \\ \boxed{V_4} \\ \hline U_5=0 \\ V_6=0 \end{array}\right\}$$

$$\rightarrow\ \begin{Bmatrix} F_3 \\ F_4 \end{Bmatrix} = \frac{AE}{L}\begin{bmatrix} 1.35 & -0.35 \\ -0.35 & 0.35 \end{bmatrix}\begin{Bmatrix} U_3 \\ V_4 \end{Bmatrix}$$

Invert and solve for the displacements U_3 and V_4:

$$\begin{Bmatrix} U_3 \\ V_4 \end{Bmatrix} = \frac{L}{AE}\begin{bmatrix} 1.35 & -0.35 \\ -0.35 & 0.35 \end{bmatrix}^{-1}\begin{Bmatrix} F_3 \\ F_4 \end{Bmatrix} = \frac{L}{AE}\left(\frac{\begin{bmatrix} 0.35 & 0.35 \\ 0.35 & 1.35 \end{bmatrix}}{((0.35)(1.35)-(0.35)(0.35))}\right)\begin{Bmatrix} F_3 \\ F_4 \end{Bmatrix}$$

$$\rightarrow\ \begin{Bmatrix} U_3 \\ V_4 \end{Bmatrix} = \frac{L}{AE}\begin{bmatrix} 1 & 1 \\ 1 & 3.85 \end{bmatrix}\begin{Bmatrix} F_3 \\ F_4 \end{Bmatrix}$$

Solve for the reactions at nodes A and C:

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_5 \\ F_6 \end{Bmatrix} = \frac{AE}{L}\begin{bmatrix} -1 & 0 \\ 0 & 0 \\ -0.35 & 0.35 \\ 0.35 & -0.35 \end{bmatrix}\begin{Bmatrix} U_3 \\ V_4 \end{Bmatrix} = \frac{AE}{L}\begin{bmatrix} -1 & 0 \\ 0 & 0 \\ -0.35 & 0.35 \\ 0.35 & -0.35 \end{bmatrix}\frac{L}{AE}\begin{bmatrix} 1 & 1 \\ 1 & 3.85 \end{bmatrix}\begin{Bmatrix} F_3 \\ F_4 \end{Bmatrix} = \begin{bmatrix} -1 & 1 \\ 0 & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix}\begin{Bmatrix} F_3 \\ F_4 \end{Bmatrix} = \begin{Bmatrix} -F_3-F_4 \\ 0 \\ F_4 \\ -F_4 \end{Bmatrix}$$

Element strain, stress:

Use the transformation matrix to obtain displacements in element coordinate frames: $\{u\}=[T]\{U\}$

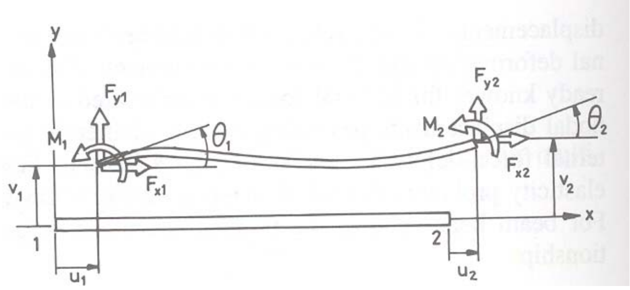
Example for a truss element "a" with nodes "A" and "B" oriented at an angle ϕ to *global* X-axis:

$$\varepsilon_a = \frac{u_B-u_A}{L_a} = \frac{(U_B-U_A)\cos\phi+(V_B-V_A)\sin\phi}{L_a}$$

$$\sigma_a = \varepsilon_a E_a$$

Stiffness Matrix Method – Beam Elements (Rigid-Jointed Frames):

Recall that a "truss" is a pin-joined member that has 2 DOF (translation) at each node, but is not able to transmit moments. A "beam", on the other hand, can transmit moments (i.e. it can *bend*). A frame structure with "welded" (i.e. rigid) joints, would then be considered to be made up of "beam elements", which are able to "stretch" (or compress), as well as "bend". So, a rigid-jointed frame structure could have 3 DOF at each node – axial displacement (u), transverse displacement (v), and angular rotation (θ). This is illustrated in the below figure.



What do the element displacement and force vectors look like?

$$6 \text{ total DOF} \ldots \{u\} = \begin{Bmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \end{Bmatrix} \qquad 6 \text{ forces/moments} \ldots \{f\} = \begin{Bmatrix} F_{x1} \\ F_{y1} \\ M_1 \\ F_{x2} \\ F_{y2} \\ M_2 \end{Bmatrix}$$

What does the *element* stiffness matrix look like?

It is 6 x 6 matrix.
It is made up of elements for axial stiffness (AE/L) <u>and</u> bending stiffness (EI/L^3)

The bending stiffness terms are developed using matrix analysis (see Hughes).

$$\left[k^e\right] = \begin{bmatrix} \alpha & 0 & 0 & -\alpha & 0 & 0 \\ 0 & 12\beta & 6\beta L & 0 & -12\beta & 6\beta L \\ 0 & 6\beta L & 4\beta L^2 & 0 & -6\beta L & 2\beta L^2 \\ -\alpha & 0 & 0 & \alpha & 0 & 0 \\ 0 & -12\beta & -6\beta L & 0 & 12\beta & -6\beta L \\ 0 & 6\beta L & 2\beta L^2 & 0 & -6\beta L & 4\beta L^2 \end{bmatrix} \qquad \text{where } \alpha \equiv \frac{AE}{L} \text{ and } \beta \equiv \frac{EI}{L^3}$$

The rest is the same as before! (…transformation, assembly, partitioning, inversion, etc.)

Stiffness Matrix Analysis – 7 Step Process (Summary):

The following provides a summary of the process for solution of a problem using the stiffness matrix method:

1. Define structural model
    - Free body diagram
    - Geometry → Transformation Matrix
    - Material properties
2. Calculate each global *element* stiffness matrix [K^e]
3. Assemble the global *structure* stiffness matrix [K]
4. Apply BCs to {F} and/or {U}, then partition
5. Solve for the *global* displacements and reactions
    - Global displacements in non-BC DOFs (nodes)
    - Reaction forces at BCs (nodes)
6. Transform the element coordinate system, solve for *element* nodal displacements
7. Solve for *element* strains, stresses and factors of safety

The Deacon's Masterpiece

Finite element analysis has provided the naval architect the opportunity, like never before, to *optimize* a ship's structure. With the clear graphical output it is easy to see where the stress hot spots and cool areas are. By adding, removing, changing the thickness, or rearranging components, it is possible to generate a uniform stress field on a part, theoretically making the factor of safety the same throughout! Theoretically that means when the part fails, it fails everywhere at once! This is the proverbial "perfectly engineered part". The classic question is whether that is the right idea!

Back in 1858, Oliver Wendell Holmes (poet and savior of the USS Constitution) wrote a poem asking just that question.

The Deacons Masterpiece

Have you heard of the wonderful one-hoss shay,
That was built in such a logical way
It ran a hundred years to a day,
And then, of a sudden, it - ah, but stay,
And I'll tell you what happened without delay,
Scaring the parson into fits,
Frightening people out of their wits,
Have you ever heard of that, I say?

Seventeen hundred and fifty-five,
Georgius Secundus was then alive,

Snuffy old drone from the German hive.
That was the year when Lisbon-town
Saw the earth open and gulp her down,
And Braddock's army was done so brown,
Left without a scalp to its crown.
It was on the terrible Earthquake-day
That the Deacon finished the one-hoss shay

Now in building of chaises, I tell you what,
There is always somewhere a weaker spot,
In hub, tire, felloe, in spring or thill,
In panel, or crossbar, or floor, or sill,
In screw, bolt, thoroughbrace, - lurking still,
Find it somewhere you must and will,
Above or below, or within or without,

And that's the reason, beyond a doubt,
A chaise breaks down, but doesn't wear out.

But the Deacon swore (as Deacons do),
With an "I dew vum," or an "I tell yeou,"
He would build one shay to beat the taown
'N' the keounty 'n' all the kentry raoun';
It should be so built that it couldn' break daown:
"Fur," said the Deacon, "'t 's mighty plain
Thut the weakes' place mus' stan' the strain;
'N' the way t' fix it, uz I maintain,
Is only jest
T' make that place uz strong uz the rest."

So the Deacon inquired of the village folk
Where he could find the strongest oak,
That couldn't be split nor bent nor broke,
That was for spokes and floor and sills;
He sent for lancewood to make the thills;
The crossbars were ash, from the strightest trees,
The panels of white-wood, that cuts like cheese,
But lasts like iron for things like these;
The hubs of logs from the "Settler's ellum,"
Last of its timber,--they couldn't sell 'em,
Never an axe had seen their chips,
And the wedges flew from between their lips,
Their blunt ends frizzled like celery tips;
Step and prop-iron, bolt and screw,
Spring, tire, axle, and linchpin too,
Steel of the finest, bright and blue;
Thoroughbrace bison-skin, thick and wide;
Boot, top, dasher, from tough old hide
Found in the pit when the tanner died.
That was the way he "put her through."
"There!" said the Deacon, "naow she'll dew!"

Do! I tell you, I rather guess
She was a wonder, and nothing less!
Colts grew horses, beards turned gray,
Deacon and Deaconess dropped away,
Children and grandchildren - where were they?
But there stood the stout old-one-hoss shay
As fresh as on Lisbon-earthquake-day!

Eighteen Hundred; it came and found
The Deacon's masterpiece strong and sound.
Eighteen hundred increased by ten;--
"Hahnsum kerridge" they called it then.
Eighteen hundred and twenty came;--
Running as usual; much the same.
Thirty and forty at last arrive,
And then came fifty, and Fifty-five

Little of all we value here
Wakes on the morn of its hundredth year
Without both feeling and looking queer.
In fact, there's nothing that keeps its youth,
So far as I know, but a tree and truth.
(This as a moral that runs at large;
Take it, - You're welcome. - No extra charge.)

First of November - the-Earthquake-day,
There are traces of age in the one-hoss-shay,
A general flavor of mild decay,
But nothing local, as one may say.
There couldn't be, - for the Deacon's art
Had made it so like in every part
That there wasn't a chance for one to start.

For the wheels were just as strong as the thills,
And the floor was just as strong as the sills,
And the panels just as strong as the floor,
And the whipple-tree neither less nor more,
And spring and axle and hub encore,
And yet, as a whole, it is past a doubt
In another hour it will be worn out!

First of November, 'Fifty-five!
This morning the parson takes a drive.
Now, small boys, get out of the way!
Here comes the wonderful one-hoss shay,
Drawn by a rat-tailed, ewe-necked bay.
"Huddup!" said the parson. Off went they.
The parson was working his Sunday text,
Had got to fifthly, and stopped perplexed
At what the - Moses - was coming next.
All at once the horse stood still,
Close by the meet'n'-house on the hill.
First a shiver, and then a thrill,
Then something decidedly like a spill,
And the parson was sitting up on a rock,
At half-past nine by the meet'n'-house clock,
Just the hour of the Earthquake shock!
What do you think the parson found,
When he got up and stared around?
The poor old chaise in a heap or mound,
As if it had been to the mill and ground!
You see, of course, if you're not a dunce,
How it went to pieces all at once,
All at once, and nothing first,
Just as bubbles do when they burst.

End of the wonderful one-hoss shay,
Logic is logic. That's all I say.

As naval architects, do we want the perfectly engineered structure that fails catastrophically all at once? Or perhaps do we want some warning? Does it make a difference if performance is paramount, such as in the America’s Cup?

It might be worthwhile to recall the Code of Ethics for Engineers.

Code of Ethics for Engineers (from National Society of Professional Engineers)

Engineers, in the fulfillment of their professional duties, shall:

1. **Hold paramount the safety, health and welfare of the public.**
2. **Perform services only in areas of their competence.**
3. **Issue public statements only in an objective and truthful manner.**
4. **Act for each employer or client as faithful agents or trustees.**
5. **Avoid deceptive acts.**
6. **Conduct themselves honorably, responsibly, ethically, and lawfully so as to enhance the honor, reputation, and usefulness of the profession.**

Does that shed any light on the appropriate design strategy?! Hopefully!

Some thought-provoking questions:

What would you do if your boss tasked you to design a ferry with a FOS of 1.1, because it would be less expensive to build, therefore making your company more competitive?

What if, after the launching of a vessel you designed, you realized that you goofed in your assumptions and the actual factor of safety will be a lot less than you originally estimated?

What would you do if your client, the shipbuilder, tells you to substitute materials or equipment for less-expensive versions, even though you know them to be nondurable or unreliable?

These and other “gray area” questions are common in our industry, and present challenges that are often hard to deal with. Stick to your ethics and the Code, and do what you feel is the right thing.

Now, back to learning about ship structural design!

Weekly Assignment #7: FEA Beam Bending Analysis

Deliverables:

Submit an engineering report comparing your results from Assignment #5 with your results using the COSMOS/M finite element analysis program. Note that the load applied in the FEA model is 3000 lb (per support). As a *minimum*, include the following in your report:

(a) Stress plots. Provide one each VonMises, σ_x, and τ_{xy} (provide nodal values, deformed, perspective top view). Discuss the stress "hot spots" (positive and negative) in each stress plot (maximum/positive stresses are red, minimum/negative stresses are dark blue). Explain why the VonMises stress "hot spots" where the loads are applied are not real (not correct). Where in the beam is the maximum "strain energy", and therefore where is it likely to yield first?

(b) Strain plots. Provide two ε_x (provide one for nodal values and one for element values, both deformed, perspective top view). What is the difference between the two plots?

(c) Calculation of minimum FOS. Show all of your calculations and explain clearly how you have done it. Hint: see (a) above.

(d) A detailed discussion of the theoretical stress calculation (from Assignment #5) vs. FEA predictions. Specifically, compare the theoretical (Euler) maximum bending stress (σ_{max}) with FEA predicitions (σ_x) at gage locations 1, 2, and 5. To do this, you will need to *list* the *stress components* for the specific nodes and/or elements where the strain gages are located (explain clearly your choice of the appropriate corresponding FEA element or node number for each strain gage location). Discuss the differences in stress at the three gage locations (#1, 2, and 5). What causes these differences (be specific)?

(e) A detailed discussion of the strain gage experimental results (from Assignment #5) vs. FEA predictions. Specifically, compare the *normal strains* for gage locations #1, 2, 5, 6, and 7. To do this, you will need to *list* the *strain components* for specific nodes and/or elements where the strain gages are located (explain clearly your choice of the appropriate corresponding FEA element or node number for each strain gauge). For the hatch corner locations, you will need to use the "plane strain transformation equations" (see your EN221 text) to compare normal strains at the orientation of the gauges (± 45 degrees). *For extra credit, use the stress components, generalized Hooke's Law, then the strain transformation equation (verify the FEA strains* ε_x, ε_y and γ_{xy}*).* Show all of your calculations in comparing the FEA to the experiment, and explain clearly how you do it. Discuss the results (comparisons).

Note: This is an individual project. Each student must do and submit their own work (although *collaboration* is encouraged, as you might learn from each other)!

Suggested Approach for Using COSMOS/M FEA:

Welcome to the wonderful world of finite element analysis (FEA)! FEA is a growing technique used to analyze structures. While FEA can be very accurate, it can also be incredibly inaccurate if used incorrectly, and its misuse has caused some very large accidents. A routine step when starting a problem on a program you have never used is to run a validation study. You will do that by comparing your beam calculations to FEA predictions. You will use the COSMOS/M finite element software demonstrated in class. The FEA preprocessor is GeoSTAR and will be your sole portal to the program. Four subversions exist for this program and are indicated by numbers after the. These related to the number of allowable DOFs you can have ("GeoSTAR 128K" models can have up to 128,000 DOFs). Given the size of our models you will use the version designated "GeoSTAR 128K". Note that a model built in one version is not compatible with other versions.

Here is a step-by-step process for this lab. You may also wish to try some of the tutorials built into the program. What you are going to do in this lab is import an existing model and then analyze it. In future labs you will build the models, and finally in this course you will design using FEA.

1. After opening the program you will need to open a file (*.gen). Each COSMOS "project" creates up to 60 files, so it is important to put each project in its own folder. So, the first time you will need to create a new folder for each group and then open a new file. It is a good idea to use eight letters or less for your model name. An

example might be “beampm1”, which is the type of problem, followed by the analyst’s initials, followed by the revision number (as you progress you will probably have multiple versions of the model). You will get a question whether you want to create the file. Answer yes.

2. You should now see a black screen with an axis. At the bottom is a console bar. You can enter commands by either typing them in or with the pull-down menus.

3. For printing it is best to change the background color to white and the lettering to blue or black. Do that by using the B-C button on the left, for the background color. To output any pictures the best way to save them is as a tiff file. Do this by using the Control-Devices-DeviceFile-TIFFFile command, and then viewing the result in MS Photo Editor. You can then change the file type to jpg to reduce its size. This is the best way to include the pictures in your reports.

4. The next thing to do is to create the model. The general modeling steps are to: create the geometry, define the material properties, select the element type (group), define the real constants, create the elements and nodes, input the boundary conditions and loads, specify the analysis type and run it! In this case however, you will import a model that was already created. You will then run it and analyze the results.

5. Go to the course Blackboard page and download the file name “BeamJS1.gfm” and place it in your working folder. The *.gfm extension indicates a particular format for this program. You can open the file with a text editor and see the commands that create the model. Save the file in your directory.

6. Use the File-Load command to open the file. Note that when the dialog box opens it is looking for a *.geo extension. Change that to *.gfm. Accept the defaults and the model should read in.

7. You will need to resize your screen to fit the model, click the “Auto” button on the left. You now should have the entire model in front of you. Everything that was used to create the model is shown, including: points, surfaces, nodes, elements, loads, and boundary conditions. Hit the clear screen button (CLS) in the bottom left to remove everything, then the Meshing-Elements-Plot command to plot the elements, and the Meshing-Nodes-Plot command to see the nodes.

8. To see the values associated with the model, use the PropSets-List Material Props to see the values used for the aluminum, and the PropSets-List Real Constants for the thickness, and the PropSets-List Element Groups for the type of element used. In this case we are using a shell element.

9. The LoadsBC-Structural menu will show you the forces (by using the Plot command) and will give you their value (by using the List command). To figure out which node is which, select the Meshing-Nodes-Identify command and pick the node using the mouse. The LoadsBC-Structural-Displacement menu will give you the option of showing the boundary conditions. The model should be complete, so we can now run it!

10. Before the first time you run it you need to select the analysis method. Since it is not moving we will choose a static analysis, and since the deformations are small, we will use a linear analysis. Go to the Analysis-Static-StaticAnalysisOptions menu, and by selecting all the defaults you will set up your run. To be safer however, select the Soft Spring Flag on. That will remove all the zeros from the stiffness matrix and should avoid minor singularities. Once you have set this command you will not need to reset it if you modify the model.

11. Analysis-Static-RunStaticAnalysis will initiate the run. If it doesn’t run, try Analysis-RunCheck to identify potential errors. The run should take less than a minute.

12. After a successful run use Results-Plot-DeformedShape to see if you get what you expected! To see the displacement, strain and stress results you will need to activate that plot. For example, to see the averaged directional strains (ε_x, ε_y, etc.) your two-step process will be to first go to Results-Plot-Strain and select EPSX and element strain and then hit the contour button. Choose the deformed plot and then look at the results. The legend on the side shows the maximum and minimum values. You may find that there are only colors and no numbers. That is because the writing color is the same as the background color. Change the plot lettering to blue or black by selecting Results-Setup-Color/ValueRange, continue to the second menu and set Chart Color to blue or black. In the Results menu you can also plot the stresses! To get the most accurate stress output you will want to use nodal stresses rather than element stresses. The nodal stresses are interpolated at each point while the element stresses and strains are averaged over the element size. Note that if the element size is the same as

the strain gage size then the element strain should be exactly the same as the strain gage reading! To find the stress or strain at a particular element location you can find the element number through the element identify command, then use the results-list command to obtain the values.

If you get really screwed up, then delete all the files in your directory (except the GFM file) and start over! Some things you might experiment with (after creating the GFM file) include animate, buckling, etc. Under the Control-Measure command is Find Mass Prop. That will give you the mass (not weight) of the model in the appropriate units. That is a good check to see if you have modeled it correctly.

When you have questions, don’t hesitate to stop by, call, etc.!
Note! The software is provided with numerous tutorials for additional practice!

Tertiary Structure: Plates

The focus in this course so far has been to analyze beam structures, where we defined a beam as a structure that has one dimension (x-axis) larger than the other two (y and z axes). This allowed us to ignore Poisson's effects, which is the main difference between plates and beams. Plates are structures that have two dimensions that are similar in magnitude and one (thickness) that is much less. Many of the same theories used for beams can be modified slightly to apply to plate structures. The other big difference between beams and plates is that for plates we always have to consider loads in more than one direction. This means almost every plate structure we analyze will require us to use the VonMises equivalent stress.

We will use plate analysis to design plating for the hull, decks and bulkheads. While point loads are possible, most plate loading is due to hydrostatic pressure (external or internal). On the bottom plating the pressure will be uniform, but on the topsides (side shell plating) it will be linearly varying.

Boundary Conditions and Plate Bending:

Plates have many of the characteristics that beams have. Initial deflection is proportional to load, giving a linear response that is relatively easy to calculate. Larger deflections have a nonlinear elastic component called *membrane stresses* that increase apparent strength and stiffness. Plastic deformation follows. Plates can also fail in buckling. As with beams, the boundary conditions strongly influence the failure loads.

A typical plate on a ship is one of the hull bottom plating. It is bounded on its four sides by longitudinals and frames. These stiffeners are much stiffer than the plate, and carry the load from the plate to the bulkheads and/or web frames. As far as the plates are concerned, the stiffeners are rigid structures. As a rigid structure they can be modeled using the standard boundary conditions. Most commonly these are simply-supported, pinned or fixed. The former allows translations in plane but not out-of-plane and allows rotations about the axis (like a hinge on a roller). The second allows no translations but does allow translations (like a fixed hinge). The third allows neither translation nor rotation.

One concept to understand in plate bending is how much stress is generated in the different directions of a plate. Stress, by definition, is a function of strain (radius of curvature for bending). Plate analysis is mostly about bending loads, causing an uneven distribution of strain. Take for example a plate that is 4 feet long (in the x-direction) and 2 feet wide (in the y-direction) that has a point load in the middle that is normal to the plate (in the z-direction). The plate deflects such that the location of the point load has the most deformation. Looking at the stresses along the x-axis and the y-axis, it is interesting to note that the y-axis stresses will be larger. Why? Since the maximum deflection is the same along both x and y axes, but the x dimension is larger, it figures that along the y-axis the radius of curvature is tighter, giving a larger strain and a higher stress. This in general is true; *the stress will be higher in the shorter plate dimension*.

Typically we refer to a plate having thickness denoted by *t*, a length or span (longer dimension) denoted *a*, and a short width or breadth (shorter dimension) denoted *b*.

Small Deflection Plate Theory:

We might think of a plate bending like a row of beams, with the beams glued or welded together at their adjacent boundaries. Recall that Hooke's law for 1-D (like a beam) is:

$$\sigma = E \cdot \varepsilon$$

Recall also for a beam that:

$$\sigma_x = \frac{My}{I}$$

Combining:

$$\varepsilon_x = \frac{My}{EI}$$

We see that EI is a "bending stiffness" for a beam. The main assumptions for beam bending were that plane sections remained plane, and that no transverse strain or stress existed, so no transverse bending.

For a 2-dimensional plate bending we must limit the deflection to small values (less than ¾ of the plating thickness t). Unlike a beam, for a plate this is necessary because membrane stresses develop for larger deflections (due to the Poisson effect). Additional assumptions for this "small deflection plate bending" are:

- No strain or stress exists through thickness
- There is bending strain (or stress) on the top and bottom, but the strain (or stress) at the neutral axis is zero.
- Plane sections remain plane
- Material is isotropic
- Material behavior remains in the linear elastic range

Consider a very thin beam of width b (perhaps made with a very narrow plate) vs. a very wide plate of width b and length a, unsupported along its sides, but both with simply-supported end conditions, and both loaded with a point load at the center. The beam bends downward in a curve as you would imagine. However, due to the Poisson effects, the deformed plate looks like a Pringle! For the beam, the "bending stiffness" EI can be calculated as:

$$EI_{beam} = E\frac{bt^3}{12}$$

However, for the wide plate, the equivalent "bending stiffness" is

$$EI_{plate} = E\frac{bt^3}{12\left(1-\upsilon^2\right)}$$

A parameter which gives the "bending stiffness" per unit width b is denoted D is known as the "flexural rigidity" of the plate, which using the above gives:

$$D \equiv \frac{EI_{plate}}{b} = \frac{Et^3}{12\left(1-\upsilon^2\right)}$$

For different materials, υ ranges from 0.01 -0.45 (although some composite laminates can be 0.15 to 0.8). A typical value is 0.3. So, which is stiffer, the beam or the plate? Since υ is greater than 0, then the plate is thicker than the beam. Thus, a wide plate is stiffer than a row of beams (having the same total width b. Why? Because of the Poisson effect.

For a plate of width b and length a, the governing equation for bending deflection can be derived in a similar (just more complex) approach to the differential equation for beam bending – using static equilibrium of a differential element of plating (see Hughes). The governing equation is the 4th order partial differential equation known as the Biharmonic Equation:

$$\nabla^4 w = \frac{p}{D}$$

where w is the lateral (out of plane) deflection of the plate, p is the lateral pressure (load) on the plate, and D is the "flexural rigidity" of the plate (as defined above).

In rectangular x-y Cartesian coordinates, the Biharmonic Equation is:

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{p}{D}$$

Like any differential equation, the solution depends upon the characteristics of the load (p) and the boundary conditions (BCs) along all of the edges of the plate. In general however, what is the form of the solution such that you can take the fourth derivative and still get a reasonable solution? Sketch the forms (profile view)…

Simply-supported: Clamped (fixed-fixed): Pinned:

Note that in FEA, simply-supported boundary conditions cannot be applied on all edges, or you get rigid-body motion! At least one node must be constrained in all translations (typically 1 edge pinned).

Solutions to the Biharmonic Equation with a uniformly distributed load p (uniform pressure) for different boundary conditions have been solved using some advanced mathematics (by some very smart people). The two most important we will use. These are for simply-supported and clamped (fixed) boundary conditions.

The solution for simply-supported edges (all 4 edges simply-supported) was given by Navier, who applied a uniform load p as a Fourier Series (a summation of sinusoids of different frequencies). The maximum deflection (which occurs at the center of the plate is:

$$w_{\max(ss)} = k_1 \frac{5pb^4}{384D}$$

where k_1 is a parameter which depends upon the "aspect ratio" of the plate (a/b), where a = long side and b = short side.

The solution for the clamped boundary condition (all 4 edges clamped) was given by Levy:

$$w_{\max(\text{clamped})} = k_2 \frac{pb^4}{384D}$$

where k_2 is a parameter which depends upon the aspect ratio a/b.

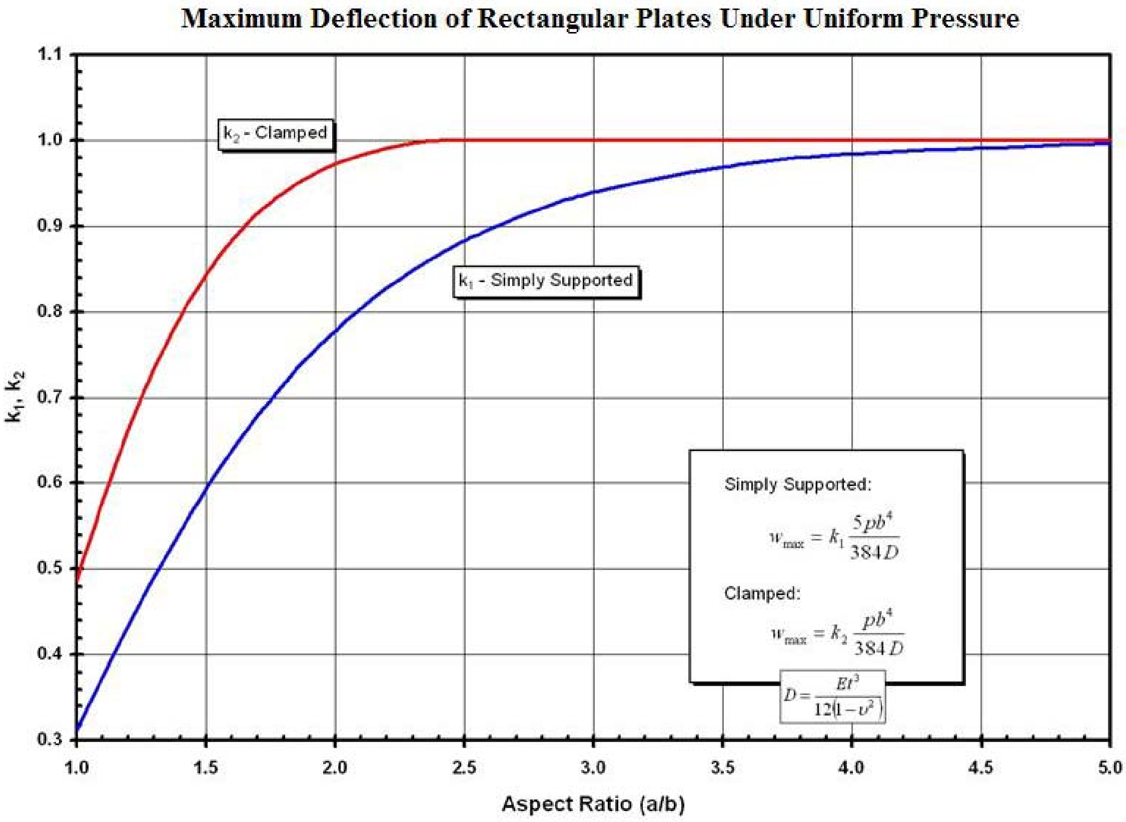
For both simply-supported and clamped boundary conditions, the maximum stress can be written:

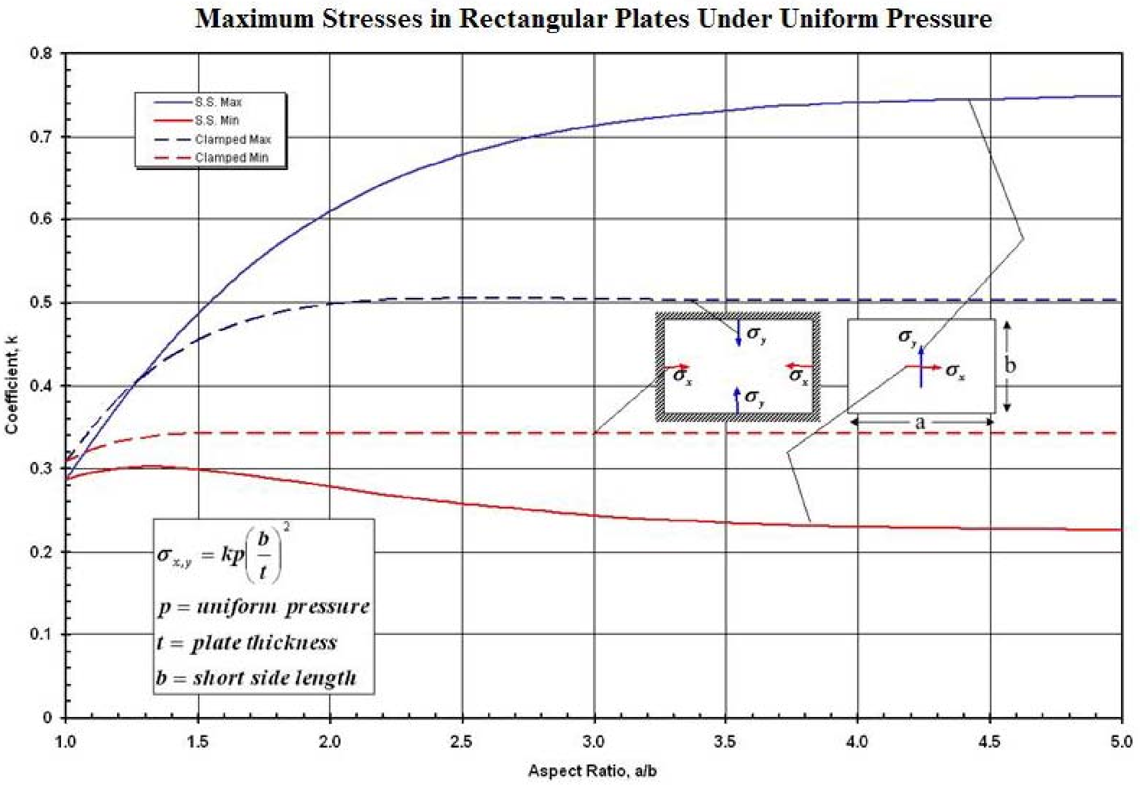
$$\sigma_{\max} = k \cdot p \cdot \left(\frac{b}{t}\right)^2$$

Where k is a parameter which depends upon the aspect ratio a/b AND the boundary conditions.

For all of the above, use the figures below and on the next page. Note however that the locations of the maximum stress components for simply-supported and clamped BCs are not the same! Note also the directions of the maximum stress components for each BC. For the simply-supported BC, the maximum stress components occur simultaneously at the center of the plate, thus VonMises must be used to determine equivalent stress and FOS.

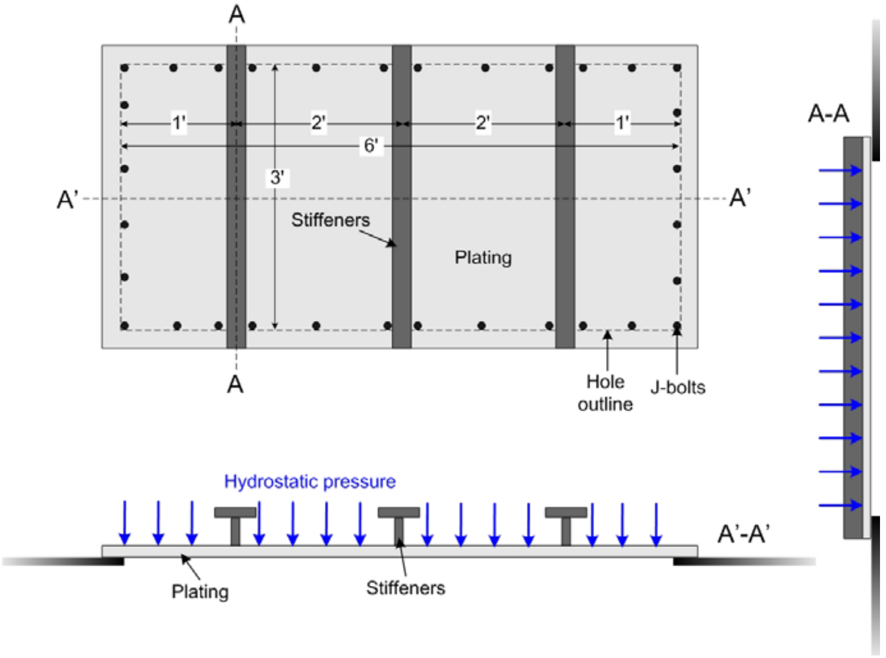
Small deflection plate bending is a conservative method for predicting maximum bending stress and deflection in a plate that works in most ship design cases, particularly stress values well below yield. The common approach is that the design engineer starts by assuming that the deflection is going to be small and uses this theory. **If the predicted deflection is greater than ¾ t, then they make the panel smaller (by reducing stiffener spacing), make the plate thicker, or go to large deflection theory!**




Maximum Stresses in Rectangular Plates Under Uniform Pressure
S.S. Max
S.S. Min
Clamped Max
Clamped Min
$\sigma_{x,y} = kp\left(\frac{b}{t}\right)^2$
p = uniform pressure
t = plate thickness
b = short side length
σ_y
σ_x
σ_x
σ_y
σ_y
σ_x
b
a
Coefficient, k
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Aspect Ratio, a/b
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0

In-class Exercise: Stiffened Plate Salvage Patch Design

Work in teams of two and consider the following ship structural design problem. You are the DCA on a DDG-51 class destroyer. While on patrol in a war zone, your ship strikes a floating mine, which blows a hole in a bow compartment, causing the compartment to flood. It is desired to apply an external patch over the hole so that the compartment can be dewatered using salvage pumps. The hole in the plating has been trimmed by divers to an opening which is nearly rectangular, of dimensions 6 ft wide by 3 ft high. It is expected that the patch would have to withstand a hydrostatic pressure of 30 feet of seawater. One candidate patch built by HT3 Smuckatelli is made of ½" A36 steel plate with three MT 2x6.5 (T-stiffeners) as shown in the figure below. The patch is to be installed by securing it with J-bolts around the edges (therefore, assume simply-supported edges). Calculate the maximum stress in the plating and the stiffeners, and the minimum FOS vs. material yield. For the MT 2 x 6.5 T-stiffeners, you can use the following approximate dimensions: d = 2.0", t_w = 0.25", b_f = 3.94", t_f = 0.37".



Weekly Assignment #8: Small Deflection Plate Bending

Deliverable:

An engineering report comparing three methods of determining the stress in a plate due to an out-of-plane load.

Approach:

Using small deflection plate theory, determine the minimum plate thickness needed for a 6-ft by 3-ft steel plate to carry a hydrostatic pressure head of 20 ft of salt water. Assume the steel is Grade A36 (however, for this problem only, use a Poisson ratio of 0.30). Clearly state your final, recommended thickness.

1. To "bound the problem", first evaluate using simply-supported boundary conditions all around using the graphs to do the calculations. Assume the plate is available in 1/16" increments. Calculate the resulting maximum VonMises stress and FOS based on your final plate thickness.

2. Re-evaluate your results in problem 1 assuming that the boundary conditions are clamped (fixed) all around and choose what your final plate thickness (only one!) will be based on the simply-supported and fixed results.

3. Using the spreadsheet program on the course Blackboard page (for simply supported plates) analyze the plate with the final thickness you have chosen. Prepare graphs showing the deflection, magnitude of the x-direction stress and magnitude of the y-direction stress across the middle of the plate in both the short and long directions. (This means two graphs total, one for the short direction and for the long direction) Use two different Y-axes on your graphs (one for deflection and one for stress). Determine the maximum Von Mises stress.

4. Repeat problem 3 for the plate thickness you chose in problem 2, using the clamped spreadsheet program to perform the analysis (also available on the corse Blackboard page). Again produce two graphs, or put the plots on the two graphs in problem 3 (that would mean four plots per graph).

   Note: You should be aware that the curves on the previous pages in the notes for Small-Deflection Plate Theory assume a Poisson ratio of 0.30 (this is not stated explicitly on the curves). Since A36 steel has a Poisson ratio of 0.26 per MATWEB, there would be a small difference between the results using the curves and the spreadsheet (and the FEA below). So, for comparison in this problem only, you should use a Poisson ratio of 0.30 throughout. Assuming that you are careful in extracting coefficients from the curves, you should see that all of the methods (curves, spreadsheet, FEA) are within a few psi (stress) of each other (less than 1% difference)! Consider using a set of dividers to carefully extract coefficients from the curves.

5. Compare your results (displacement, stress, FOS) in a tabular format, with % variation, to the FEA results (Von Mises and deflection). Here are some guidelines for the finite element analysis

   (1) Start by creating a new folder and a new model name. Open a file, change the background color, etc.

   (2) The general modeling steps are to: create the geometry, define the material properties, select the element type (group), define the real constants, create the elements and nodes, input the boundary conditions and loads, specify the analysis type and run it!

   (3) Start by creating points to describe the boundaries. Use Geometry-Points-Define to specify the coordinates. Be careful to stay consistent with your units throughout the process! If you make a mistake just reenter the point's number and coordinates.

   (4) You will only see a couple points at a time, so to resize your screen to fit the model, click the "Auto" button on the left. Next is to create the surface from the four points. Use Geometry-Surfaces-Define-by-4Pt to create the surface. You can either enter the point numbers or click on the screen. The "Pic" icon on the left may need to be on to pick off the screen. Make sure you either go clockwise or CCW to create a rectangle!

   (5) Define the material properties by using the PropSets-PickMaterialLib and the first option will give you "allow steel". Note that no strengths are given. You will need to calculate the FOS manually after reviewing the stress output. Unfortunately, the "alloy steel" in the library uses a Poisson ratio of 0.28. You can

change the Poisson ratio to 0.30 by choosing PropSets-MaterialProperty, and manually entering (changing) the Poisson ratio (change all three) as 0.30. If you do this, your FEA should be within a few psi of the curves and spreadsheet results!

(6) PropSets-ElementGroup will define the type of element. We will use the Shell4 element. Accept all the default values for the element.

(7) PropSets-RealConstants will define the thickness. Make sure the associated element group is the same number as you selected for the EG. At this point that should be 1. The first real constant will be your initial plate thickness. After you have run the model you can redefine that real constant to improve your design. Accept the other defaults.

(8) To create the elements (EL) and nodes (ND) we will mesh the surface. Meshing-ParametricMesh-Surfaces will create the best mesh in this case. Accept the defaults, except for the number of elements on each side of the surface. As we know, mesh density is important to the results. You will generally need at least 20 elements between supports for out-of-plane loads. To improve accuracy you would like to have your mesh comprised of nearly square elements.

(9) With the elements created we need to define the boundary conditions (BC). The four edges of the surface (SF) are curves (CR). You can see those by choosing curve plot from the Geometry-Editing menu. You can define the BCs from the LoadsBC menu using either the curves or the nodes. LoadsBC-Structural-Displacement-DefinebyCurves will allow you to set the BCs. Make sure you consider all the DOFs so that you don't get rigid body motion!

(10) LoadsBC-Structural-Pressure-DefinebySurfaces will allow you to apply the pressure load. Pressure direction 4:normal will place the pressure on the correct face, which is face #5. You can use the zoom buttons on the left to take a closer look at an element.

(11) Analysis-Static-StaticAnalysisOptions, with all the defaults will set up your run. To be safer however, select the Soft Spring Flag on. That will remove all the zeros from the stiffness matrix and should avoid minor singularities. Once you have set this command you will not need to reset it if you modify the model.

(12) Analysis-Static-RunStaticAnalysis will initiate the run. If it doesn't run, try Analysis-RunCheck to identify potential errors.

(13) Plot the results as you did for the beam. Create tiff files and copy them into your report. Use a white background color.

Hint: If you did all the calculations, steps and getting the results correctly, all your results should be within 1.5%!

Large Deflection Plate Bending

Small-deflection plate bending theory works well if the deflection is less than about ¾ of the plate thickness. This generally occurs when the material is relatively "stiff", with relatively small spans, when moderate factors of safety are used (greater than 2), and when the boundary conditions are "mostly fixed" (no translations at the ends - pinned or better). This is generally OK for more than 90% of ship structural design.

If the deflection of the plate exceeds about ¾ of the thickness, then the edges of the plate try to "pull-in". If the edges are constrained (pinned, fixed, welded, etc.), then there is resistance to this "pull-in", and we get "membrane" or "in-plane" effects, which create tensile forces throughout the plate. Simple every-day examples of membrane effects include sailboat sails, guitar strings, and balloons.

An interesting result of this membrane effect is that it increases the effective stiffness of the structure to bending. Structures with in-plane tensile forces which increase effective stiffness are called "membrane structures". For these types of structures, because deflections can be "large" (greater than ¾ the thickness), we must resort to a "large deflection plate bending method", which is a non-linear method.

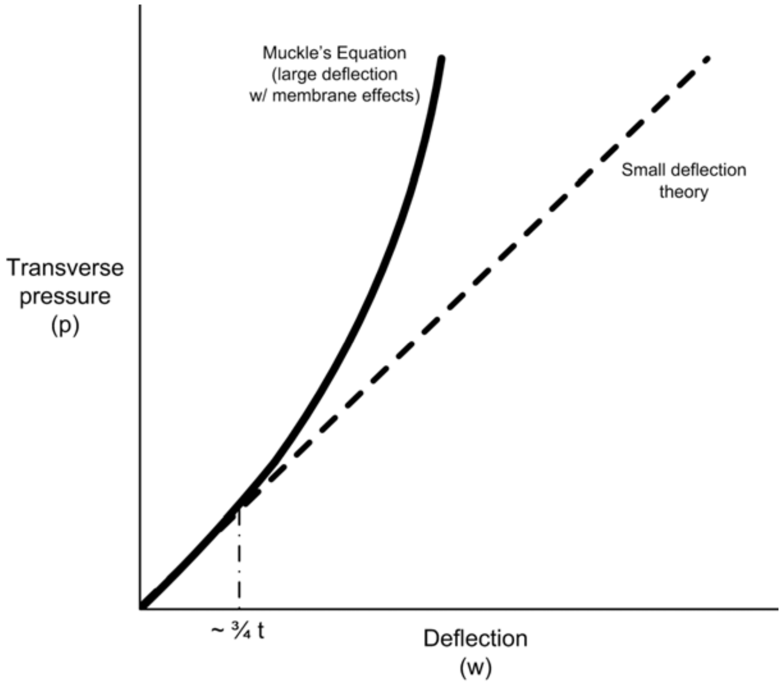
Generally, large deflection methods account for non-linear stiffness (i.e. the structure gets "stiffer" as the deflection increases). This occurs because of the edge pull-in effects, as well as an increase in stiffness with the larger curvatures. The result is an increase in the *apparent* strength of the structure (i.e. the structure "appears" stronger because strains are lower due to the higher effective stiffness).

The equations for plates subject to large deflections and membrane effects are more complex than for small deflection bending. The basic equations are discussed in detail in Hughes.

A simplified result was developed for large aspect ratio plates (a/b greater than about 2), without initial deflection, with edges pinned (fixed from translation but not rotation). This is referred to by many Naval Architects as "Muckle's Equation":

$$\frac{E\pi^5}{8b^4}\left(\frac{t}{4}w^3 + \frac{2t^3}{12\left(1-\upsilon^2\right)}w\right) = p$$

Note that there is a linear term plus a nonlinear term in w. A comparison of this large-deflection result (bending plus membrane effect) and the small deflection result (bending only) is given in the figure below. Note that for larger deflections, the plate is able to carry more load (pressure) for a given displacement w. This illustrates the apparent strength increase due to the membrane effect. Note also that for small deflections (less than about ¾ of the plating thickness) the two are very close (thus the reason small deflection theory works well for small deflections less than ¾ t).



Effect of initial deformation:

We have seen that the apparent stiffness (and apparent strength) of a plate in bending increases with deflection. The more curvature and deflection a plate has, the more membrane effect, the "stiffer" the plate is!

With this apparent strengthening of the plate, the question might arise as to whether it makes sense to "pre-deflect" a plate. The figure below (from Hughes) addresses this question by comparing the stress-pressure relation for large aspect ratio (a/b greater than about 2) simply-supported plates. The upper curves are for zero initial deflection (i.e. not "pre-deflected"). As the lateral load (pressure) increases, the bending stresses with membrane effects included are less than with small-deflection theory, because there is less deflection and therefore less bending! The lower curves are for "pre-deflected" plate (with initial deflection equal to ½t). The mean magnitude of stress is significantly reduced for given lateral load (pressure) due to the immediate impact of membrane effects limiting further deflection (and stress).

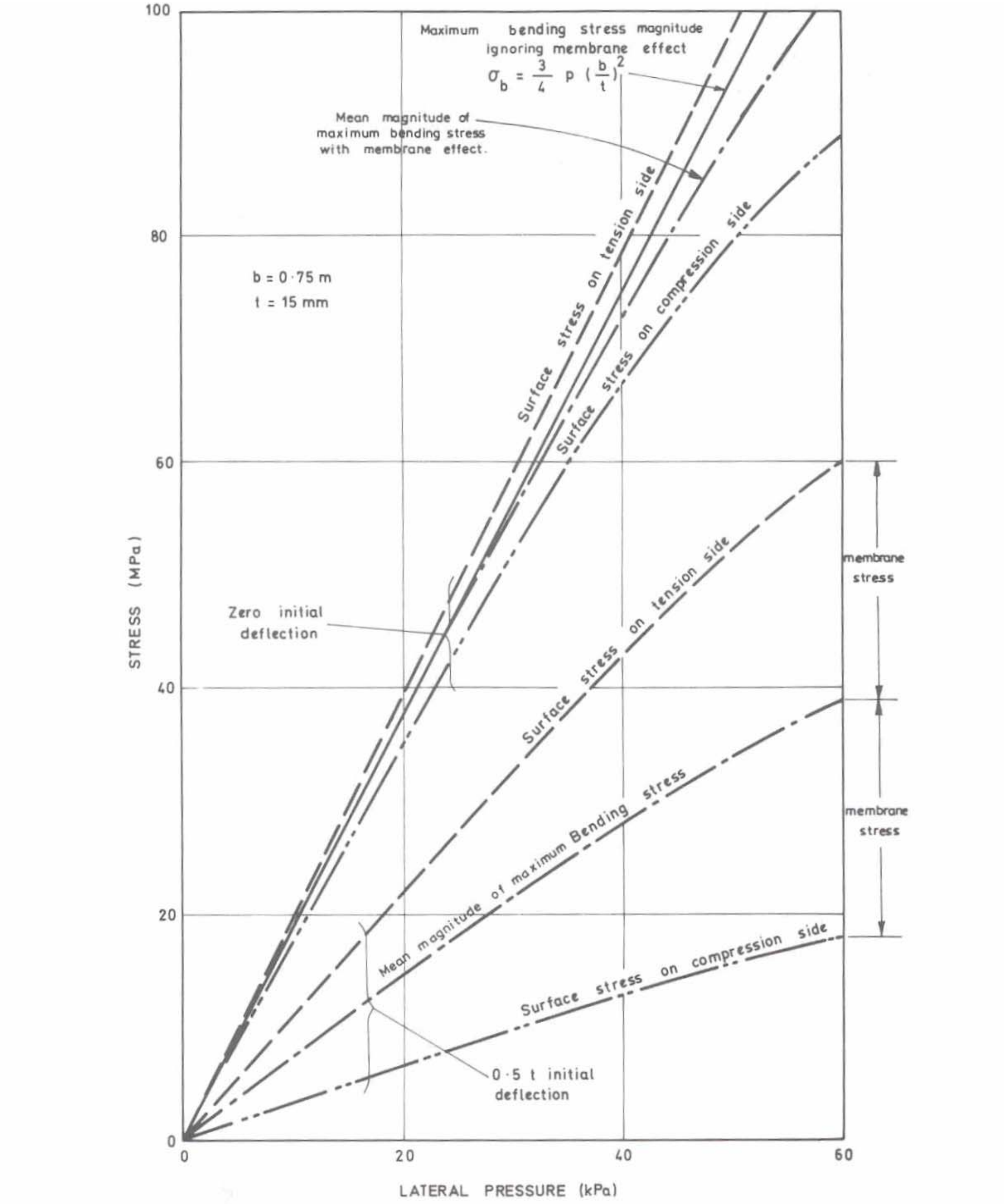
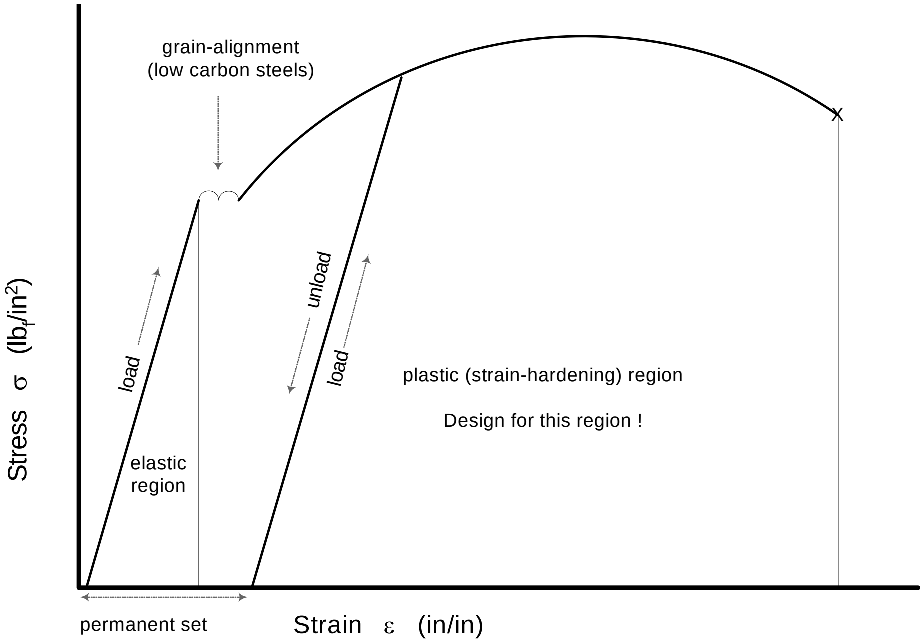


Figure 9.8 Membrane effect for long simply-supported plates.

In “real life” stiffened plating on ships has some amount of inward “dishing”, or pre-deflection, due to shrinkage of the weld metal (which pulls the plating inward) or “permanent set” (which will be discussed subsequently). This gives some ships the “hungry horse look”. Interestingly, while undesirable from an aesthetics and hydrodynamics stand point, the inward dishing or pre-deflection is actually beneficial from the stand point of strength vs. lateral loads (pressure) and in-plane tension. However, while beneficial vs. lateral loads and in-plane tension, the pre-deflection may lead to premature buckling for plates subject to compressive loads. For this reason, it is general practice in naval architecture to try to limit pre-deflection in shell plating to avoid buckling. That said, pre-deflection is used frequently in transverse bulkheads and platform deck plating.

Plates Loaded Beyond Their Elastic Limit ("Elastic-Plastic" Design)

While greater than 90% of ship structural design is done limiting material behavior within the linear elastic range, there are a number of situations for which it might be acceptable (and beneficial) to design for the "strain-hardening" region of the material (this is limited to ductile materials of course – steel and aluminum are examples). This type of design analysis is referred to as "elastic-plastic" design analysis. As a review from your Strength of Materials class, the figure below illustrates "strain-hardening" of a low-carbon steel when loaded beyond its elastic region.



So why would we design to this material region? The answer is that if we are confident of the maximum loads and fatigue life we can allow for some strain-hardening of the material (which will increase our yield strength). With a higher yield strength and a fixed factor of safety that we must meet, we can use thinner plate. That will save us weight and building cost!

Note however that elastic-plastic design is a "one-time" design. In naval architecture, we may use this to account for an initial permanent set in the plate (which we will discuss subsequently) or to account for a reduced fatigue life (damage accumulation).

Design for Permanent Set:

The most widely used application of elastic-plastic design for naval architects is to design for a "permanent set". This essentially allows a ship to have slightly "dished" plating (for shell plating this is commonly referred to as a "hungry horse" look, as the ship's frames stand out like ribs). For hydrodynamic drag and/or aesthetic reasons however, maximum allowable permanent set (w_p) is normally limited in a ship's specification, usually as some fraction of frame spacing or longitudinal stiffener spacing. Typically, specified limits are s/50 → s/250, where "s" is the shortest dimension of the stiffener or frame spacing (i.e. the plate dimension "b").

The theory of elastic-plastic plate bending is very complex, and not amenable to analytic analysis. In ship design, we often design for a maximum allowable permanent set (w_p) using either FEA or empirical equations and design curves. One suitable method using empirical equations and design curves is given in Hughes. The design curves are given on the following page (Hughes figures 9.17a-9.17e). To use these curves we define several *dimensionless* parameters:

$\beta \equiv \frac{b}{t}\sqrt{\frac{\sigma_Y}{E}}$ dimensionless plate slenderness parameter ("slender" plates $\beta > 2.4$ and "sturdy" plates $\beta < 2.4$)

$Q \equiv \frac{pE}{\sigma_Y^2}$ dimensionless plate load parameter (note p is the lateral pressure load)

$\frac{w_p}{\beta t}$ dimensionless plate permanents set parameter (note w_p is the lateral plate permanent set)

Are ship plates typically considered "slender" or "sturdy"?
Use typical values for A36 steel (E=29 msi and yield is 34 ksi and b/t ranges from 30-100) to get an idea!

To use the permanent set design curves:

- Specify proposed plate dimensions based on frame spacing and longitudinal stiffener spacing (a, b), along with material elastic modulus (E) and yield strength (σ_Y). Determine the required hydrostatic pressure (p). Determine the maximum allowable permanent set, based on minimum plate dimension (w_p).
- Calculate the plate aspect ratio (a/b), dimensionless plate load parameter (Q), and dimensionless plate permanent set parameter ($w_p/\beta t$).
- For the calculated plate permanent set parameter ($w_p/\beta t$), select the appropriate set of design curve figure (figures 9.17a-e). Use the closest value of $w_p/\beta t$.
- For the appropriate figure, enter the curves on the vertical axis with the calculated Q. For the curve with the closest plate aspect ratio (a/b), read down to find the required value of β.
- Using the required value of β, calculate the minimum required plating thickness (t). For design, we usually specify the plating thickness as the next larger standard plate thickness.

Example:

Consider ship bottom plating with frame spacing 25 inches, longitudinal spacing 120 inches, material A36 steel (σ_Y = 36 ksi, E = 30 msi), along with a design head 45 fsw and a design specification $w_p < s/50$

Solution: $w_p = \frac{25\,\text{in}}{50} = 0.5\,\text{in} \qquad \frac{a}{b} = \frac{120\text{in}}{25\text{in}} = 4.8$

$$Q \equiv \frac{pE}{\sigma_Y^2} = \frac{(\rho g H)E}{\sigma_Y^2} = \frac{\left(64\frac{\text{lb}}{\text{ft}^3}\right)(45\text{ft})\left(30{,}000{,}000\frac{\text{lb}}{\text{in}^2}\right)}{\left(36{,}000\frac{\text{lb}}{\text{in}^2}\right)^2}\left(\frac{1\,\text{ft}^2}{144\,\text{in}^2}\right) = 0.46$$

$$\frac{w_p}{\beta t} = \frac{w_p}{b\sqrt{\frac{\sigma_Y}{E}}} = \frac{0.5\text{in}}{25\text{in}\sqrt{\frac{36{,}000\text{psi}}{30{,}000{,}000\text{psi}}}} = 0.58 \quad \rightarrow \quad \text{Use Fig. 9.17c}$$

With Q = 0.46 and a/b = 4.8 ≈ 5.0, $\beta = 3.4 = \frac{b}{t}\sqrt{\frac{\sigma_Y}{E}} = \frac{25\text{in}}{t}\sqrt{\frac{36{,}000\text{psi}}{30{,}000{,}000\text{psi}}} \rightarrow t = 0.254\text{in}$

We should specify 5/16" (12.5#) plate, but we might be able to justify using ¼" (10.2#) plate

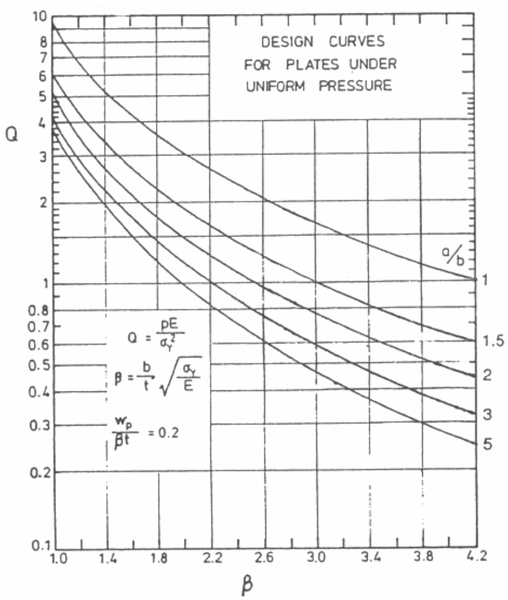


Figure 9.17a

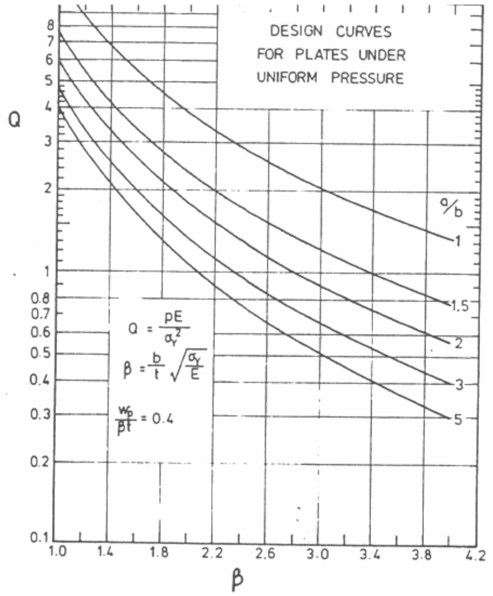


Figure 9.17b

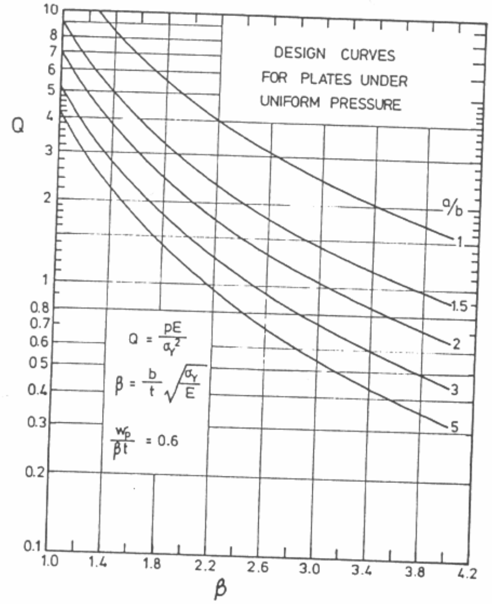


Figure 9.17c

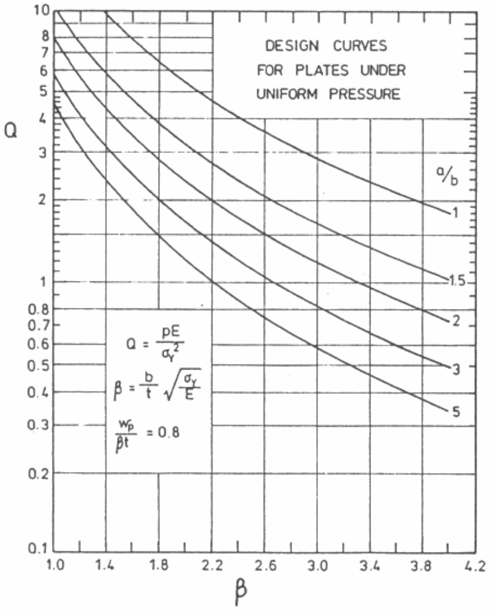


Figure 9.17d

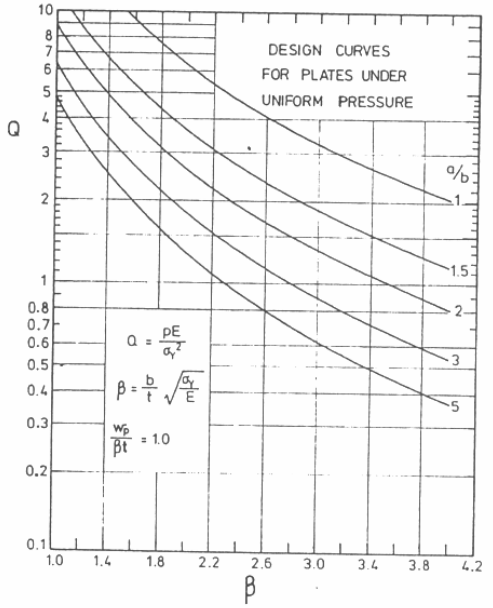


Figure 9.17e

Secondary Structure

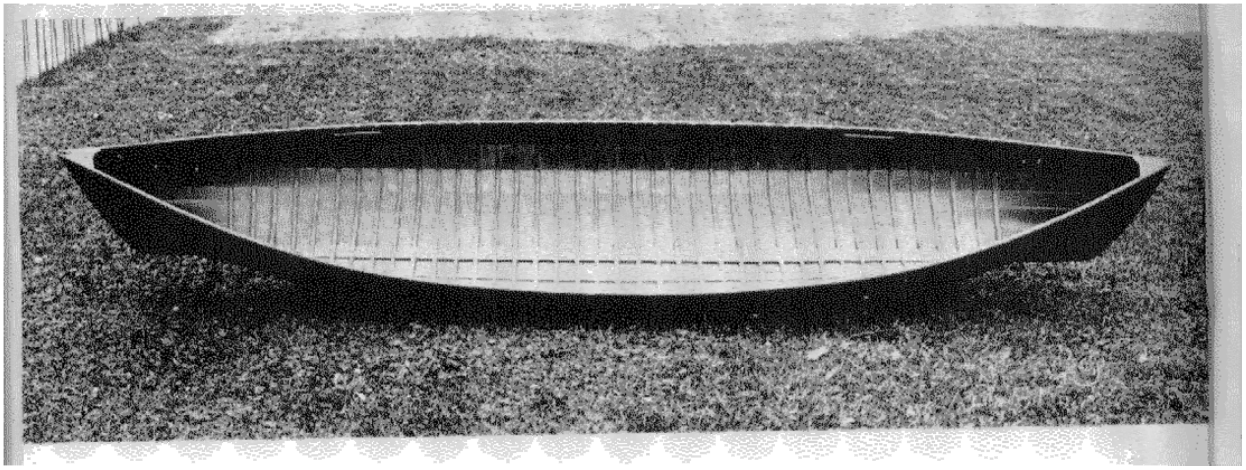
When we started this course we mentioned that ship structures can be categorized by how they are loaded. Primary structure was related to the ship globally bending due to cargo and waves. Tertiary structure was the plating bending due to predominantly hydrostatic pressure loads. The third type of structure is called secondary structure and includes stiffened panels.

Stiffened Panels in Bending:

A stiffened panel includes the plates and the stiffeners connected to the plates. As we saw early on, the stiffeners could be longitudinals, stringers, frames, girders, floors, etc. A stiffened panel is usually considered bounded by a very stiff structure such as a deep web, bulkhead, or vertical plating such as the topsides or superstructure. Depending on how stiff the CVK is, it might also be considered a stiffened panel boundary.

One of the lessons we learned when we studied plates was that plates bend relatively easily, and to avoid large deflections we need to either make the plate fairly thick or have small plate dimensions. The basic trade-off is that the design approach of thin plate with many stiffeners is lighter but not as puncture tolerant as a thick plate with fewer stiffeners. Depending on labor and the availability of automated welding machines, either approach might be less expensive to manufacture.

One extreme example of a light weight ship structure is Sairy Gamp, a 9.5 foot, 10 pound canoe built in 1883 for the noted outdoorsman George Washington Sears for long-distance travel. The canoe is owned by the Smithsonian Institution. In the picture below, notice the extremely close frames. The challenge in building a boat like this (she is a Rushton canoe) is that the frames were steamed to bend, yet many would break. The limit is how small the builder could make the frames. What makes this boat interesting from a ship structures point of view is that similarly sized boats on the market today range from 16 to 50 pounds![1]



We can analyze stiffened panels using a number of different approaches. The most common are: FEA, grillage theory, orthotropic plate theory, and beam-on-elastic foundation method. Section 3.8 of PNA goes into some detail on each method. Today, FEA is commonly used, and looking at the last three, the most common is the beam-on-elastic foundation method, which is often used by civil engineers.

Orthotropic plate theory blends the stiffeners with the plate to get effective E and I values for an equivalent plate. E does not have to be equal in the longitudinal and transverse directions. Commodore Schade developed a number of charts to assist with this method and it can be applied relatively quickly. For this reason it is still used by naval architects. It gives a good idea of the predicted deflections and plating stress, but does not work as well for the stiffeners. It is however a method with wide applications in composite materials structures.

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[1] Eric Marx, ENA Class of 2007, decided to test the limits of construction, and in an EN495/496 project he designed, built and successfully demonstrated a 10 foot canoe using carbon fiber, epoxy and foam core. One of his conclusions was that the final displacement (weight) of 9 pounds was probably 30% higher than it could have been!

The grillage method breaks down the stiffeners and plating into individual beams with point loads at the intersections. As each beam can be represented by a simple equation similar to those used in the stiffness matrix method, a series of equations for deflection at each intersection can be created that must be in equilibrium. These simultaneous equations can then be solved. If this sounds like a good application of computers, it is!

We will use the FEA method in this class, along with a first-principles approach. In practice, either FEA or the Classification Society Rules are used. The typical approach used by ABS for example, starts with finding the plate thickness using a modified first-principles approach (an empirically-modified small deflection plate bending method), then calculating the section modulus of the stiffener (with an effective breadth of attached plating) needed to support the bending moment.

Weekly Assignment #9: Stiffened Panel Design for Bending

Task:

In teams of two, design a 3' x 6' stiffened panel of 10# (1/4") A36 steel plate subjected to 20 feet of hydrostatic pressure. Use a minimum FOS is 2.0 vs. material yield. You may use up to *four* stiffeners running in any direction, but they are limited to 4" in height and 3" in width. They may be flat bars, T's, or angles (L's). The minimum thickness for any web, flange or plate is 10# (1/4"). Try to make the panel as light weight as possible while still meeting the FOS requirement.

Deliverable:

Submit a design report (one report per team) clearly explaining your panel's design. At a *minimum*:

Include in your report complete and detailed rationale for your selection of plating thickness, stiffeners type, size, geometry, and arrangement.

Also include and discuss FEA displacement and stress plots for both simply-supported and fixed (clamped) edge BCs. Provide color versions of the two deformed stress plots. Identify your maximum stress, and your final FOS. Be very careful that you model the boundary conditions correctly! Look at the deformed plot to determine if the movement of the stiffeners makes sense, as they are continuous beyond the boundaries!

Also include a calculation of your panel's weight, and compare it to the previous lab result (unstiffened plate). *Special note: the team with the lowest weight panel will receive 5% extra credit!*

Also include an *engineering sketch* (not necessarily to scale) of your final design. Show as many views as needed to clearly convey all of the necessary information to clearly define the panel's geometry. Be neat and complete!

Suggested Approach for Using COSMOS/M FEA:

1. The same modeling techniques apply as they did in the last lab. The general modeling steps are to: create the geometry, define the material properties, select the element type (group), define the real constants, create the elements and nodes, input the boundary conditions and loads, specify the analysis type and run it!

2. You will use the same element group (Shell4) and material props as last time. What is different in this case is that you may wish to change the thickness of the various parts independently. For instance, you may find during one iteration that the web is overstressed, but the plate and flange are not. To change just the web thickness you will need to give those elements a unique real constant set.

3. To create a stiffener you will first create a surface and then mesh it as you did before. You must make sure that the nodes and element edges of the panel and the nodes and element edges of the stiffener join up, or the two parts won't work together in carrying the load! Start by creating two points on both edges of the panel where you want the stiffener to start and end. Use Geometry-Points-Generation-Point at Node to create a point located at a node. Create the other two points for the top of the stiffener by either using the Geometry-Points-Define command or the Geometry-Points-Generation-Generate command. If you use the latter method, on the first pop-up screen you will enter the points you are duplicating (pic using the mouse) and on the second screen you will enter the offset value in the z-direction.

4. Create the surface using the four points. I suggest the first two points you select are those on the plate.

5. Before meshing the surface, create a new real constant set. Use the PropSets-Real Constant command. If your plate was Real Constant (RC) 1, make this one RC 2. Define your first guess thickness.

6. Now mesh the surface. Make sure you only select the new surface to mesh, and that you chose the correct number of elements on the curves so that the mesh on the new surface corresponds to the mesh on the plate! If you have the wrong mesh, you can undo it by using Meshing-Parametric Mesh-Delete Elements on Surface.

7. When you are happy with your mesh, you will need to join the two sets of elements and nodes. You do this with the Meshing-Nodes-Merge command. Use all the defaults. You should see that the number of nodes that merge are equal to the number of nodes at the junction between the two parts. If you set the merge tolerance high enough you can get your entire model to merge into one node! It is not a bad idea to save a .gfm file first!

8. Now set the BC's on the stiffener to match the model. Think about this one for a minute; how will the stiffener react if it was continuous beyond the model's boundaries? This is the issue of symmetry.

9. Now run the model and check your results! You may find the stress is now much lower in the plate, but is too high in the stiffener! This is normal! Keep going! A design hint: locate the stiffeners across the places of largest deflection, and keep the stiffeners as short as possible.

10. To iterate by changing the RC value, just overwrite the previous value. For instance, if you now want to change the web thickness, use PropSets-Real Constants, and select the RC set for the web (in our example it was RC Set 2 with Element Group 1). The starting point will be the first RC value, and you will only need to enter the one value.

Weekly Assignment #10: Wooden Stiffened Panel Design-Build-Test Project

Objective:

In teams of three, design a stiffened bottom panel (between bulkheads) for a wooden vessel. Each section will build and test their "best" design, which will be determined by dividing the predicted maximum pressure by the panel's weight. The panel will be subjected to a uniform in-plane compressive load (1000 pounds). A constantly increasing transverse pressure (representing hydrostatic load) will be applied until failure.

Ship Characteristics:

The ship you are designing the bottom panel for is a 25' sportfishing craft. It has a draft of 4 feet, 3 feet of freeboard, and 9 foot beam. The ABS-required bottom pressure works out to 8 psi, which includes a slamming DAF factor of 1.5.

Materials:

You are limited in material selection! The plating will be 60" x 36" and can be either 3/8", 1/2" or 3/4" thick Douglas Fir exterior grade plywood. To make the stiffeners you can use up to two 8' "2x4s" cut to any length. Note that a "2x4" is actually 1.5" x 3.5". You can orient the stiffeners in any pattern you wish, but the maximum height of the panel must be less than 4.5". We will not be able to scarf the 2x4s. The stiffeners will be connected to the plating with epoxy. Use the following material properties for the analysis:
Doug Fir: E_x = 1.75 msi, E_y = 250 ksi, υ_{xy} (Poisson's) = 0.45, compressive strength = 6430 psi, tensilestrength = 4100 psi (see FEA notes below), and weight density = 30 lb/ft^3.

Loading and Boundary Conditions:

In addition to the 8+ psi normal pressure load, the panel will be loaded in compression parallel to the long direction. You will need to determine what the failure mode and load will be for the load. Because of the testing machine limitations, the boundary conditions are a little different than what we have done before, and do not actually represent the boat. Make one of the short edges pinned. The other short edge will be free to compress (roller), but will be constrained from lateral or vertical motion. The two long edges will be free (no constraints at all).

Deliverables:

By 0800 on Monday of the Panel Construction Lab week, each team must submit a Design Report, including an engineering sketch, and all calculations (including FEA results). Your engineering sketch must include all the necessary information for a carpenter to build the panel. The report must also include the following:

1. FEA plots for stress in the x (60") and y (36") directions (note VonMises does not apply to plywood). For stress plots, apply a uniform lateral pressure of 8+ psi and a uniform in-plane compressive load of 1,000 pounds. Annotate the BCs used on the edges of the plate (plywood) and at the ends of the stiffeners. Think about these carefully.
2. FEA plot of the lowest buckling mode (displacement plot). Show the BCs used on the edges of the plate (plywood) and at the ends of the stiffeners. Include an annotation (calculation if required) of critical buckling load (in pounds). The *buckling factor* is shown on the displacement plot after running the buckling module.
3. A summary table showing the failure pressure for yield and buckling, the predicted weight (in pounds), and the ratio of the lowest failure pressure divided by the weight.

During the first five minutes of the Panel Construction Lab (Tuesday), we will compare each team's results. We will then begin to build the "best" design (one for each section). The team in each section whose design we build will earn an extra 10 quiz points! In addition, the team whose design has the highest strength to weight ratio after the testing will get an extra five lab points. Note that your design will be disqualified if you do not submit all the required documentation, and/or if your design cannot be built from your engineering sketch.

FEA Notes:

For the most part this FEA is very similar to the bottom panels from earlier projects. The only significant difference is that wood is orthotropic. To capture these effects in the model we need to make three changes: first, we need to specify the element group for a laminated material, rather than an isotropic material; second, we need to specify the material properties rather than just pick a material; third, we need to specify the laminate.

SURFACE Construction: Make sure the first point is (0,0,0) and the second is (60, 0, 0). This will insure that the element coordinate system matches the global coordinate system. We will discuss this later.

EGROUP: Select the SHELL4L element rather than the SHELL4 element when doing the plywood. Accept all the defaults, except that you need to specify how many layers are in your laminate. We are using 5-ply plywood. The lumber is a SHELL4 element.

MPROP: Use the PROPSETS-Material Property command to enter the properties. You will only have one material property set for everything, but the properties themselves will need to be defined. You will need to enter EX, EY, NUXY, & DENS (which is mass density rather than weight density). Make sure you are consistent with your in-lb-sec units!

REAL CONSTANTS: You will need two real constant sets. The first will be for the plywood and the second for the lumber. The plywood real constant will be tied to EGROUP 1 and the lumber to EGROUP 2. For the plywood, accept the first default on thickness (1E-6) and the second default on temperature. Then you will enter the values describing each ply. Each ply is the same thickness, so take your plywood thickness and divide by 5. In all cases the material number of the set is 1. The ply orientation will be 0 degrees for the first, third and fifth plies. It is 90 for the second and fourth. An annoying aspect of this program is that you can only enter 10 real constants at a time. To enter the others, run the command again, making sure you are still on Real Constant and Element Groups 1, then start the real constant list at 11. For the lumber will only need the thickness of the 2x4.

Buckling Failure Modes

As you learned in EN222, buckling is the collapse of a structure due to instability. A rope for instance has virtually no buckling resistance, while a steel rod has some. Buckling can be either elastic, where it returns to its original shape after it is buckled, or inelastic, where it does not. Both elastic and inelastic buckling are usually considered "stiffness driven failures", but elastic buckling is not usually catastrophic to a ship. Normally we design ship structures for no buckling, elastic or inelastic, but as designers we need to decide appropriate factors of safety for different components. For example, would it be worse to have elastic buckling of plating or stiffeners? How about inelastic buckling of either?

Buckling of Columns – "Ideal" Columns

We first consider a simple "ideal" column, the same as you did in EN222.

First, we define the maximum load that the column can sustain without collapse as the "ultimate load" (P_{ult}), and the maximum stress, as the average stress at the buckling load, as the "ultimate stress" or "ultimate strength" of the column (σ_{ult}).

As you should recall, the "critical buckling load" (P_{cr}) is the maximum compressive load the column can sustain without buckling. For an "ideal" column, you should recall that the critical buckling load (called the "Euler critical buckling load") is

$$P_{cr} = \frac{\pi^2 EI}{L_e^{\ 2}} = P_E$$

where L_e is the effective length of the column, which depends on the type of end supports (end conditions). This can also be written as $L_e = kL$, where k is an "effective length factor". Recall that k = 1 if both ends are pinned, k = 0.5 if both ends are fixed, and k = 2 if one end is fixed and the other free. Thus

$$L_e = kL \qquad k = 1\,(\text{pinned - pinned}),\ k = 0.5\,(\text{fixed - fixed}), k = 2\,(\text{fixed - free})$$

Special note: For welded ship stanchions and stiffeners treated as columns, we usually use an intermediate value

$$k = \sqrt{2} \approx 0.707 \ \ (\text{welded ship stanchions or stiffeners})$$

The "critical buckling stress" (σ_{cr}) is the average stress at the critical buckling load. You should recall that the critical buckling stress (called the "Euler critical buckling stress") is

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 EI}{AL_e^{\ 2}} = \sigma_E$$

For "practical" critical buckling stress, we often use radius of gyration defined by $I = \rho^2 A$ or $\rho = \sqrt{\frac{I}{A}}$. Then the critical buckling stress is written:

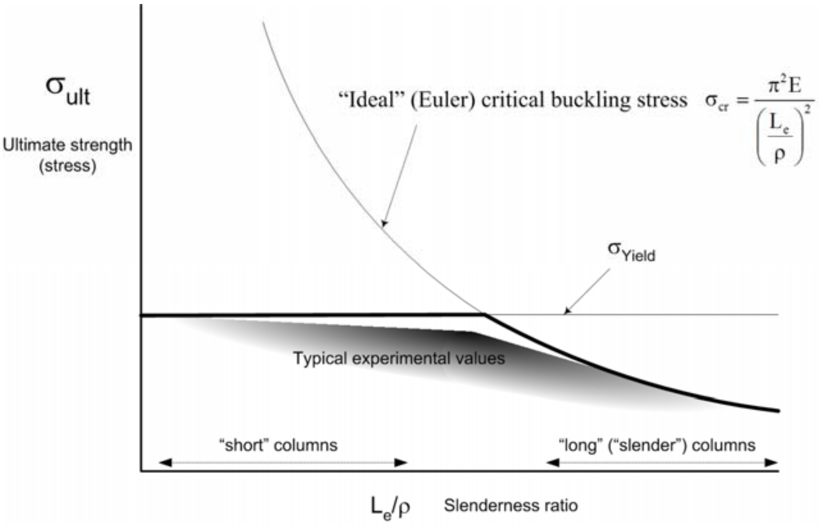
$$\sigma_{cr} = \frac{\pi^2 E}{\left(\frac{L_e}{\rho}\right)^2}$$

Note: Tubes (pipes) are commonly used as structural columns. Their radius of gyration is $½(r_o^{\ 2}+r_i^{\ 2})^{0.5}$

where $\left(\frac{L_e}{\rho}\right)$ is the "slenderness ratio" of the column.

It is useful to plot the "ultimate strength curve" for a column. Here we plot the ultimate strength (stress) vs. the slenderness ratio (see the figure on the next page). Note that we have plotted the critical buckling stress (σ_{cr}) as well as a horizontal line for the yield strength. Not that the ultimate strength of the column (for a given slenderness ratio) will be the *lesser* of σ_{cr} or σ_Y. Thus, we say that for "short" columns (L_e/ρ small, $\sigma_Y < \sigma_{cr}$), the column will fail due to compressive yield (i.e. $\sigma_{ult} \approx \sigma_Y$). Conversely, for "long" or "slender" columns (L_e/ρ large, $\sigma_Y > \sigma_{cr}$), the column will fail due to elastic buckling (i.e. $\sigma_{ult} \approx \sigma_{cr}$).

In an ideal world, you might think that we should design our columns so that $\sigma_Y = \sigma_{cr.}$ This would lead to an "optimal" design (minimum weight). However, since elastic buckling of a column is catastrophic compared to compressive yielding, we generally desire to ensure that buckling occurs after yielding. Thus, we use a factor of safety for buckling of 3.0 (vs. 2.0 for yielding).



As can be seen in the figure, very short columns (also called "compression blocks") fail by yielding, which very long columns fail by elastic buckling. In the intermediate range (between "short columns" and "slender columns") lies a range of values of slenderness ratio for which *real columns* fail at a stress below both yield and critical buckling stress for the ideal column. Columns in this range of slenderness ratio are called "intermediate columns", and tend to fail due to inelastic stability – meaning that there is a partial yielding of the material, usually due to small irregularities or "eccentricities" in the geometry of the column. This will be discussed in greater detail subsequently.

Example:

Consider a simple column deck support (stanchion) made using a wide flange I-beam W6 (see the next page) made of HTS (σ_Y = 47 ksi), with a compressive load P = 100,000 lb, and a length L = 12 ft. Assume that the load P is applied as an "ideal" compressive load (no eccentricities), and that there are no "shear lag" effects (i.e. kL/b large so that $b_e/b \approx 1$).

Solution:

First not from the Wide Flange Properties table (see next page), W6 has ρ = 1.52 inch (lesser of $\rho_{1\text{-}1}$ and $\rho_{2\text{-}2}$) Thus:

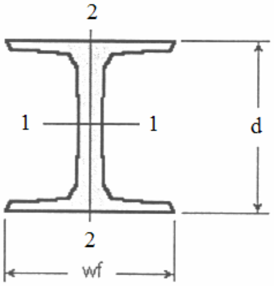
$$\sigma_{cr} = \frac{\pi^2\left(30{,}000{,}000\,\frac{\text{lb}}{\text{in}^2}\right)}{\left(\frac{(0.707)(144\text{in})}{1.52\text{in}}\right)^2} = 66.0\text{ ksi} \qquad \text{(note here } \sigma_Y < \sigma_{cr}\text{)}$$

$$\text{Compressive stress } \sigma = \frac{P}{A} = \frac{100{,}000\text{lb}}{7.34\text{in}^2} = 13.6\text{ ksi}$$

$$\text{FOS}_{\text{buckling}} = \frac{\sigma_{cr}}{\sigma} = \frac{66.0\text{ ksi}}{13.6\text{ ksi}} = 4.8 \qquad \text{(note > 3.0)}$$

$$\text{FOS}_{\text{Yield}} = \frac{\sigma_Y}{\sigma} = \frac{47.0\text{ ksi}}{13.6\text{ ksi}} = 3.4 \qquad \text{(note > 2.0)}$$

"W" Wide Flange Properties



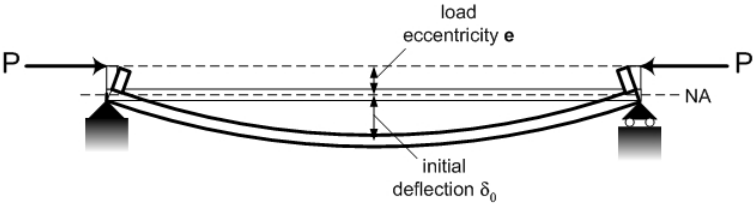
d = Section depth, in
wf = Flange width, in
ws = weight per foot of steel sections, lb per foot
A = area, in^2
I = moment of inertia, in^4
ρ = radius of gyration, in
Z = section modulus, in^3

S	d	wf	ws	A	I1-1	ρ1-1	Z1-1	I2-2	ρ2-2	Z2-2
W36	36-3/4	16-5/8	300	88.3	20300	15.2	1110	1300	3.83	156
W36	36-3/4	12-1/8	210	61.8	13200	14.6	719	411	2.58	67.5
W33	34-1/8	15-7/8	241	70.9	14200	14.1	829	932	3.63	118
W30	31	15-1/8	211	62.0	10300	12.9	663	757	3.49	100
W30	30-1/4	10-1/2	132	38.9	5770	12.2	380	196	2.25	37.2
W27	27-3/4	14-1/8	178	52.3	6990	11.6	502	555	3.26	78.8
W27	27-1/4	10-1/8	114	33.5	4090	11.0	299	159	2.18	31.5
W24	25	13	162	47.7	5170	10.4	414	443	3.05	68.4
W24	24-1/4	9-1/8	94	27.7	2700	9.87	222	109	1.98	24.0
W24	23-3/4	7	62	18.2	1550	9.23	131	34.5	1.38	9.80
W21	22	12-1/2	147	43.2	3630	9.17	329	376	2.95	60.1
W21	21-5/8	8-3/8	93	27.3	2070	8.70	192	92.9	1.84	22.1
W21	21	6-1/2	57	16.7	1170	8.36	111	30.6	1.35	9.35
W18	19	11-1/4	119	35.1	2190	7.90	231	253	2.69	44.9
W18	18-1/2	7-5/8	71	20.8	1170	7.50	127	60.3	1.70	15.8
W18	18	6	46	13.5	712	7.25	78.8	22.5	1.29	7.43
W16	17	10-3/8	100	29.4	1490	7.10	175	186	2.51	35.7
W16	16-3/8	7-1/8	57	16.8	758	6.72	92.2	43.1	1.60	12.1
W16	15-7/8	5-1/2	31	9.12	375	6.41	47.2	12.4	1.17	4.49
W14	22-3/8	17-7/8	730	215	14300	8.17	1280	4720	4.69	527
W14	14-5/8	14-3/4	132	38.8	1530	6.28	209	548	3.76	74.5
W14	14-1/2	10-1/8	82	24.1	882	6.05	123	148	2.48	29.3
W14	13-7/8	8	53	15.6	541	5.89	77.8	57.7	1.92	14.3
W14	14-1/8	6-3/4	38	11.2	385	5.87	54.6	26.7	1.55	7.88
W14	13-7/8	5	26	7.69	245	5.65	35.3	8.91	1.08	3.54
W12	14-3/8	12-5/8	190	55.8	1890	5.82	263	589	3.25	93.0
W12	12-1/4	10	58	17.0	475	5.28	78	107	2.51	21.4
W12	12-1/4	8-1/8	50	14.7	394	5.18	64.7	56.3	1.96	13.9
W12	12-1/2	6-1/2	35	10.3	285	5.25	45.6	24.5	1.54	7.47
W12	12-1/4	4	22	6.48	156	4.91	25.4	4.66	.847	2.31
W10	11-3/8	10-3/8	112	32.9	716	4.66	126	236	2.68	45.3
W10	10-1/8	8	45	13.3	248	4.32	49.1	53.4	2.01	13.3
W10	10-1/2	5-3/4	30	8.84	170	4.38	32.4	16.7	1.37	5.75
W10	10-1/4	4	19	5.62	96.3	4.14	18.8	4.29	.874	2.14
W8	9	8-1/4	67	19.7	272	3.72	60.4	88.6	2.12	21.4
W8	8	6-1/2	28	8.25	98.0	3.45	24.3	21.7	1.62	6.63
W8	8-1/4	5-1/4	21	6.16	75.3	3.49	18.2	9.77	1.26	3.71
W8	8-1/8	4	15	4.44	48.0	3.29	11.8	3.41	.876	1.70
W6	6-3/8	6-1/8	25	7.34	53.4	2.70	16.7	17.1	1.52	5.61

Buckling of Columns – "Eccentricity"

Real columns have irregularities in their geometry called "eccentricities", which also affect the buckling load and stress. Eccentricities can be caused by the following:

(a) Initial deflection of the column (due to residual stresses due to welding, imperfect manufacture, or permanent set from previous load application)
(b) Load eccentricity (line of action of compressive load is off-set from the centroidal axis of the column
(c) Lateral loads (transverse bending and deformation – this creates what we call a "beam-column")



The effect of the eccentricity is to produce a combined compression and bending on the column. The effect of this is essentially to "magnify" the effect of the compressive load. The maximum compressive stress, which occurs locally on the "inside" compression flange of the column can be written:

$$\sigma_{\max} = \frac{P}{A} + \frac{My}{I} = \frac{P}{A} + \frac{P\,\Delta\,\phi}{SM_{comp}}$$

where $\Delta = e + \delta_0$ (load eccentricity + initial deflection) (i.e. the "total" eccentricity)

SM_{comp} = section modulus (to the compression side)

$$\phi \approx \underbrace{\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_E}}\right)}_{\text{Initial deflection eccentricity}} \approx \underbrace{\frac{P_E}{P_E - P}}_{\text{Load eccentricity}} = \text{the "Eccentricity Magnification Factor"}$$

It is assumed that the column with eccentricity will collapse when the maximum compressive stress reaches the yield stress (i.e. when $\sigma_{max} = \sigma_Y$). Note that this is considered inelastic (plastic) buckling.

Note also that the effect of eccentricity (initial deflection or load eccentricity) is essentially to "pre-buckle" the column. However, you see that the Euler buckling load comes in as the "eccentricity magnification factor", the effect of which is to magnify the bending moment created by the eccentricity. This is an important point, since the column will actually deflect more (than the initial deflection) due to the magnification effect.

Example:

Consider the same column as the last example, but assume 3" eccentricity on SM_{min} side (P = 200,000 lb)

Solution:

$$P_E = \sigma_E A = (66{,}000\text{psi})(7.34\text{in}^2) = 484{,}440\,\text{lb}$$

$$\phi \approx \frac{P_E}{P_E - P} = \left(\frac{484{,}440\text{lb}}{484{,}440\text{lb} - 100{,}000\text{lb}}\right) = 1.26$$

$$\sigma_{\max} = \frac{100{,}000\text{lb}}{7.34\text{in}^2} + \frac{(100{,}000\text{lb})(3\text{in})(1.26)}{5.61\,\text{in}^3} = 13.6\text{ ksi} + 67.4\text{ ksi} = 81.0\text{ ksi}$$

$$FOS_{Yield} = \frac{\sigma_Y}{\sigma_{\max}} = \frac{47.0\text{ ksi}}{81.0\text{ ksi}} = 0.58 \quad (\text{note} < 2.0) \qquad FOS_{buckling} = \frac{\sigma_{cr}}{\sigma} = \frac{66.0\text{ ksi}}{13.6\text{ ksi}} = 4.8 \quad (\text{as before})$$

Buckling of Columns – "Beam-Columns"

A beam-column is a column that is loaded by an axial compressive load P, plus a lateral load w. The lateral load induces a bending moment as a beam, which is *additive* to the bending moment induced by the axial compressive load with its eccentricities.

The maximum bending moment is the sum of the bending moment due to the lateral load (M_w) plus the bending moment due to the axial compressive load with its eccentricity (eccentricity of load and/or initial deflection). The total eccentricity now includes the axial compressive load eccentricity (e), an initial eccentricity (δ_0), plus an additional eccentricity due to the bending induced by the lateral load (δ_w):

$$M_{max} = M_w + \phi \cdot P \cdot \Delta = M_w + \phi \cdot P \cdot \left(e + \delta_0 + \delta_w\right)$$

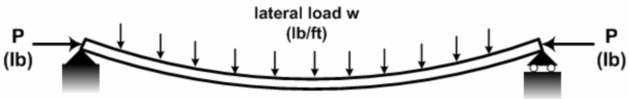
To find the maximum stress in the beam-column, we apply the column bending equation, including this additional bending moment. However, in this case, to find the maximum stress we must calculate the stress on both the top and bottom flanges of the beam-column:

$$\sigma_{top} = -\frac{P}{A} \pm \frac{M_{max}}{SM_{top}} \qquad \sigma_{bottom} = -\frac{P}{A} \pm \frac{M_{max}}{SM_{bottom}}$$

The ± indicates that the bending portion of the stress is dependent upon the side of the beam-column on which the lateral load w acts.

The simplest beam-column is one which has pinned ends on both ends, and is subject to a uniform lateral load w (lb/ft). In this case

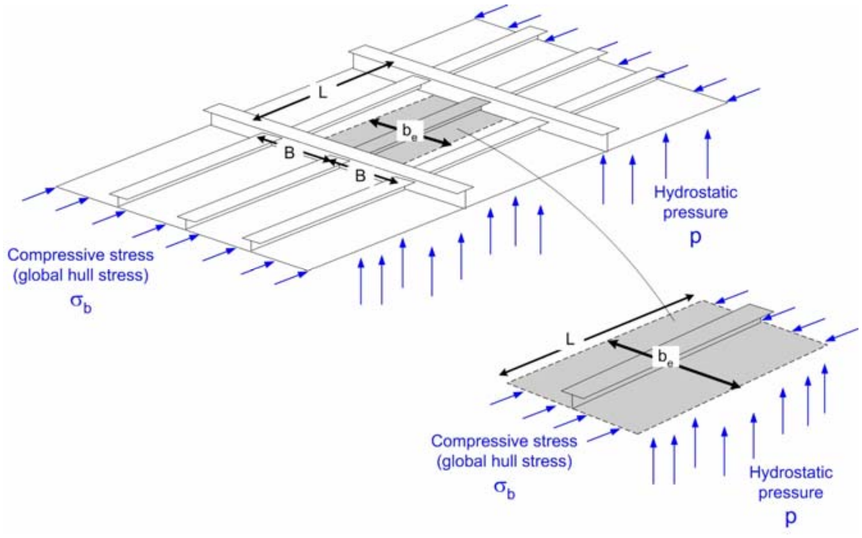
$$M_w = \frac{wl^2}{8} \quad \text{and} \quad \delta_w = \frac{5\,wl^4}{384\,EI}$$



Note that a spreadsheet is a nice tool for evaluating beam-columns. This will be illustrated on the example on the following page.

It is assumed that, like the column with eccentricity (initial or load), the beam-column will collapse when the maximum compressive stress reaches the yield stress (i.e. when $\sigma_{max} = \sigma_Y$). Note that this is considered inelastic (plastic) buckling.

One very important application of beam-columns in ship structural design is a ship stiffener (such as a bottom longitudinal or stringer), including an "effective width" of attached plating. When subject to a hydrostatic pressure load (lateral load) in addition to a compressive hull girder compressive load (i.e. ship in a hogging condition), the longitudinal along with an "effective width" of attached plating acts as a beam-column (see figure below). Note the similarity of this to the simple bending approach taken for the stiffener design previously.



The "effective width" of attached plating (b_e) is either the stiffener spacing B or $1.9\ t_p\sqrt{\frac{E}{\sigma_Y}}$ *(whichever is less).*

The latter is approximately equal to $60 \cdot t_p$ for mild steel ($E \approx 30$ msi, $\sigma_y \approx 30$ ksi), so this value used to be specified by ABS in their rules for steel ships. Note also the term "effective width" is used (not "effective breadth" which was due to shear lag). This will be explained more fully subsequently when we discuss buckling of plates.

For the beam-column made up of the stiffener and attached plating, we include only the "effective width" of attached plating (b_e) in the calculation of A, I, and SM (both flange and plate sides). This is illustrated in the following example.

Example:

Consider a beam-column made up of ship longitudinal and attached plating, subject to a hydrostatic head of 20 feet of sea water (fsw) and simultaneously to a compressive stress (from global hull girder bending) of 15,000 psi. Assume the material is A36 steel, frame spacing is 72 inches, longitudinal spacing is 24 inches, stiffener has a depth 4 inches and flange width 4 inches, and all plate thickness is ½ inch (plating, web, flange). Determine the factors of safety vs. buckling and yield of this beam-column.

Solution:

First, we must find the area properties of the stiffener with its attached effective width of plating:

$$1.9\ t_p\sqrt{\frac{E}{\sigma_Y}} = 1.9\left(0.5\text{in}\right)\sqrt{\frac{29{,}000{,}000\text{psi}}{36{,}300\text{psi}}} = 26.85\ \text{in}$$

So b_e = 24 inches (the stiffener spacing is 24 inches, which is less than 26.85 inches)

Using our section modulus spreadsheet, we find the following effective area properties:

$A = 15.75\ \text{in}^2$, $I = 32.68\ \text{in}^4$, $SM_{\text{flange}} = 9.28\ \text{in}^3$, $SM_{\text{plate}} = 33.34\ \text{in}^3$

Calculate the elastic buckling stress (Euler buckling stress):

$$\sigma_{cr} = \sigma_E = \frac{\pi^2 EI}{AL_e^{\ 2}} = \frac{\pi^2\left(29{,}000{,}000\text{psi}\right)\left(32.68\text{in}^4\right)}{\left(15.75\text{in}^2\right)\left(0.707 \cdot 72\ \text{in}\right)^2} = 229{,}199\ \text{psi}$$

Calculate the compressive load P, the Euler buckling load P_E, and the eccentricity magnification factor ϕ:

$$P = \sigma_b A = \left(15{,}000\text{psi}\right)\left(15.75\text{in}^2\right) = 236{,}250\,\text{lb} \qquad P_E = \sigma_E A = \left(229{,}199\text{psi}\right)\left(15.75\text{in}^2\right) = 3{,}609{,}884\,\text{lb}$$

$$\phi \approx \frac{P_E}{P_E - P} = \left(\frac{3{,}609{,}884\ \text{lb}}{3{,}609{,}884\ \text{lb} - 236{,}250\ \text{lb}}\right) = 1.07$$

Calculate the maximum stress on the *compression* side of the beam-column:

$$w = \left(\rho g H\right)\left(B\right) = \left(64\frac{\text{lb}}{\text{ft}^3}\right)\left(20\text{ft}\right)\left(2\text{ft}\right)\left(\frac{1\text{ft}}{12\text{in}}\right) = 213.33\frac{\text{lb}_f}{\text{in}}$$

$$M_w = \frac{wL^2}{8} = \frac{\left(213.33\ \text{lb/in}\right)\left(72\text{in}\right)^2}{8} = 138{,}240\ \text{inlb}$$

$$\delta_w = \frac{5\ wl^4}{384\ EI} = \frac{5\left(213.33\text{lb}/\text{in}\right)\left(72\text{in}\right)^4}{384\left(29{,}000{,}000\text{lb}/\text{in}^2\right)\left(32.68\text{in}^4\right)} = 0.08\text{in}$$

$$M_{\max} = M_w + \phi \cdot P \cdot \Delta = M_w + \phi \cdot P \cdot \left(e + \delta_0 + \delta_w\right) = 138{,}240\text{inlb} + \left(1.07\right)\left(236{,}250\text{lb}\right)\left(0 + 0 + 0.08\text{in}\right) = 158{,}400\ \text{inlb}$$

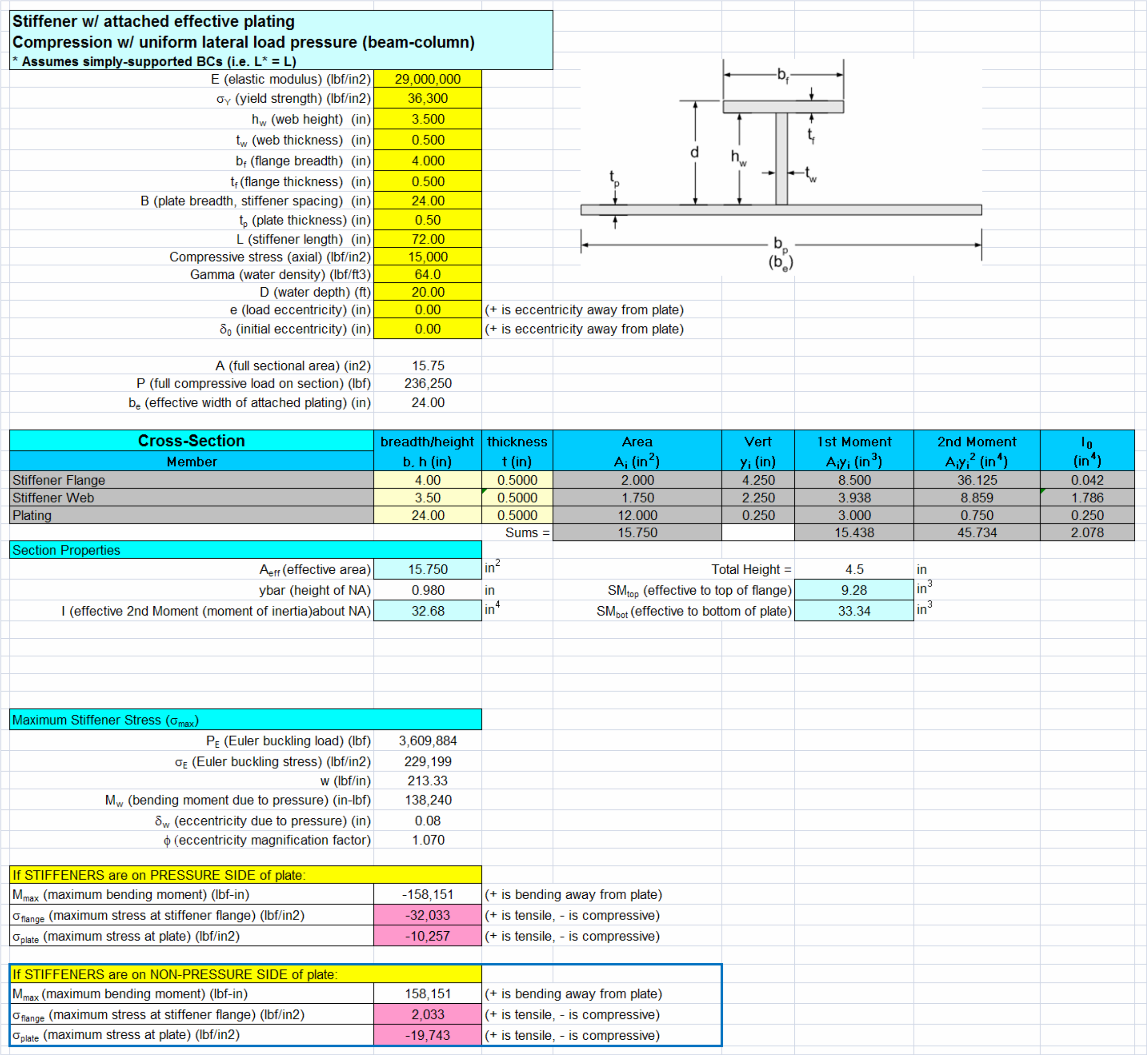
$$\sigma_{\text{plate}} = -\frac{P}{A} - \frac{M_{\max}}{SM_{\text{plate}}} = -\frac{236{,}250\ \text{lb}}{15.75\ \text{in}^2} - \frac{158{,}400\ \text{inlb}}{33.34\ \text{in}^3} = -15.0\ \text{ksi} - 4.8\ \text{ksi} = 19.8\ \text{ksi (tension)}$$

$$\sigma_{\text{flange}} = -\frac{P}{A} + \frac{M_{\max}}{SM_{\text{flange}}} = -\frac{236{,}250\ \text{lb}}{15.75\ \text{in}^2} + \frac{158{,}400\ \text{inlb}}{9.28\ \text{in}^3} = -15.0\ \text{ksi} + 17.1\ \text{ksi} = 2.1\ \text{ksi (tension)}$$

Calculate the factors of safety:

$$FOS_{\text{Yield}} = \frac{\sigma_Y}{\sigma_{\max}} = \frac{36.3\ \text{ksi}}{19.8\ \text{ksi}} = 1.83 \quad (\text{note} < 2.0) \qquad FOS_{\text{buckling}} = \frac{\sigma_{cr}}{\sigma} = \frac{229\ \text{ksi}}{15.0\ \text{ksi}} = 15.3 \quad (\text{note} > 3.0)$$

As a design tool, it is very useful to set up a spreadsheet to do this beam-column calculation. Below is a screen capture of such a spreadsheet. By careful inspection of this spreadsheet, you should realize that the same spreadsheet could be used for design of a ship stiffener for compression and/or bending. Specifically, it can be used for simple bending (due to hydrostatic pressure alone) by setting the compressive stress to zero, and using the "effective breadth" of attached plating (from the Shade curve) in lieu of the "effective width". Note that the cell which calculates the "effective width" of attached plating uses an "IF" statement to find the correct value (see the top of the previous page, and use the "Help" menu in Excel). Note also that this spreadsheet can be used for the case when the stiffeners are on the non-pressure side of the plating (normal hull stiffeners), or for the case when the stiffeners are on the pressure side of the plating.



Stiffener w/ attached effective plating
Compression w/ uniform lateral load pressure (beam-column)
*** Assumes simply-supported BCs (i.e. L* = L)**

E (elastic modulus) (lbf/in2)	29,000,000	
σ_Y (yield strength) (lbf/in2)	36,300	
h_w (web height) (in)	3.500	
t_w (web thickness) (in)	0.500	
b_f (flange breadth) (in)	4.000	
t_f (flange thickness) (in)	0.500	
B (plate breadth, stiffener spacing) (in)	24.00	
t_p (plate thickness) (in)	0.50	
L (stiffener length) (in)	72.00	
Compressive stress (axial) (lbf/in2)	15,000	
Gamma (water density) (lbf/ft3)	64.0	
D (water depth) (ft)	20.00	
e (load eccentricity) (in)	0.00	(+ is eccentricity away from plate)
δ_0 (initial eccentricity) (in)	0.00	(+ is eccentricity away from plate)
A (full sectional area) (in2)	15.75	
P (full compressive load on section) (lbf)	236,250	
b_e (effective width of attached plating) (in)	24.00	

Cross-Section	breadth/height	thickness	Area	Vert	1st Moment	2nd Moment	I_0
Member	b, h (in)	t (in)	A_i (in^2)	y_i (in)	$A_i y_i$ (in^3)	$A_i y_i^2$ (in^4)	(in^4)
Stiffener Flange	4.00	0.5000	2.000	4.250	8.500	36.125	0.042
Stiffener Web	3.50	0.5000	1.750	2.250	3.938	8.859	1.786
Plating	24.00	0.5000	12.000	0.250	3.000	0.750	0.250
		Sums =	15.750		15.438	45.734	2.078

Section Properties

A_{eff} (effective area)	15.750	in^2	Total Height =	4.5	in
ybar (height of NA)	0.980	in	SM_{top} (effective to top of flange)	9.28	in^3
I (effective 2nd Moment (moment of inertia)about NA)	32.68	in^4	SM_{bot} (effective to bottom of plate)	33.34	in^3

Maximum Stiffener Stress (σ_{max})

P_E (Euler buckling load) (lbf)	3,609,884
σ_E (Euler buckling stress) (lbf/in2)	229,199
w (lbf/in)	213.33
M_w (bending moment due to pressure) (in-lbf)	138,240
δ_w (eccentricity due to pressure) (in)	0.08
ϕ (eccentricity magnification factor)	1.070

If STIFFENERS are on PRESSURE SIDE of plate:

M_{max} (maximum bending moment) (lbf-in)	-158,151	(+ is bending away from plate)
σ_{flange} (maximum stress at stiffener flange) (lbf/in2)	-32,033	(+ is tensile, - is compressive)
σ_{plate} (maximum stress at plate) (lbf/in2)	-10,257	(+ is tensile, - is compressive)

If STIFFENERS are on NON-PRESSURE SIDE of plate:

M_{max} (maximum bending moment) (lbf-in)	158,151	(+ is bending away from plate)
σ_{flange} (maximum stress at stiffener flange) (lbf/in2)	2,033	(+ is tensile, - is compressive)
σ_{plate} (maximum stress at plate) (lbf/in2)	-19,743	(+ is tensile, - is compressive)

Weekly Assignment #11: Stiffened Panel Construction & Stanchion and Beam-Column Design/Analysis

During the lab period this week you will build the “best” of the stiffened panel designs from your section!

Submit the following design/analysis exercises in a “technical report” format:

1. You have been asked to design a deck support stanchion. The distance between the two decks is 7.5 feet, however the deck girder above (a T-beam) is 6” high. The compartment above is 6’ x 10’ x 7’ and has a permeability of 85%. For a conservative estimate, you figure that the stanchion has to hold the full weight of the flooded compartment with a heave DAF of 1.25. For the stanchion construction, the fabricator has told you that the best they can do will mean that the stanchion will have up to 1/8” of initial deflection, and the shipfitters have promised the maximum offset will be 1/8” from the stanchion on the deck below (eccentric load). You have a T-beam that you can cut to length (3” flange, 3” web, both 10# A36). Will it safely work? Is a cylinder a better structural shape for such a stanchion? Why? Assuming a minimum of 1/8” wall thickness, what is the minimum diameter that would give a safe design?

2. You are asked to check the design of a beam-column made up of ship longitudinal and attached plating, subject to a hydrostatic head of 30 feet of sea water (fsw) and simultaneously to a compressive stress (from global hull girder bending) of 15,000 psi. Assume the material is A36 steel, frame spacing is 72 inches, and longitudinal spacing is 36 inches. The hull plating is 30.6# and the longitudinal stiffeners are ST6x17.5 (d = 6.00 in, t_w = 0.43 in, b_f = 5.10 in, t_f = 0.55 in).

    a. Determine the factors of safety vs. buckling and yield of this beam-column using “hand calculations” (similar to the example on page 95). Clearly explain what you are doing (use words and sentences).

    b. Create a spreadsheet to perform the beam-column calculation. Make the spreadsheet robust enough so that you can input any material, geometry, or load parameters (shown in the yellow cells in the example on the previous page), and calculate the maximum stress in the plating and stiffener. You may use the example as a guide. Hint: Use the example on the previous page to “validate” your spreadsheet, then use it to solve this problem. Note: This spreadsheet should be very useful for your Midship Section Design Project!

Buckling of Plates - Elastic Plate Buckling Under Uni-axial Compression

Unlike columns, plates rarely fail in compressive yield - rather they usually buckle. Consider a plate loaded in compression, simply-supported at the loaded ends, but free on the unloaded edges (see the figure below). Note that the edge dimensions are defined such that "b" is the dimension of the loaded edge, and "a" is the unloaded edge. Note also that since here we are considering a very "wide" plate, we have here that b>>a.

If we consider this to be a "very wide column", then the critical buckling stress would be the Euler buckling stress

$$\sigma_{cr} = \frac{\pi^2 EI}{AL_e^{\;2}} = \frac{\pi^2 E}{\left(\frac{L_e}{\rho}\right)^2}$$

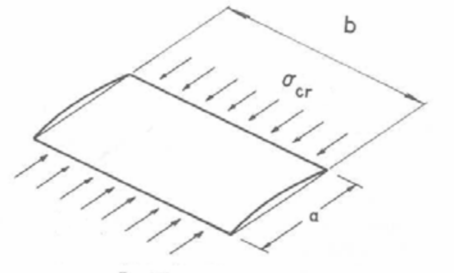
However, for a wide plate, we have a Poisson effect (similar to the Poisson effect for plate bending), and the critical buckling stress for the wide plate is

$$\sigma_{cr} = \frac{\pi^2 D}{a^2 t} = \frac{\pi^2 E}{12(1-\upsilon^2)}\left(\frac{t}{a}\right)^2$$

Recall that D is the flexural rigidity of the plate

$$D = \frac{EI_{\text{effective}}}{b} = \frac{Et^3}{12(1-\upsilon^2)}$$

Note that the ratio (a/t) for the plate plays the same role as the "slenderness ratio" (L/ρ) for the column, and therefore buckling of a plate is geometry-driven. If the plate is also subject to bending (such as an eccentricity) this is simply a plate bending case, so σ_Y controls the strength (like an eccentric column)!



Buckling of wide column

Similar to the lateral plate bending case, there are closed-form solutions, which depend upon boundary conditions (edge conditions) and characteristics of the applied loads.

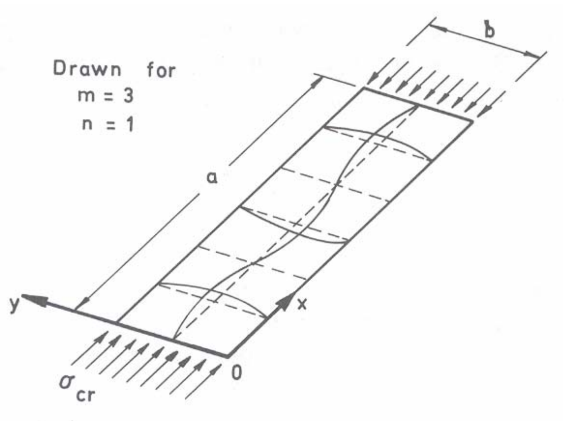
For "regular" plates with aspect ratio (a/b), the solution can be written in a very convenient form known as "Bryan's Equation"

$$\sigma_{cr} = k\frac{\pi^2 D}{b^2 t} = k\frac{\pi^2 E}{12(1-\upsilon^2)}\left(\frac{t}{b}\right)^2$$

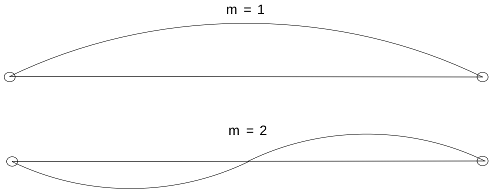
where k is a coefficient which depends on the aspect ratio (similar to the coefficients for bending). Specifically, for plates which are simply-supported (as shown in the below figure) then

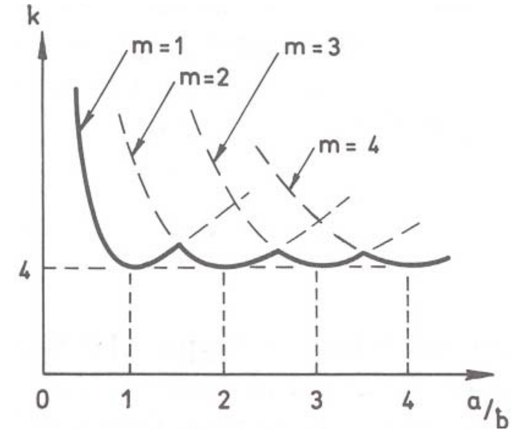
$$k = \left(\frac{mb}{a} + \frac{a}{mb}\right)^2$$

where "m" is the number of "buckling ½ waves" in the x-direction (direction of load), m = 1, 2, 3, etc., and the minimum value of k will give the lowest buckling capacity. For a ≥ b, there will be 1 wave in the y-direction (this gives the minimum buckling energy and therefore minimum σ_{cr}). The coefficient "k" can be plotted vs. aspect ratio (a/b) for simply-supported BCs as shown in the below figure (a similar figure is given in PNA figure 64).



Buckled shape of a long plate.





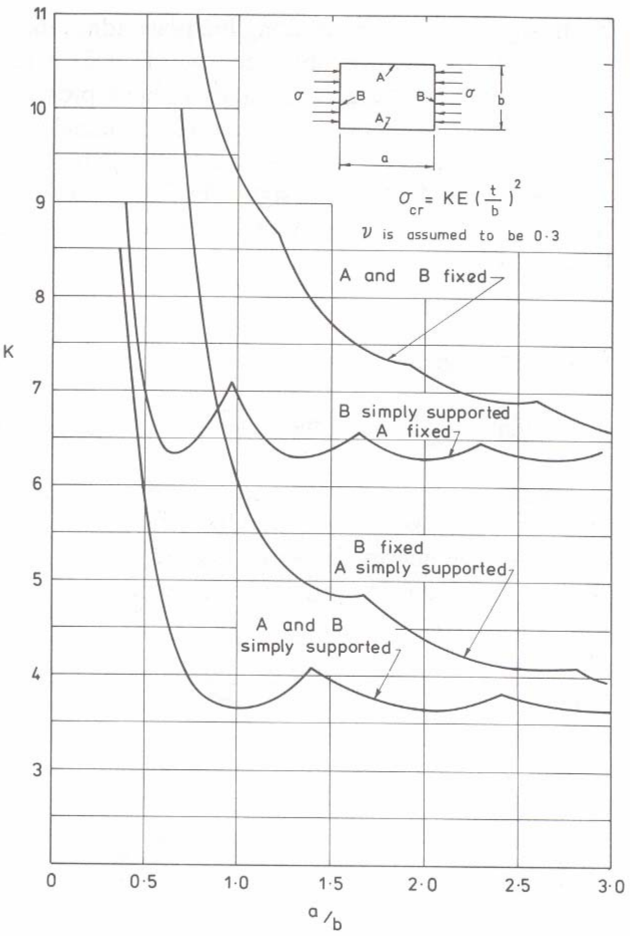
Note that the buckling stress is lowest when the number of "buckling ½ waves" is equal to the aspect ratio (i.e. m = a/b and n = 1). This is the *preferred buckling shape*! Basically, the plate will try to "buckle in squares" (i.e. with an aspect ratio of one). You can theoretically increase the buckling stress by using a/b ≈ 1.5, 2.5, 3.5, etc. However, if the plate is "pre-deflected" or forced to some alternate shape, it may seek that alternate mode shape.

The above assumes that the plate is supported on all four edges. While this is a good assumption for watertight plates such as the hull and deck, it does not consider possible buckling of, for instance, a T-stiffener that has a stanchion landed on it. In cases where the loaded edge is unsupported the minimum k value can be as low as 0.4!

An alternate form of Bryan's equation that is often used for design (we will use this form) is

$$\boxed{\sigma_{cr} = KE\left(\frac{t}{b}\right)^2}$$

Here, as you can see by comparing with the original form, the coefficient K includes the Poisson ratio. So it is important to realize that this form assumes a value for Poisson ratio ν = 0.3 (which is not always valid, but works with steel)! The value of the coefficient "K" can be plotted for different edge conditions (BCs) as shown in the below figure. Note that the lowest buckling stress for the simply-supported edge conditions (all edges) is about 3.62, so this value is often used as the "lower bounds" on elastic buckling design of plates.



Buckling stress coefficient "K" in the design formula for flat plates under uni-axial compression

Example:

Consider a simply-supported plate under uni-axial compression, with the axial compressive stress σ_a = 15,000 psi (note the subscript "a" is for "axial"). The plates thickness is 3/4", and its dimensions are a = 72" and b = 36". Find the FOS vs. buckling.

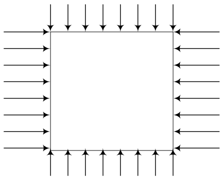
Solution:

$$\sigma_{cr} = (3.62)\left(30{,}000{,}000\frac{\text{lb}}{\text{in}^2}\right)\left(\frac{0.75\text{in}}{36\text{in}}\right)^2 = 47.0\ \text{ksi}$$

$$\text{FOS}_{\text{buckling}} = \frac{\sigma_{cr}}{\sigma_a} = \frac{47.0\ \text{ksi}}{15.0\ \text{ksi}} = 3.1$$

Buckling of Plates – Elastic Buckling Due to Biaxial Compression, Shear, and Bending

Unfortunately for the ship structural designer, much of the time there are multiple loads along both edges of a plate. These can be caused by the longitudinal hogging and sagging loads, the transverse hydrostatic-induced loads, and the lateral loads caused by the direct application of hydrostatic pressure on one side of the plate. With all of these loads, it is no surprise that we may have a reduced buckling capacity! In particular, an in-plane compressive load on a plate that is restricted from lateral expansion by an adjoining plate will also have a transverse stress develop due to Poisson's effects (as illustrated in the figure below).



For many relatively simple load situations the designer can use approximate closed form solutions. For more complex problems, finite element analysis is used. In both cases the solution is very dependent on the loads and boundary conditions assumed. In that regard, FEA has a distinct advantage as it is relatively easy to vary the loads and boundary conditions.

Biaxial Compression:

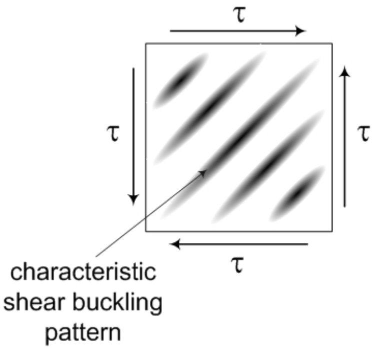
The general equation for the bi-axial compression situation (illustrated in the figure above) is the same as for uni-axial compression, but it gets a bit more complex as the buckling can be induced from either the "a" or "b" direction. This means that the ratio of each direction's stress compared to that direction' buckling stress, and the combination of the two directions, are both important. Additionally, the aspect ratio still plays a significant part. For instance, if the applied stress parallel to the x-axis, σ_{ax}, is at 95% of the critical buckling stress in that direction, $(\sigma_{ax})_{cr}$ then it is easy to visualize that it would not take much applied compressive stress in the y-direction, σ_{ay} , to get the plate to buckle. Although we will not cover this phenomenon in any greater detail in this class, the student is referred to Hughes for a more detailed explanation, including methods for predicting using "hand calculations". Of course, it is most suitable to analyze these problems using FEA.

Note that if edge loads are not uniform, such as due to shear flow, then we also have shear to think about!

Pure Shear:

In ship structures, the plating is commonly subjected to large shear loads. This occurs mostly in the side shell plating of the ship, which usually carries the greatest amount of shear stress, but can also occur in the deck plating due to shear flow.

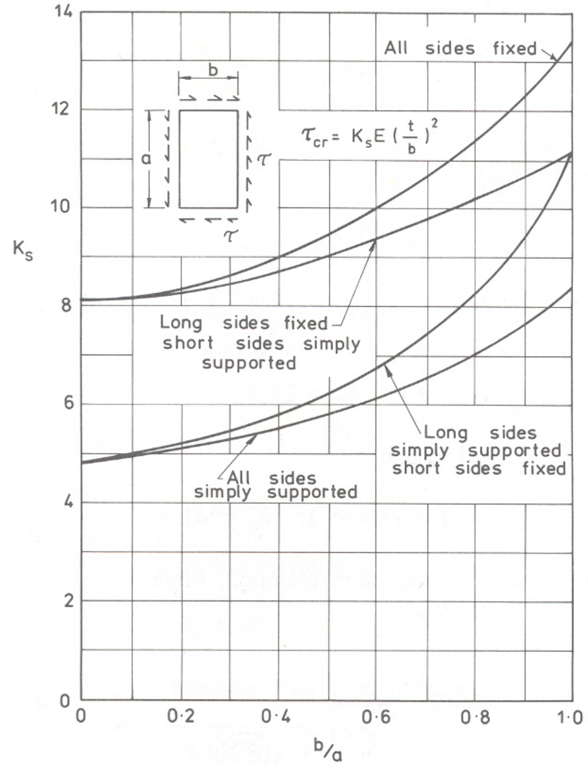
For pure shear, characteristic buckling occurs at 45^0 to the x-y axis (see the figure below). However, the precise wave length and buckled shape due to pure shear will depend upon the aspect ratio (a/b) and the boundary conditions. You can experiment with this phenomenon by applying shear forces to a piece of paper with your hands. Notice the diagonal "wrinkling" of the paper?



The general equation for predicting the critical buckling stress due to shear is almost identical to "Bryan's Equation" discussed for the uni-axial case. The "design" form of this equation is:

$$\tau_{cr} = K_s E\left(\frac{t}{b}\right)^2$$

where K_s is a coefficient which depends on the aspect ratio of the plate and the boundary conditions. The figure below (taken from Hughes) provides Ks. Note that this figure plots Ks as a function of the inverse aspect ratio (b/a) instead of the aspect ratio (a/b)! Also note that, as was the case for the "design form" of Bryan's equation, this assumes a Poisson ratio $\nu = 0.3$ (OK for steel or aluminum).



As was the case for uni-axial compression, we desire a factor of safety of at least 3 vs. shear buckling:

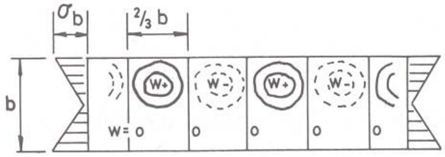
$$FOS_{\tau-buckling} = \frac{\tau_{cr}}{\tau} \geq 3.0$$

Example:

> Consider the same plate in the previous example, but say it is subject to a pure shear stress τ = 15 ksi. What is the FOS vs. shear buckling?
>
> Solution:
>
> $$\frac{b}{a} = \frac{36in}{72in} = 0.5$$
>
> From the figure above, assuming simply-supported BCs: Ks = 0.57
>
> $$\tau_{cr} = (5.7)\left(30{,}000{,}000\frac{lb}{in^2}\right)\left(\frac{0.75in}{36in}\right)^2 = 74.2\text{ ksi}$$
>
> $$FOS_{\tau-buckling} = \frac{\tau_{cr}}{\tau} = \frac{74.2ksi}{15ksi} = 4.9 \quad \text{(OK)}$$

Pure In-Plane Bending:

The web of an I-beam (or the side-plate of a box-girder!) sees applied stress that varies across the “plate” of the web (i.e. tension is maximum on one edge and max compression on the other). This variation in compressive stress across the web can lead to buckling of the web. The part of the web seeing the compressive stress buckles in an alternating ½ wave pattern (see the figure below).



This can be illustrated using a piece of paper. Lay the paper on a table and apply a “in-plane bending load” to the paper using your hands. Notice the alternating ½ wave pattern on the compression side of the paper.

Although we will not cover this phenomenon in any greater detail in this class, the student is referred to Hughes for a more detailed explanation, including methods for predicting using “hand calculations”. Of course, it is most suitable to analyze these problems using FEA.

Buckling of Plates – Plates Subject to Combined Loads (in-plane and lateral loads)

Many plates are subject to combined in-plane loads (tension or compression) and lateral loads (hydrostatic pressure). For example, bottom plating (between frames and longitudinal) are subject to tensile and compressive stresses from global hull girder bending plus hydrostatic pressure. Plating at other locations may also see these combined loads (deck plating, bulkheads with tanks, etc.).

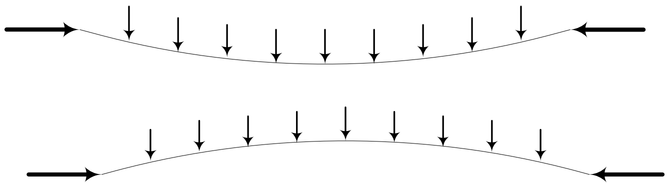
Our approach to designing these plates depends upon the types of loads. For in-plane tension plus lateral pressure, if deflections are small (we can use small-deflection theory), then we can simply use superposition of the stresses calculated separately, and σ_{VM}. However, if deflections are large (membrane effects), then the best solution is to use a nonlinear FEA method.

For in-plane compression plus lateral pressure, we cannot simply use superposition of the stresses, since the in-plane compression will have the effect of magnifying the lateral deflection (similar to the beam-column). The details of this approach are provided in Hughes, with only the basic result provided here. The outcome is determined by the *ratio* of in-plane compressive load to lateral pressure. There are two general possibilities:

1. If the compressive load is low enough that buckling would <u>not normally</u> occur on its own (without the hydrostatic pressure), then the in-plane compression acts as a magnifier of the lateral deflection due to the pressure (similar to the beam-column). If this is the case, we might consider two additional possibilities:

    a. If the in-plane compressive stress is “relatively small”, then it acts only as a multiplier for the out of plane bending due to the lateral pressure. In this case, we can use figures 12.18 and 12.19 from Hughes (see next page).
        i. We first use figure 12.18 to check the maximum deflection (note that σ_e is the Euler buckling stress for the plate (which can be found using Bryan’s Equation).
        ii. Next, we use figure 12.19 to calculate the maximum bending stresses at the center of the plate in the x and y directions (these bending stresses include the magnification effect).
        iii. Next we add the in-plane compressive stress to the bending stress in the x or y direction as appropriate.
        iv. Finally, we find the equivalent vonMises stress at the center of the plate, and determine the factor of safety vs. material yield.

    b. If the in-plane compressive stress is “relatively large”, then it is likely that the plate would buckle or collapse with the addition of the lateral pressure load. In this case, it may buckle in a different “mode shape” than the normal lowest energy mode due to the deflection caused by the lateral pressure. For example, for a/b = 2 it would buckle in a single ½ wave (bowl) shape vice two ½ waves as it would otherwise. Another possibility is that it might “snap through” to its normal lowest energy mode in spite of the existence of the lateral pressure. This “snap through” can be very violent (and loud). Another condition which can cause “snap through” is when a “pre-deflected” shape is opposite to the lateral pressure – then “snap through” may also occur (examples of this are bilge radius plate or submarine hull plating).



2. If the compressive stress is high enough that buckling would <u>likely</u> occur on it own (without the lateral pressure), then it <u>will</u> buckle with lateral pressure!

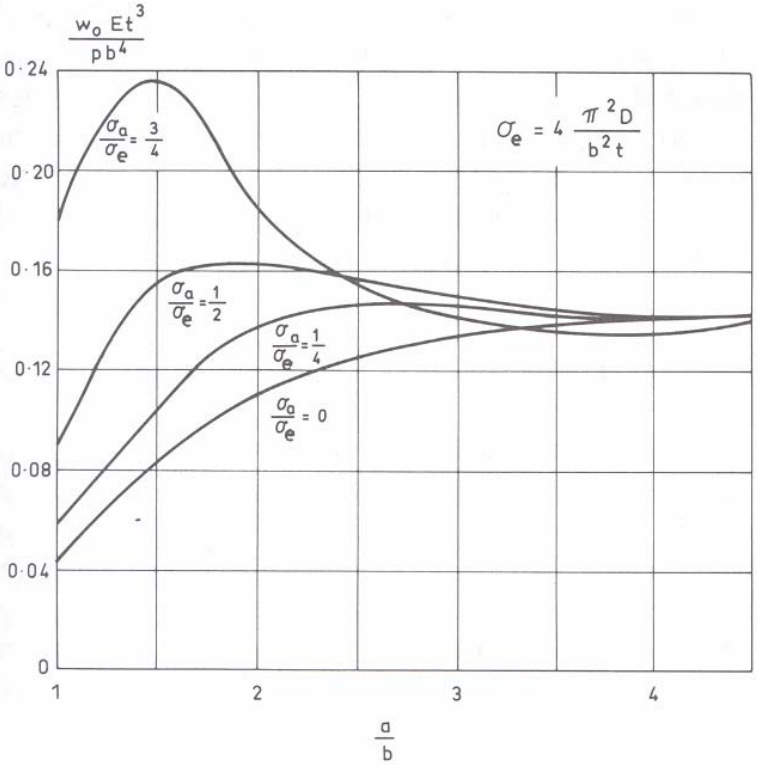


Figure 12.18 Maximum deflection due to uniaxial compression and lateral pressure; simply supported plate [13].

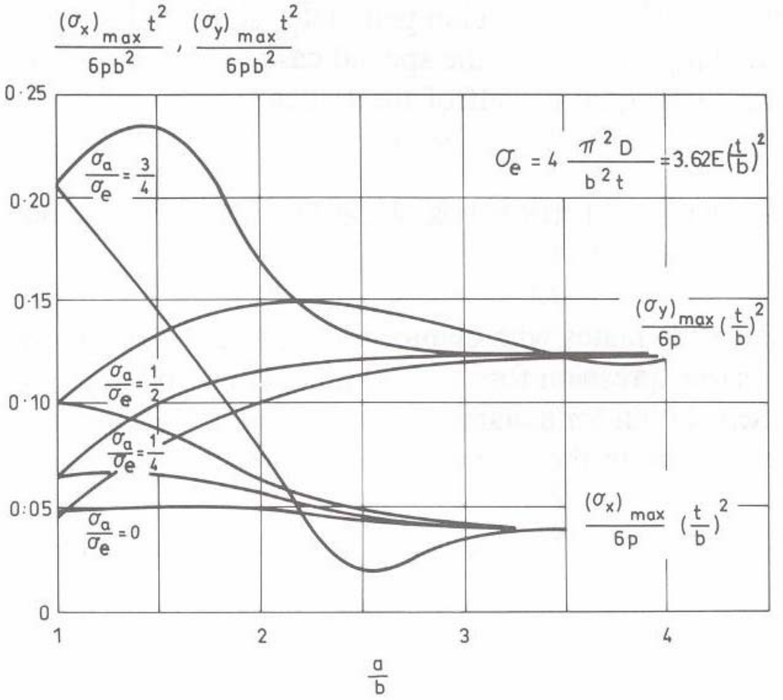


Figure 12.19 Maximum bending stress due to uniaxial compression and lateral pressure; simply supported plate

In-class exercise:

> Consider a ship's bottom plating subjected to a hydrostatic pressure head of 30 feet of sea water (fsw) and simultaneously to a compressive stress (from global hull girder bending) of 12,000 psi. Assume the material is A36 steel, frame spacing is 48 inches, longitudinal spacing is 24 inches, and that the plating thickness is ½ inch. Determine the factors of safety vs. material yield of this plate.

Weekly Assignment #12: Stiffened Panel Testing and Midship Design Project Procedure

During lab this week we will test the panel you built!

In addition, you will start your Midship Design Project!

Objective: To tie together all the topics covered in the course by accomplishing the structural design of the midship section of a medium-sized steel vessel.

Due Dates:

The beginning of week 14 – Detailed Procedure
The beginning of week 15 – Bending Moments / Global Section Modulus Spreadsheet
The beginning of week 16 – Final Bottom and Side Plating Calculations
The end of week 16 – Completed Report, Specifications and Drawing

Deliverable for this week: (note: the first three assignments may not be returned in a timely manner; keep copies! Only one submission per team is needed.)

> Detailed Procedure – Based on the short procedure outlined in class, develop a detailed procedure for your midship design. Use a numbered list format. Include which calculations you will use at the different stages. For example, "2a. Perform plate buckling analysis on bottom plating for hogging condition." This procedure should fill up 2-3 typed pages. Include the drawing, report and specification steps in your procedure.

Grading: 15% each for the first three submissions, 55% for the final submission. Factors will include completeness, neatness, accuracy, clarity of description and peer evaluation feedback.

Other Requirements:

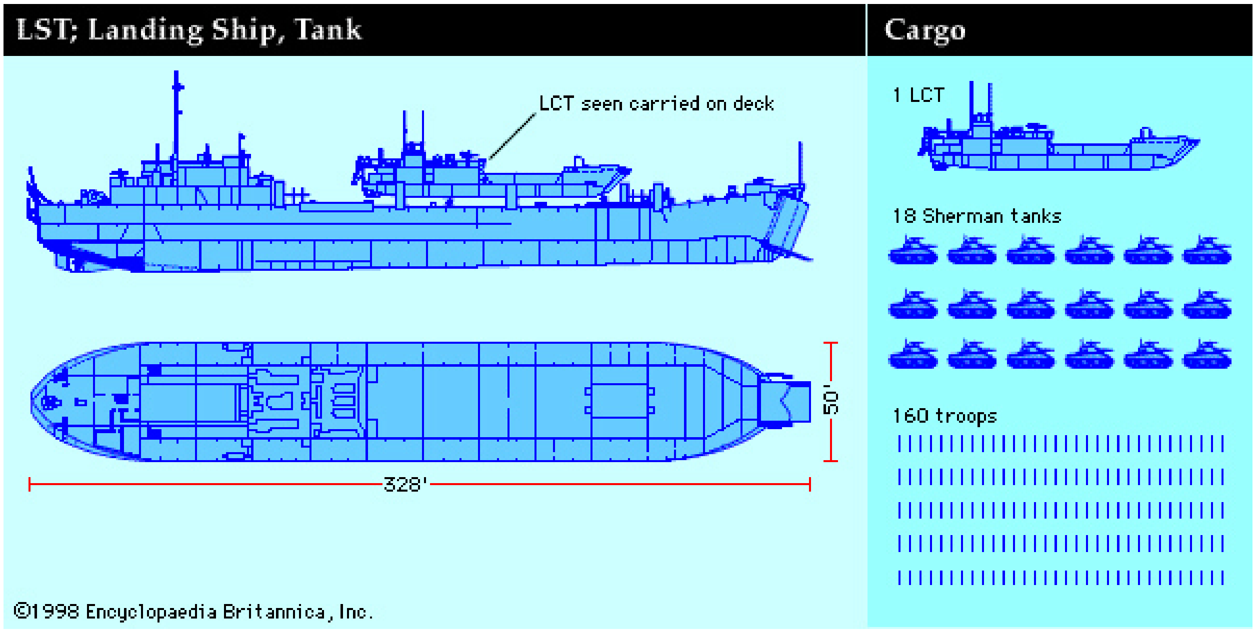
1. All plating must be standard sizes (1/16" increments from 3/16"), and no larger than 50'x12' (so that it can fit on a flatcar)(specify the weld locations and type on the drawing). The plating and stiffeners must have a yield no greater than HTS.
2. The final report must be in a binder and must have a Table of Contents and Cover Memo (signed by each team member). Organize the binder to follow the evaluation form.
3. Teams will generally be three students.
4. When the second and final deliverables are submitted, each team member must electronically submit a peer evaluation form. This is located on the course web page.

Design Head for this project (from ABS):

General equation, $p=\rho gh$ where ρ is the mass density of seawater, g is the acceleration due to gravity and h is the height of the water column.
The general industry practice is to give a value for h that is based on the maximum water column height, which is to the bottom of the canoe body. "h" is then reduced appropriately for the topsides and deck, based on the vessel's depth and freeboard.

$h=cF(3T+0.14LOA+5.3\text{ ft})$, where c is a slamming factor and is 0.8 from Station 0 to Station 0.5, c is 1.2 from Station 0.5 to Station 4, c tapers from 1.2 at Station 4 to 0.7 at Station 10. F is a function of frame spacing, but is approximately equal to 0.55 for normal frame spacings.

The Ship: An LST!



LST Design and Construction (from http://www.insidelst.com)

The Design:

Designed by the Navy's Bureau of Ships, the WW2 LST had a 328-foot length, 50-foot beam, a flat-bottom, a sloping keel, giving a 7-1/2-foot maximum draft (forward), and a 14-foot maximum draft (aft), with about a 16-foot freeboard. The tank deck dimensions were 230-feet long, 30-feet wide, and 12-feet high. It had 3/8" plating for the hull (shell plating), rather than the planned original 1/4", and the plating under the bow was 1" thick. The firm of Gibbs and Cox, New York, completed the actual design details, and became the contractor charged with the procurement of materials/equipment, and they selected the Dravo Corporation as the first contractor.
John C. Neidermair, the Principal Naval Architect, insisted the design should contain no more shapes and sizes of plates than you have fingers on your hand -- five of each! The LST required 30,000 parts, including such items as steering gear, stern anchor gear, armament, snaking winch, appliances, refrigeration plant, ladders, doors, pumps, engines, stanchions, main generator and power distribution switching gear.


LST 1080 with pontoons circa 1955 (photo by Morris Smith)

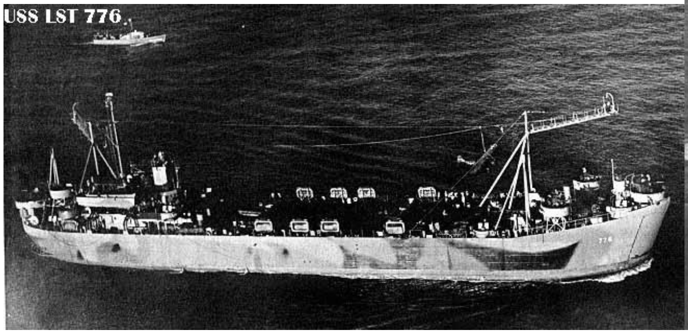
The LST was designed to ground evenly (from bow to stern) on a beach with a slope of about one-foot for every fifty-feet (the design gradient). Each propeller (screw) is protected by a skeg which extends forward from it and provides a sturdy "runner" beneath its blades. The twin rudders are mounted directly behind the screws, and thus achieve maximum effectiveness as a result of the propeller discharge. The propellers were spaced almost 40-feet apart and set up clear of the base line of the hull. The sea chests, or intakes for sea water, were located on the sides of the hull.

The LSTs actually took more man-hours to build than the Liberty and Victory (cargo) ships, which were almost 5 times larger; about 10,600 tons deadweight to the LST's 2,180 tons. Some compromises in construction difficulty/cost/time constraints were made. Using a flat-plate design to form the turn at the bilge would have added 16-25% more water resistance, and adversely affected the 10-knot speed desired. The decision was a curved bilge. Constructing a curved (cambered) main deck would have drained water more readily, but added to the construction time and cost. The decision was to build a flat main deck.

The first LSTs were designed with an elevator to carry equipment between the tank deck and the main deck. This was a time-consuming process which the "LST-511 class" improved upon by replacing the elevator with a ramp that was hinged at the main deck for this purpose. It permitted vehicles to be driven from the main deck to the leading edge of the tank deck, across the bow ramp to the beach or causeway. Some other modifications, with the "LST-542 class", included the installation of the (Conn) navigation bridge [3rd level above the main deck] atop the Captain's sea cabin, the installation of a water distillation plant with a capacity of 4,000 gallons per day, removal of the tank deck ventilator tubes from the center section of the main deck, strengthening the main deck to carry an LCT (Landing Craft, Tank), and an upgrade in armor/armament.

The ship could carry 1,060 tons of diesel fuel in ballast tanks. Draft forward could be as little as 1-1/2-feet when unloaded and unballasted. The LSTs were actually loaded for ocean passage to as much as 1,600 tons on the tank deck, and 300 tons on the main deck, and resulted in 33.3 tons per inch of immersion. The draft forward with a 1,900-ton payload was about 8-feet. For beaching, the **designed** load was 500 tons, and the actual load was an average of 700 tons.

The design was excellent in many ways. The hull was double-bottomed, and water-tight compartments lined the entire sides of the ship, offering some protection to the engine room spaces. The double-bottom compartments contained salt-water, diesel fuel, and fresh water, in a grouped arrangement order progressing from the bow to the stern. The LSTs were often modified. Below are two versions of the LST aircraft carrier! In the first picture the aircraft took off and landed using a cable system swung out over the port side! This method was popular in the Pacific Theatre. The more conventional approach on the right is from the Med.



Construction:

Production of the "LST-1 Class" was begun late in l942 and continued through 1943. The "LST-511 Class" was built in 1944, with the "LST-542 Class" being built in 1944 and l945.

Production costs averaged $1.6 million per ship for the LST-542 class, versus $1.4 million for the LST-1 and LST-511 classes.

Brief Statistics:

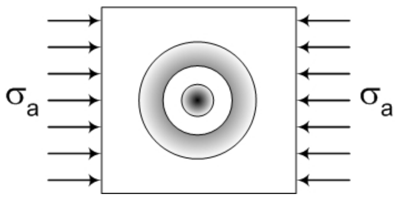
Hull	All-welded steel 3/8” thick
Length	327’ 9”
Beam	50-feet
Displacement empty / full	1,650 / 3640 (LST 1) 4080 (LST 511)
Design draft forward / aft	6’ 8-1/2” 13’ 0-3/8” [Displacement @ D.D. 3,590 tons]
Empty draft forward / aft	1'6" / 10'6"
Full load draft forward / aft	8' / 14'6"
Beaching trim forward / aft	3' to 6' / 10' to 13'
Freeboard	16'6"
Engines	2 (GM V-12 Diesel - 12-567A)
Screws	2
screw horsepower	1,700

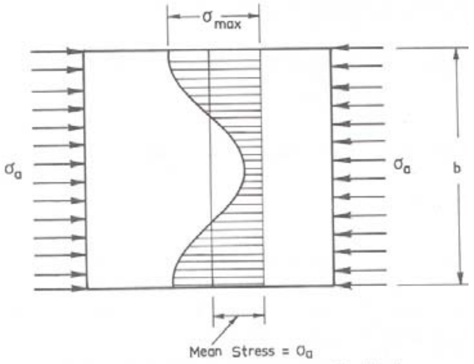
Ultimate Strength of Plates

As discussed previously, the collapse of columns occurs either by elastic buckling if the column is slender (Euler or "ideal" column buckling) or by the commencement of yielding which so seriously decreases the column bending stiffness that collapse by inelastic buckling follows almost immediately. In the case of plating, the mechanism of collapse is more complex. Specifically, collapse of plating depends upon type of loading, boundary conditions, aspect ratio, initial distortion, and even residual stresses.

For "slender" plates subject to an increasing compressive uni-axial stress (without lateral load), the plate goes through a number of phases prior to ultimate failure or collapse. At low levels of stress, the plate remains flat, as the stress remains below the elastic buckling stress and below the yield stress. The first noticeable response when the stress reaches the elastic buckling stress is that the plate buckles, and as a result the center portion of the plating partially "escapes" from the axial shortening being applied by the compressive stress. As a result the center buckled portion of the plate "sheds" some of the load, which is transferred to the edges of the plate. In this post-buckled state, the outer edges of the plate are carrying higher stress (higher than the mean stress σ_a) and the center portion carries a lower stress. This is illustrated in the figure below. As the compressive stress continues to increase, eventually the stress on the outer edges of the plate reaches the material yield stress, and the edges yield (plastic deformation). Eventually, as the center portion of the plate also yields, and the plate collapses.

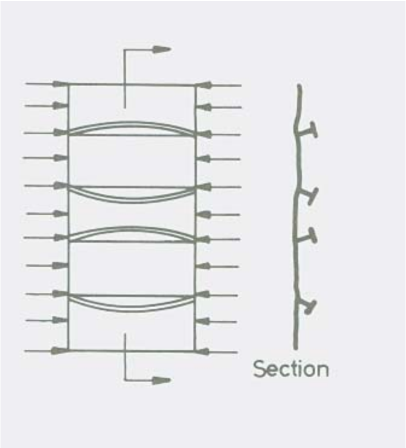
The extent to which a plate will buckle and collapse depends upon several factors, including load (magnitude, type, etc.), plate geometry (late aspect ratio a/b, thickness, initial distortions), boundary conditions, and manufacturing variances (welds, voids, etc.). Because of this highly variable nature, evaluation of collapse of plate structures is usually investigated using finite element analysis (FEA).





Torsional Buckling of Stiffeners – Tripping

A stiffener may buckle by twisting (or *rotating*) about its line of attachment to the plating. This is referred to as tripping. This is not the same as the column or beam-column type of buckling we have discussed previously for stiffeners, and it is not the same as local compressive buckling of the web or buckling of the flange. Tripping of several stiffeners in a uniaxially-loaded panel is illustrated in the below figure. Note that the plate may also rotate to some extent to accommodate the stiffener rotation – this is not plate buckling, but rather part of the stiffener tripping phenomenon. Tripping could be elastic or plastic, but usually plastic and catastrophic.



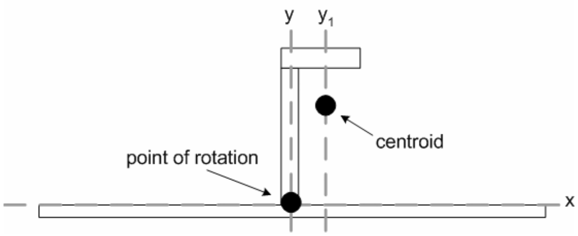
The key to minimizing the possibility of this torsional buckling mode is to maximize the torsional stiffness (GJ) of stiffener. Note that G (shear modulus or modulus of rigidity) is already high for steel. Therefore, the best way to minimize tripping is by maximizing J (polar moment of inertia) of stiffener.

A slightly simplified solution for stiffener tripping is presented in Hughes. The result is that for stiffener tripping due to a uni-axial compression:

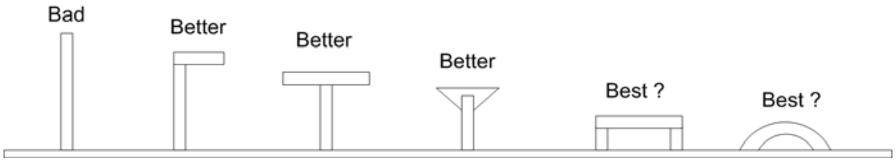
$$\sigma_{cr-tripping} = \frac{1}{I_{sp}}\left[\pi^2 E\left(\frac{h}{a}\right)^2 I_{sz}\right]$$

where

h = distance (height) from plate to *shear center* of stiffener
A_x = stiffener area (flange + web)
I_{sp} = *polar* moment of inertia of stiffener about *center of rotation* ($I_x + I_y$)
I_{sz} = moment of inertia of stiffener about axis t*hrough centroid of stiffener and parallel to web* (I_{y1})



Taking a look at this solution, you should be able to see that the design goal to prevent tripping is that we desire a stiffener to be short (small I_{sp}) and wide (large I_{sz}). The below figure illustrates the idea. What are problems with "Best" stiffeners on the right? When are they used?



Note that the desire for stiffener to be short (small I_{sp}) is counter to the desire to maximize vertical moment of inertia (I_x) which we desire for vertical bending stiffness (the main purpose of the stiffener). Therefore, designing a stiffener to resist tripping is partially a trade-off or compromise with the stiffener's main function to resist vertical bending. One solution to this is to install "tripping brackets" at several spots along the length of the stiffener to resist tripping.

A simplified solution for stiffener tripping applies to open thin-walled stiffener sections (T's, L's, and flat bars). In this case h ≈ d (web height), b_f = flange width, A_f = flange area, A_w = web area, and we can simplify

$$I_{sp} = d^2\left(A_f + \frac{A_w}{3}\right) \text{ and } I_{sz} = b_f^2 \frac{A_f\left(\frac{A_x}{3} - \frac{A_f}{4}\right)}{A_x} \cong \frac{b_f^2 A_f}{12}$$

This results in a simplified equation for critical stress for stiffener tripping for open thin-walled stiffener sections:

$$\sigma_{cr-tripping} = \frac{\pi^2 E}{12 + 4\frac{A_w}{A_f}}\left(\frac{b_f}{a}\right)^2$$

A few additional notes: Increasing flange width too much may lead to (local) flange buckling. However, flange buckling can be prevented by maintaining an adequate width-to-thickness ratio – a good guideline is: $b_f/t_f \leq 14$ (for mild steel), $b_f/t_f \leq 10$ (for Alum, HTS, HYs). For steel, all *standard* sections are "compact sections" and have adequate width-to-thickness ratios. This is not true for aluminum, however, and this should be checked carefully. Actually for designing stiffeners with aluminum, you should refer to the "Aluminum Design Manual" (The Aluminum Association) for complete design guidelines.

Buckling of Stiffened Panels

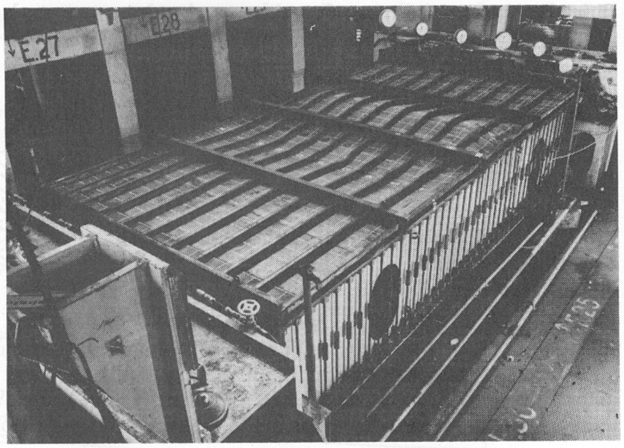
We have discussed buckling of columns, which we have applied to the design/analysis of stanchions between decks, as well as stiffeners with effective width of attached plating, including stiffeners under combined axial compression due to global hull girder bending and lateral hydrostatic pressure (beam-columns). We have also discussed buckling of plating (between stiffeners), which we have applied to design of ship's bottom, side and deck plating, including plating under combined axial compression due to global hull girder bending and lateral hydrostatic pressure.

Panels are defined here as plating plus attached stiffeners, which extend between major supports. Here the term "major supports" refers to very stiff longitudinal or transverse structure such as bulkheads, web or ring frames, hull chines, etc.

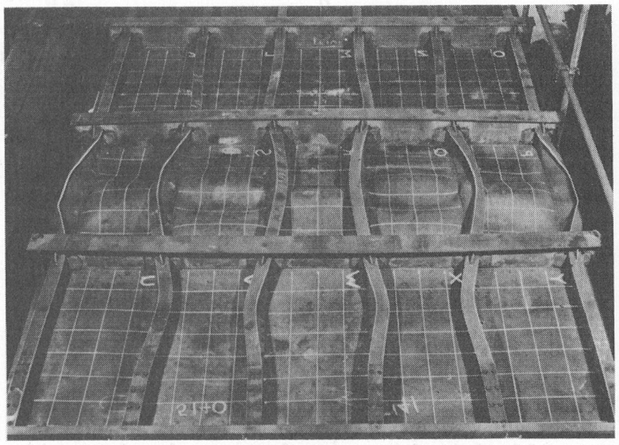
A panel can buckle in a number of ways:

- Elastic buckling of plating between stiffeners. This is also known as "dimpling". This is least likely to be a problem (except for fatigue considerations). In PNA, this is referred to as "mode 1 buckling".
- Elastic and inelastic buckling of stiffeners between frames (with effective width of attached plating). This may not be catastrophic unless an entire row of stiffeners (between frames) buckles together inelastically, in which case it may lead to collapse of the entire panel between frames. In PNA, this is referred to as "mode 2 buckling".
- Elastic or inelastic torsional buckling of stiffeners between frames ("tripping"). This is almost always catastrophic and can rapidly lead to total panel collapse. In PNA, this is referred to as "mode 3 buckling".
- Elastic or inelastic buckling of an entire panel made up of several frame bays. This is also known as "global buckling", "gross panel buckling", or "overall grillage buckling". In PNA, this is referred to as "mode 4 buckling".

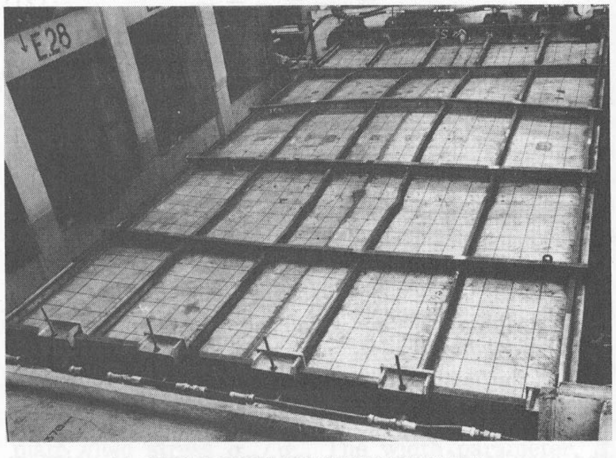
The figures below (from PNA Volume 1) provide photographs illustrating modes 2, 3, 4 (shown in a ship panel test machine).



(a) INTERFRAME FLEXURAL BUCKLING (MODE 2)



(b) INTERFRAME LATERAL-TORSIONAL BUCKLING (MODE 3)



(c) OVERALL GRILLAGE BUCKLING (MODE 4)

As a matter for design priority, which of these "buckling modes" would we want to fail first? The answer is of course "…it depends!" Elastic plate buckling (mode 1 or "dimpling") is generally not catastrophic, as the stiffeners are designed to carry the load if the plating buckles, so it is preferred for elastic plate buckling to occur first. Elastic buckling of stiffeners between frames with effective width of attached plating (mode 2) is more noticeable but relatively easy to fix (by adding additional stiffeners or in the field with stanchions or wood supports) but may become catastrophic if allowed to progress. Stiffener tripping, which generally can rapidly become inelastic (plastic), almost always progresses leading to total panel collapse, so this should be last. Thus, we employ a design goal (recommendation):

$$\sigma_{cr-stiffener} \geq 1.1 \cdot \underbrace{\sigma_{cr-plate}}_{FOS>3} \quad \text{and} \quad \sigma_{cr-tripping} \geq 1.1 \cdot \sigma_{cr-stiffener}$$

Our general recommended method is as follows:

(1) Calculate critical buckling stress for plate $\sigma_{cr-plate}$

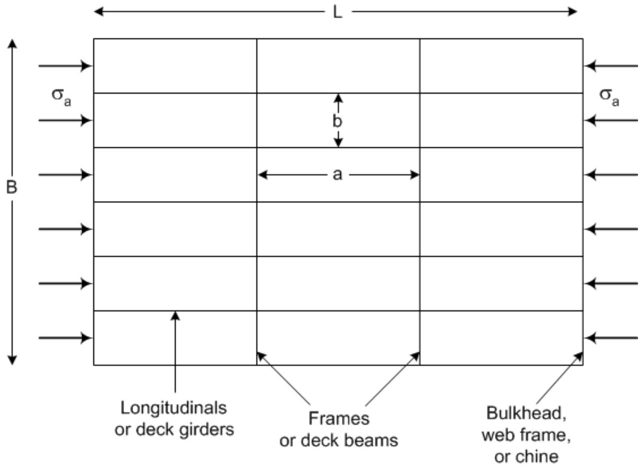
(2) Calculate critical buckling stress for stiffener (with effective width of plating) $\sigma_{cr-stiffener}$

(3) Calculate critical buckling stress for stiffener tripping $\sigma_{cr-tripping}$

(4) Make sure $\sigma_{cr-stiffener} \geq 1.1 \cdot \underbrace{\sigma_{cr-plate}}_{FOS>3}$ and $\sigma_{cr-tripping} \geq 1.1 \cdot \sigma_{cr-stiffener}$

Buckling of a "Gross Panels" ("Grillage Buckling"):

Larger panels require additional cross-stiffening. These are often referred to as "grillage panels" or "gross panels". This cross-stiffening includes intermediate *transverse* stiffeners (frames or deck beams) in addition to the longitudinal stiffeners (longitudinals, stringers, or deck girders).



For lateral loads on these "gross panels", the stiffeners usually carry more of the load. Usually if the stiffeners are OK for bending, then they are usually OK for buckling. In-plane loads are more challenging. The plate carries most of the in-plane load (stress), and may dimple (plate buckling).

Stiffeners in line with the primary in-plane compressive loads (longitudinals and deck girders) offer the greatest resistance to buckling! Transverse stiffeners (frames and deck beams) primarily provide *intermediate* support for the longitudinal, stringers and deck girders – to prevent out-of-plane deflections and rotations and reduce tripping. Thus the frames and deck beams must be stiff *relative* to longitudinal, stringers and deck girders. Thus, to avoid overall "gross panel" buckling, the intermediate *transverse* members (frames and deck beams) should be *stiffer* than the longitudinal, stringers and deck girders that they support!

Minimum transverse rigidity/stiffness to prevent "grillage" buckling (uniaxial compression):

Hughes provides a relation for the ratio of transverse to longitudinal stiffness necessary to prevent "gross panel" buckling. This is a semi-empirical, and compares rigidity of transverse stiffeners to rigidity of longitudinal stiffeners:

$$\frac{\gamma_y}{\gamma_x} = \frac{B^4}{\pi^2 C a^4}\left(1 + \frac{1}{P}\right)$$

where:

$\gamma_y = \frac{E I_y}{D a}$ = stiffness ratio of transverse stiffener, including effective width of plating

$\gamma_x = \frac{E I_x}{D b}$ = stiffness ratio of longitudinal stiffener, including effective width of plating

B = width of entire cross-stiffened panel
L = length of entire cross-stiffened panel
a = spacing of transverse stiffeners (frames or deck beams)
b = spacing of longitudinal stiffeners (longitudinals or deck girders)
$C = 0.25 + 2 / N^3$
N = number of "sub-panels" longitudinally $(N = L / a)$
P = number of longitudinal stiffeners $(P = B / b - 1)$
(note: for the example on the previous page, N = 3 and P = 5)

As long as $\gamma_y / \gamma_x > 1$ then the compressive strength of the cross-stiffened panel is determined mainly by the compressive strength of the longitudinally-stiffened "sub-panels" (i.e. *between transverse stiffeners)*. If $\gamma_y / \gamma_x > 1$, then it is sufficient to consider only buckling of the longitudinally-stiffened "sub-panels" (between frames or deck beams).

Note: In designs where there is a longitudinal bulkhead supporting the main deck and bottom panels, then the cross-stiffened panel width B should be taken as the distance between the longitudinal bulkhead and the deck edge or sideshell (chine), or the distance between longitudinal bulkheads.

Weekly Assignment #13: Midship Design Moments and Section Modulus & FEA Buckling Demonstration

For your Midship Design Project:

> Bending Moments/Global Section Modulus Spreadsheet – Provide your Maxsurf/Hydromax weight, buoyancy, load, shear and bending moment curves (for stillwater, hogging and sagging conditions). Provide the Maxsurf body plan, isometric view and hydrostatic information of your vessel. Use the ABS moments used in the homework (handout) for comparison. Provide a copy of the section modulus spreadsheet with sufficient structure to meet your design criteria. You must show your equations and explain what you are doing! Be very clear on why you are performing the calculations, what loads you are using, what your criteria for acceptance is and what your final design is.

FEA Buckling of Plates and Panels (In-class Exercise/Demonstration):

1. Using the analytical expressions (Bryans Equation) to predict the failure load (in pounds) of 4'x4', 4'x6' and 4' x 8' simply supported 30.6# A36 plates. The load is applied parallel to the "long" direction in each case.
2. Compare your hand calculations with finite element analysis. Identify the failure modes (yield, 1st-mode buckling, etc.). Plot compressive stress and deformed plots, and compare to your "hand" calculations.
3. Determine the effectiveness of adding a flat bar stiffener to the 4' x 6' plate to increase buckling capacity. Test two potential stiffeners and choose the "better" design. Both stiffeners will be made of 40.8# HTS. One will span down the centerline along the "a" (long) axis, and the other will go down the middle of the "b" (short) axis. The shorter one will be 6" tall and the longer one must be the same weight as the shorter one. Assume the stiffeners are welded to other supports.
4. Briefly discuss the effectiveness of stiffeners to prevent buckling.

Buckling FEA Suggestions:

By now you are getting comfortable with running finite element analysis! Buckling should not create too many more issues as it is just an extension of static analysis. The major differences from what you have done before are in the modeling and analyzing the results.

1. Build your plate using points and a surface.
2. Create your element group (SHELL4), material props and real constants.
3. You will need a mesh of about 1"x1" to get adequate resolution.
4. When putting the stiffeners on, don't forget to merge the nodes.
5. Set your boundary conditions to match how you think the actual structure will respond. You will want rollers at one end to allow compression while the other is fixed in all translations. It is very important that you compare the theoretical and practical boundary conditions. Specifically, what happens with lateral translations of the two side edges? In a simply-supported boundary condition they are free to translate laterally. Is that the case in real ship structures? You should test the results by varying the boundary conditions and comment on the differences. Add the forces in a way that duplicates reality. Keep in mind that you can't have boundary conditions that restrict the forces! Beware of unintended eccentricities.
6. Run the static case first to determine those results. Then run the buckling. Use the Analysis-Frequency/Buckling-Buckling Options and accept the defaults. Then use Analysis-Frequency/Buckling-Run Buckling.
7. To see your buckling results look at the deformed (not displacement) plot. The load factor is shown. That is the multiplier on the loads you put in. Make sure you identify on your plot what loads you applied to the analysis!
8. To find the stress in a particular element (your end stresses may be distorted due to the local loads), first identify which element it is using Meshing-Elements-Identify, and then list the element stress for that element using Results-List-Stress Component.

Classification Societies

The major focus of this course has been to learn the analytical methods that provide tools for us to design ship structures. We call this approach using "first principles." The assumption has been that we have not had any outside regulatory influences on our designs. That assumption does not apply to many of our designs however, as the Code of Federal Regulations (CFR) has some structural requirements. The Coast Guard is tasked with administering the maritime sections of the CFR and for some vessels the naval architect must have the plans reviewed by the Coast Guard.

In the civil construction side of engineering, the most widely known structural code is the Universal Building Code (UBC). This multi-volume manual includes "cook-book" style equations combined with tables and graphs to allow a contractor or civil engineer to quickly determine the required scantlings. When applying for a building permit the owner must submit the plans for review by the local government, which will check that the plans meet local codes. As the UBC cannot be tailored for all locations, it has sections that can be scaled to different locations and the local laws will often cite different UBC sections. The UBC is not managed by a government agency, rather it is updated by the International Conference of Building Officials (ICBO).

The maritime equivalent of the ICBO is the classification societies. These are non-profit organizations that establish guidelines for building vessels. The societies are based in the larger maritime nations (generally one per country) and have common guidelines established by the International Association of Classification Societies (IACS). Most American companies build and "classify" their ships with the American Bureau of Shipping (ABS). Internationally, other big classification societies are Det Norske Veritas (DNV), Germanischer Lloyd, and Lloyd's (to name only a few).

The genesis of the classification societies was a request from the insurance companies that ships should conform to minimum standards. As vessel types vary tremendously, there is no "Universal Shipbuilding Code". Instead, ABS has about 50 different rules and guides, ranging from steel ships to water carriers to offshore racing yachts. Many of the codes can be downloaded free of charge from the ABS website (www.eagle.org).

Some of the ABS Rules are:

- Steel Vessel Rules
- Steel Barges - Ocean Service Barges of any Length
- Steel Vessels Under 90 m - Crewboats, Tugs
- High Speed Naval Craft- Patrol Boats HSVs
- Steel Vessels for Service on Rivers, ICW - River Barges, Towboats
- Bulk Carriers for Service on the Great Lakes - Freshwater Operations
- Reinforced Plastic Vessels - F. R. P. Vessels
- Aluminum Vessels - Aluminum Commercial Vessels 100' - 500'
- Naval Vessel Rules
- MODU - Mobile Offshore Drilling Units

Some of the (less-comprehensive) Guides are:

- Motor Pleasure Yachts - Steel, Aluminum, or FRP
- High Speed Craft, - Commercial/Government Service Planing vessels
- Fishing Vessels - Steel Fishing Vessels Under 90 m (200')
- Offshore Racing Yachts - Steel, Aluminum, or FRP Sailing Vessels
- Fire Fighting Vessels - Steel Fireboats
- Crew Accommodations Guide
- Oil Recovery Vessels - Oil Spill Cleanup Vessels
- Floating Production Storage and Offloading Vessels - (FPSO)
- Small Waterplane Area Twin Hulled Vessels - (SWATH)

The Rules and Guides include sections on intact and damaged stability, structures, firefighting, etc. When you design your vessels in the capstone courses you will have to meet the ABS codes if they apply to your vessel!

Weekly Assignment #14: Midship Design Project Bottom and Side Plating and Stiffeners

For your Midship Design Project:

> Bottom and Side Plating and Stiffener Calculations – Provide the completed calculations for the bottom and side plating and stiffeners. These can be submitted as either copies of engineering paper, or as copies of a spreadsheet. If the latter, you must show your equations and explain what you are doing! Be very clear on why you are performing the calculations, what loads you are using, what your criteria for acceptance is and what your final design is.

Welding and Weld Design

The following notes on welding and weld design are provided in "bullet" format. You will also see a video in class that will cover much of this material. There is also a graphical Powerpoint presentation available on the course Blackboard page, which you should also read.

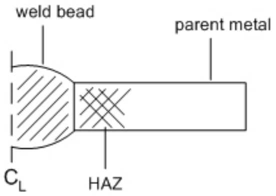
Welding:

Most common method of joining in the marine environment - ~90-95%
  Others: Riveting, bolting, soldering/brazing

Significant cause of structural failure
  Liberty ships, T2 tankers
  Alexander Kieland
  - March 1980, North Sea, Semi-submersible accommodations platform (212 people), 5 pontoon columns with cross-bracing.
  - Fatigue fracture initiated at a weld error where a hydrophone bracket was attached.
  - Propagation of initial fatigue fracture, followed by catastrophic brittle failure of one of the braces, followed by ductile overload of adjacent braces.
  - Platform quickly rolled over to 30-35^0. After 20 minutes, one of the main pontoons collapsed, causing the platform to capsize.
  - 123 lives lost.

Welding can be defined as the fusing together of two or more metals by heat and/or "pressure" (friction)

Anatomy of a weld:



HAZ (head affected zone) is the transition between liquid re-crystalization and the base (usually cold-worked) metal

  Most shrinkage occurs here, so it is the most brittle area. This is where welds usually fail
  Solutions to brittleness in HAZ:
  1. Pre-heat, post-heat (annealing)
  2. Minimize time and size of heat area → pulsed laser welding

Note that there is a stress concentration effect (SCF) at the connection of the weld bead to parent metal. Thus there is a fatigue susceptibility. This is the reason weld beads are ground and polished.

Weld processes - how heat is generated (more detail later):
  Fusion welding:
  - Gas: Oxy-acetylene → 6,000^{o}F
  - Arc: DC (or AC) – Metallic arc, gas-shielded arc, submerged arc → 10,000^{o}F
  - Electron Beam & Laser → 30,000^{o}F

  Pressure welding:
  - Friction
  - Resistance: spot, seam, butt, flash

Major weld problem: Hydrogen impurities → leads to voids, cracking, embrittlement
  Solution to hydrogen embrittlement:
  1. Use "shielded" → inert gas (nitrogen, argon, helium)
  2. Pre-heat to remove moisture from parent metal & weld rods → also good for reducing shrinkage
  3. Flux in weld rods

Weld rods:

- Filler material to fill in gaps between pieces
- Used in gas and arc welding
  - Gas uses filler material like solder does
  - W/ arc, rod is either filler (if electrode carbon), or consumable electrode
- The flux & impurities in weld rod float to the top of the bead and are called "slag". It is chipped away.

Abbreviations:

SMAW – shielded metal arc welding
TIG – tungsten electrode inert gas (tungsten electrode not consumable)
MIG – metal inert gas (electrode filler is consumable) (aka GMAW)

Weld joint types:

- Most common:
  - Butt joint
  - Corner joint
  - Tee joint
  - Lap joint, strap joint
- Other:
  - Bead weld
  - Plug weld (similar to spot weld)

Welding positions:

- Ranked from least difficult (preferred) to most difficult
  - Flat ("downhand")
  - Horizontal
  - Vertical
  - Overhead

Weld design:

1. Determine <u>if</u> the material can be welded

   Carbon equivalent: $\mathrm{CE} = \mathrm{C} + \frac{\mathrm{Mn}}{6} + \frac{\mathrm{Cr} + \mathrm{Mo} + \mathrm{V}}{5} + \frac{\mathrm{Ni} + \mathrm{Cu}}{15}$ (all are in %)

   C = carbon, Mn = manganese, Cr = chromium, Mo = molybdenum , V = vanadium, Ni = nickel, Cu = copper

   Example: Check the weldability of AISI 1020 (from Matweb)

   $\mathrm{C} = 0.23,\ \mathrm{Mn} = 0.6 \quad \rightarrow \quad \mathrm{CE} = 0.23 + \frac{0.6}{6} = 0.33$

   $$\text{ABS criteria } \begin{cases} \mathrm{CE} < 0.41 \rightarrow \text{ weldable in ambient (no pre-heat required,)} \\ 0.41 < \mathrm{CE} < 0.45 \rightarrow \text{ low carbon electrodes, pre-heat required} \\ \mathrm{CE} > 0.45 \rightarrow \text{ "special procedures" required (maybe not weldable)} \end{cases}$$

2. Determine the required weld size

   Tensile strength: based on 1 or 2 sided weld
   - Single-sided weld is maximum 80% the strength of double-sided weld
   - Warpage occurs in single-sided welds due to uneven shrinking
   - Equivalent stress in weld: $\sigma = \frac{P}{A} = \mathrm{SCF}\frac{P}{l \cdot t}$ (l = length of weld, t = thickness of plate or weld size)

   Shear stress:
   - Shear strength is based on the amount of material in the weld
   - Common approach uses shear stress calculated using the "throat area"
   - Equivalent (shear) stress in weld: $\tau = \frac{P}{A_s} = \mathrm{SCF}\frac{P}{0.707h \cdot l}$ (h = weld size or "leg" size)

   A common minimum FOS for welded joint design is 2.

   Stress Concentration Factors (SCF) for welds (from Spotts "Design of Machine Elements"):
   - Butt = 1.2
   - End of butt = 2
   - Fillet = 1.5
   - End of fillet = 2.7

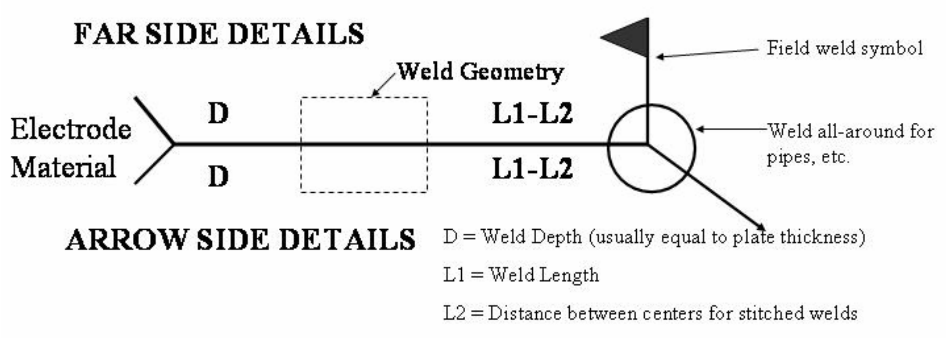
   Bending: welds should be designed so that the weld material is not in bending. For example, if a plate is joined to another plate in a T arrangement, two fillet welds will significantly reduce the stress compared to a single butt weld.

3. Specifying the weld – weld symbols

   See the handout "Deciphering Weld Symbols" (Dave Wright Welding) for more information. Some additional notes and examples are given on the following pages.

   Important note: You must use weld symbols on your midship design drawing!

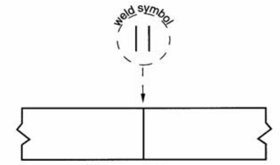
The generalized welding symbol:



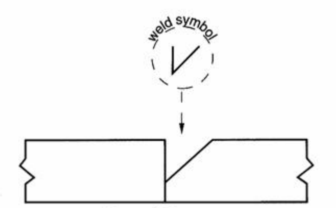
The Field Weld Symbol is a guide for installation. Shipyards normally do not use it, except in modular construction.

Welding Geometry symbols:

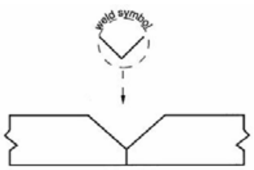
Butt Joints:



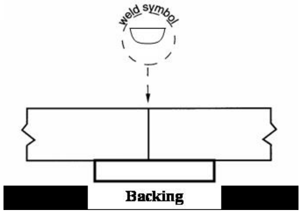
Square-Groove



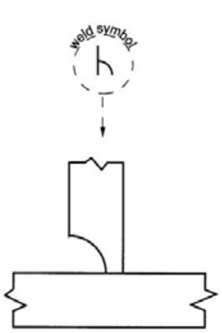
Bevel-Groove



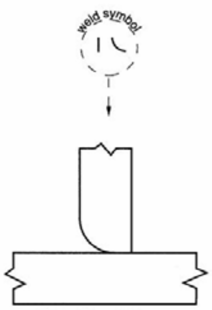
V-Groove



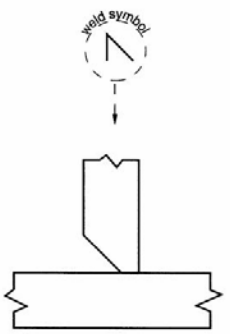
Fillet Joints:



J-Groove

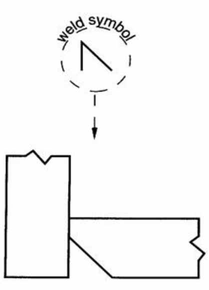


Flare-Bevel-Groove

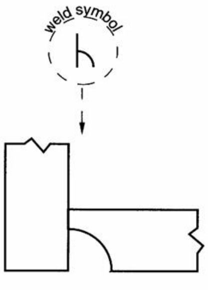


Bevel-Groove

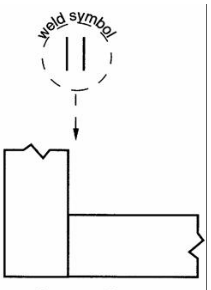
Corner Joints:



Bevel-Groove

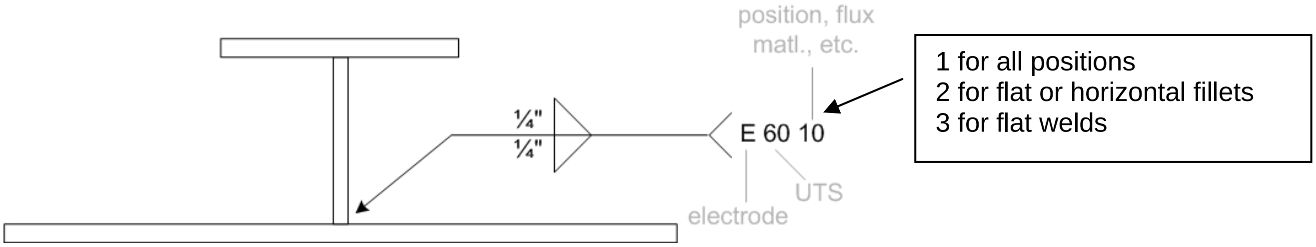


J-Groove



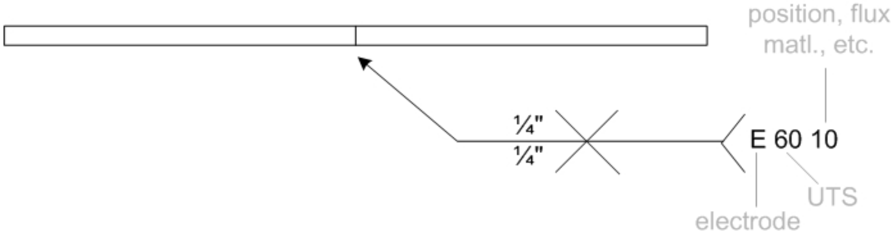
Square-Groove

Example for a fillet weld (T joint w/ double fillet weld):



1 for all positions
2 for flat or horizontal fillets
3 for flat welds

Example for a butt weld or groove weld (double v-groove weld):



Note "E60" electrode (type): $\sigma_U = 60$ ksi, $\sigma_Y = 48$ ksi, $\tau_Y = 13.6$ ksi (so, good for A36)

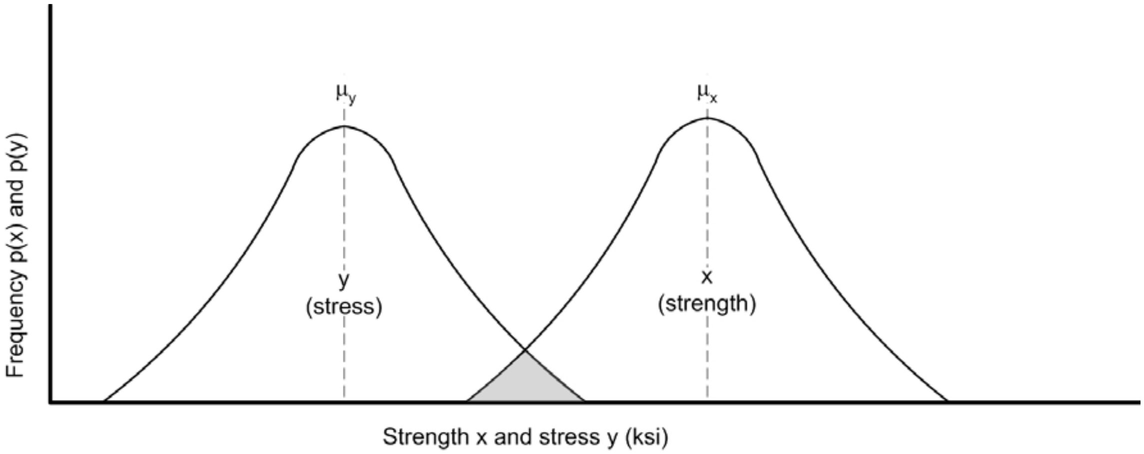
Reliability Methods

Probably the largest error in the analysis we have done in this class is our assumptions of the loads. We have assumed that our design will face a maximum load, but we have never clearly described where that load value came from. We have also mostly ignored fatigue effects! The methods we have used could easily be called “the traditional method” of ship structural design. The maximum loads came from historical correlation. If we found that a particular design method routinely developed problems, we would increase the maximum load values until the failures reached an acceptable value. This has worked because ships have evolved slowly and we have used conservative minimum factors of safety! Using this approach on new vessel types, such as the X-Craft, has produced some dramatic failures! In the past, when faced with a new type, a smart naval architect simply increased the factor of safety.

A better approach started development in the 80’s. It is called the reliability method and uses probability and statistics to better predict what the maximum loads might be. It is based on characterizing the loads in terms of statistical and probabilistic distributions and then determining what an “acceptable probability of failure” is. What is an acceptable level can be determined from historical examples and current litigation! If a new construction material is considered for example, then the new probability of failure can be calculated, and if it is not acceptable, then the structural members could be increased. Below is a copy of one of the first figures from these notes and it illustrates that we have gone full circle from an overview, through deterministic methods, and now back to reliability.

Reliability methods can yield excellent results that traditional methods would never indicate. For example, when the first carbon/epoxy America’s Cup yachts were built, they used traditional ABS methods and factors of safety (3-4 for composites). A detailed study of the aerospace-grade composites used in Dennis Conner’s syndicate indicated that as the materials had such a low variation in properties compared to that assumed by ABS (which was based on typical fiberglass boat laminates), by using reliability methods the factors of safety could be reduced to 1.25-2. That meant the next boat’s structure could be as much as 70% lighter!

Reliability methods are taught at the Naval Academy in the senior elective course EN452! If you are interested in that course, talk with your academic advisor.



Weekly Assignment #15: Midship Design Project Final Report

Due: 0800 on the first day of finals

For your Midship Design Project: Final Report

Completed Report, Specifications and Drawing – Submit your final report describing the structural design and process. Include the principal dimensions, your final procedure, copies of all your calculations, a Midship Construction Drawing (11 x 17 format using CAD, "accordion-folded" in your binder) that shows all your longitudinal stiffeners and plating, and a typical frame near midships, a listing of all structural components in the midship area (which may be on the drawing), your final weight spreadsheet, computer-generated plots, your section modulus spreadsheet, and a structural weight estimate for the midships section (the middle 20 feet of the ship).

The Future!

Congratulations on reaching the end of the Ship Structures course! No doubt you have learned a lot about how we structurally design ships. With this knowledge you can go out and design almost all the structure found on small craft and ships.

Your education can be furthered through senior electives. One common complaint about ship designers is that “they design structures that can’t be built”. To address this, an elective titled “Marine Fabrication Methods” teaches the common construction techniques.

Another issue is “what about designing a salvage solution and repair?” An elective in marine salvage will answer that question!

More advanced analytical methods are covered in the Structural Reliability course.

Best of luck and good designing!

References

Hughes, Owen, *Ship Structural Design*, SNAME. This was the EN358 textbook until this year. It has a lot of information suitable for both undergraduate and graduate courses and provides significantly more detail than PNA V1.

Tupper, *Introduction to Naval Architecture*, SNAME. This was the EN246 textbook. Chapter 7 summarizes the entire ship structures course! A great way to get the big picture of this course.

Gilmer and Johnson, *Introduction to Naval Architecture*, Naval Institute. One of the EN246 texts. It has a good overview of the ship structures design procedure.

SNAME books: *Ship Design and Construction*, *Ship Structural Design*, *Fiberglass Boat Design and Construction*, *Principles of Naval Architecture*. Naval architecture industry handbooks.

Gerr, *The Elements of Boat Strength*, McGraw-Hill, 2000. Describes common small craft building methods and includes a simple, but conservative, scantlings rule.

Higgins, R. A., *Properties of Engineering Materials*, Industrial Press. An excellent and low-cost materials science and processing reference. This is the EN380 text.

Roark's Formulas for Stress and Strain, McGraw-Hill. Everything you thought you needed to know about beam and plate analysis.

Muckle, W., *Strength of Ship Structures*, Edw. Arnold, Ltd., London, 1967. (Out of Print). A classic.

Structural Design Manual for Naval Surface Ships, NAVSEA 0900-LP-097-4010, Naval Sea Systems Command, 1976.

Evans, J.H. (editor), *Ship Structural Design Concepts*, Ship Structure Committee (SSC), Washington, DC, 1974.

J.H. Evans (Editor), *Ship Structural Design Concepts: Second Cycle*, Cornell Maritime Press, Centerville, MD, 1983.

St.Dennis, M., *On the Structural Design of the Midship Section*, DTMB Report C555, Oct. 1954.

Manual of the Properties of Combined Beam and Plate: Part I - Tees and Angles, U.S. Department of Commerce, Washington, DC. A reference manual.

Rockey, K.C., et. al., *The Finite Element Method*, Crosby Lockwood Staples, 1975.