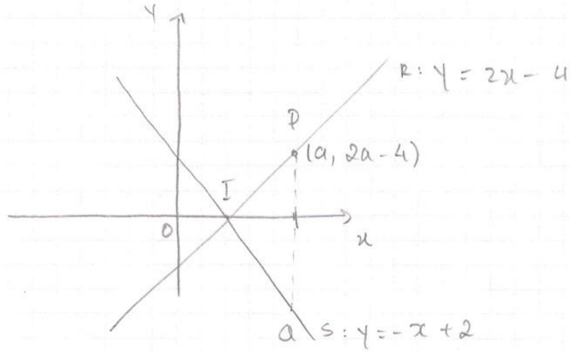
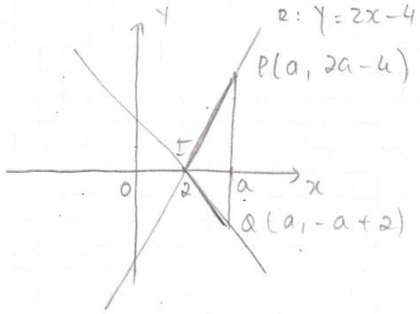
RESOLUÇÃO DO SEXTO CONJUNTO DE ITENS DO GAVE → ABRIL 2010

1.



1.1

i) para $a \geq 2$

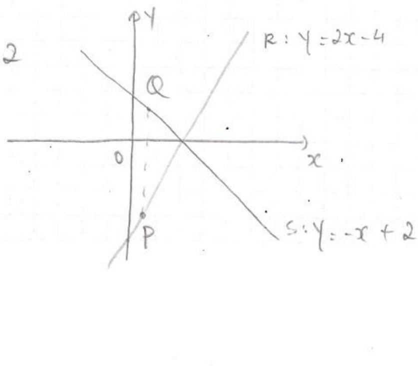


Como $P \in r$ então $P(a, 2a-4)$ e como $Q \in s$ então $Q(a, -a+2)$

Logo $\overline{PQ} = 2a - 4 - (-a+2) = 2a - 4 + a - 2 =$

$= 3a - 6 \quad , \ a \geq 0$

ii) Para $a < 2$



Logo $\overline{PQ} = -a + 2 - (2a - 4)$

$= -a+2-2a+4 = -3a+6$, para $a<2$

ou logo $\overline{PQ} = \begin{cases} 3a-6 & \text{se } a \geq 0 \\ -3a+6 & \text{se } a<0 \end{cases} = |3a-6|$ //

De a)e i.i)

ou

$\overline{PQ} = \|\overrightarrow{PQ}\| =$

$= \sqrt{0^2 + (-3a+6)^2}$

$= \sqrt{(-3a+6)^2} =$

$= \sqrt{(3a-6)^2} = |3a-6|$ //

$\overrightarrow{PQ} = Q - P =$

$= (0, -a+2) - (a, 2a-4)$

$= (0, -a+2-2a+4)$

$= (0, -3a+6)$

NOTA: i) $(-3a+6)^2 =$

$= (3a-6)^2$

ii) $\sqrt{x^2} = |x|$

1.2 $\overline{PQ} = 3 \Leftrightarrow |3a-6| = 3 \Leftrightarrow$

$\Leftrightarrow 3a-6=3 \quad \vee \quad 3a-6=-3$

$\Leftrightarrow 3a=9 \quad \vee \quad 3a=3$

$\Leftrightarrow a=\frac{9}{3} \quad \vee \quad a=\frac{3}{3}$

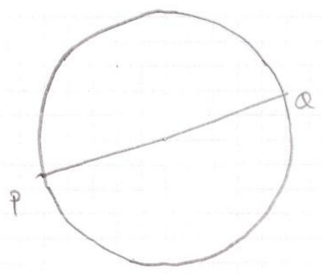
$\Leftrightarrow a=3 \quad \vee \quad a=1$

C.S. $= \{1, 3\}$

Logo $P(1, 2\times1-4) = (1,-2)$ e $Q(1, -1+2) = (1,1)$

ou $P(3, 2\times3-4) = (3, 2)$ e $Q(3, -3+2) = (3,-1)$

1.3



Raio $\frac{\overline{PQ}}{2}$

$$P_{\text{circunferência}} = \cancel{2}\pi \times \frac{\overline{PQ}}{\cancel{2}} =$$

$$= \pi \times |3a-6|$$

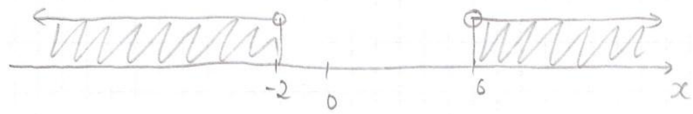
então $\pi \times |3a-6| > 12\pi$

$$\Leftrightarrow |3a-6| > \frac{12\cancel{\pi}}{\cancel{\pi}} \Leftrightarrow |3a-6| > 12$$

$$\Leftrightarrow 3a-6 > 12 \quad \vee \quad 3a-6 < -12$$

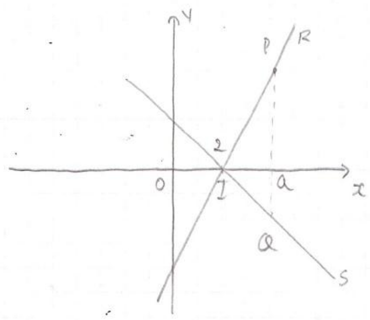
$$\Leftrightarrow 3a > 18 \quad \vee \quad 3a < -6$$

$$\Leftrightarrow a > 6 \quad \vee \quad a < -2$$



$$C.S. =]-\infty, -2[\cup]6, +\infty[$$

1.4. i) para $a > 2$



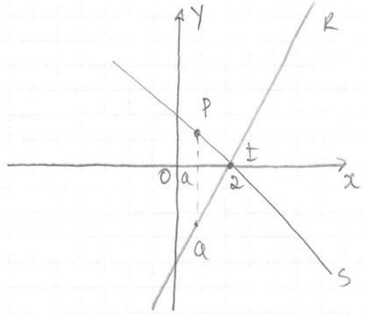
$$A_{[PQI]} = \frac{(a-2) \times \overline{PQ}}{2} =$$

$$= \frac{(a-2) \times (3a-6)}{2}$$

$$= \frac{(a-2) \times 3(a-2)}{2}$$

$$= \frac{3}{2}(a-2)^2$$

ii) para $a < 2$



$$A_{[PQI]} = \frac{(2-a)\times(-3a+6)}{2}$$

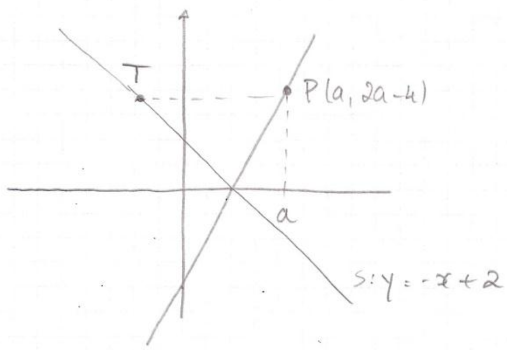
$$= \frac{-(a-2)\times(-3(a-2))}{2}$$

$$= \frac{3(a-2)^2}{2} = \frac{3}{2}(a-2)^2$$

Logo $A_{[PQI]} = \frac{3}{2}(a-2)^2 \quad , \quad a \neq 2$ //

De i) e ii)

1.5.

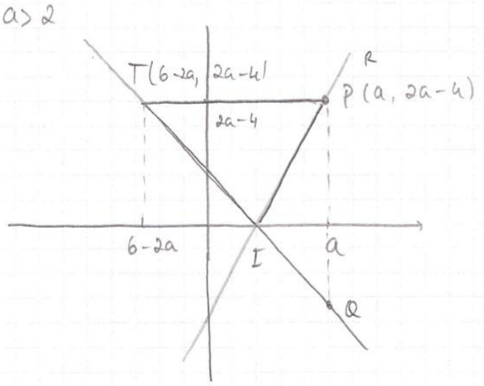


1.5.1 Ora como a ordenada de T é igual à ordenada de P então $y_T = 2a-4$, como $T \in S$ vem: $2a - 4 = -x + 2 \Leftrightarrow 2a - 4 - 2 = -x$

(sub. na equação de S) $\Leftrightarrow 2a - 6 = -x \Leftrightarrow x = 6 - 2a$

Logo $T(6-2a, 2a-4)$

1.5.2 i) Para $a>2$



$$\overline{PT} = a-(6-2a) = a-6+2a = 3a-6$$

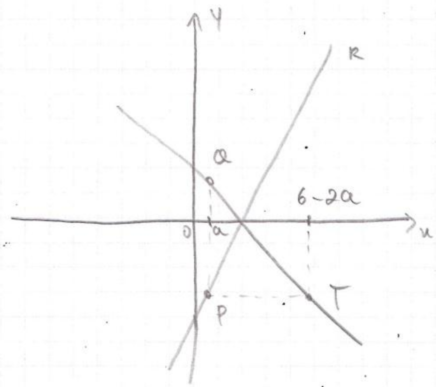
$$A_{[PQT]} = \frac{\overline{PT} \times \overline{PQ}}{2} = \frac{(3a-6)\times(3a-6)}{2}$$

$$= \frac{3(a-2)\times 3(a-2)}{2} = \frac{9}{2}(a-2)$$

e $\underbrace{\frac{9}{2}(a-2)}_{A_{[PQT]}} = 3 \times \underbrace{\frac{3}{2}(a-2)}_{A_{[PQI]}}$

Logo $A_{[PQT]} = 3\,A_{[PQI]}$ para $a>2$

ii) Para $a<2$



$A_{[PQT]} = \frac{\overline{PT} \times \overline{PQ}}{2}$ $\qquad \overline{PT} = 6 - 2a - a$

$= 6 - 3a$

$= \frac{(6-3a) \times (6-3a)}{2}$

$= \frac{-3(a-2) \times (-3(a-2))}{2}$

$= \frac{9}{2}(a-2)$

e $\underbrace{\frac{9}{2}(a-2)}_{A_{[PQT]}} = 3 \times \underbrace{\frac{3}{2}(a-2)}_{A_{[PQI]}}$

Logo $A_{[PQT]} = 3\, A_{[PQI]}$ para $a < 2$

então $A_{[PQT]} = 3\, A_{[PQI]}$ para $a \neq 2$

De i) e ii)

2. $g(x) = -x^2 + 2x + 3$

2.1 $g(x) = -2x + 7 \Leftrightarrow -x^2 + 2x + 3 = -2x + 7$

$\Leftrightarrow -x^2 + 4x - 4 = 0$

$\Leftrightarrow x = 2$

Substituindo $x = 2$ em $y = -2x + 7$ vem $y = -2 \times 2 + 7$

$\Leftrightarrow y = 3$

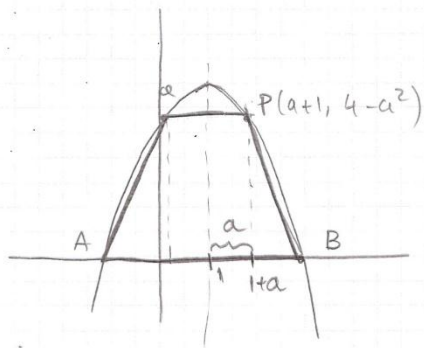
Logo a recta de equação $y = -2x + 7$ é tangente ao gráfico de g no ponto de coordenadas $(2,3)$.

2.

2.2.1 O ponto P tem abcissa $1+a$ e pertence ao gráfico de g logo $P(1+a,\ g(1+a))$. então a ordenada de P é $g(1+a)$

$$\begin{aligned} g(1+a) &= -(1+a)^2 + 2(1+a) + 3 \\ &= -(1+2a+a^2) + 2 - 2a + 3 \\ &= -1 - \cancel{2a} - a^2 + 2 - \cancel{2a} + 3 \\ &= 4 - a^2 \end{aligned}$$

2.2.2.



I) Zeros de g: $g(x)=0 \iff -x^2+2x+3=0$
$\iff x=-1 \lor x=3$

Logo $A(-1,0)$ e $B(3,0)$

II) $T(a) = A_{[ABPQ]} = \dfrac{\overline{AB}+\overline{PQ}}{2} \times h$

onde $\overline{AB}=4$, $\overline{PQ}=2a$ e $h=4-a^2$

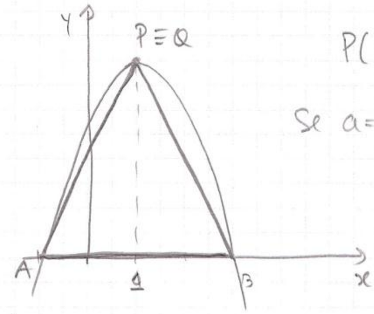
então $T(a) = \frac{4+2a}{2} \times (4-a^2)$

$= (2+a)(4-a^2)$

$= 8 - 2a^2 + 4a - a^3 =$

$= 8 + 4a - 2a^2 - a^3 \quad , \quad a \in \,]0,2[$

2.2.3 Se $a=0$, os pontos P e Q coincidem no vértice da parábola.



$P(1+a,\ 4-a^2)$

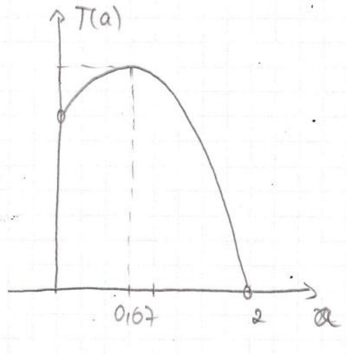
Se $a=0$ $P(1,4)$

Portanto a região sombreada é um triângulo de base $\overline{AB}=4$ e altura igual a $4-0^2=4$ logo $A_{\text{rec}} = \frac{4\times 4}{2} = 8$

Se substituirmos a por 0 em $8+4a-2a^2-a^3$, também se obtém 8.

2.2.4. Definir $Y_1 = T(a)$ Janela: $[0,2] \times [0,10]$

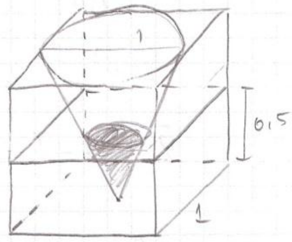
obtemos:



Logo $a \simeq 0{,}67$.

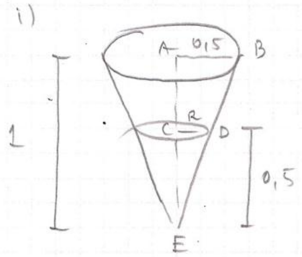
3.

3.1



$$V_{\text{líquido}} = 1 \times 1 \times 0,5 - V_{\text{cone sombreado}} = 0,5 - 0,0327$$
$$= 0,4673 \, m^3$$
$$1 \, m^3 = 1000 \text{ litros} \qquad = 467,3 \text{ litros}$$

i)



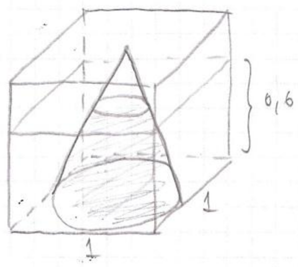
Os triângulos [ABE] e [CDE] são semelhantes pois têm dois ângulos iguais, AEB é comum e $E\hat{C}D = E\hat{A}B = 90°$

Logo $\frac{\overline{AB}}{\overline{CD}} = \frac{\overline{AE}}{\overline{CE}} \Leftrightarrow \frac{0,5}{R} = \frac{1}{0,5}$

$\Leftrightarrow R = 0,5 \times 0,5 \Leftrightarrow R = 0,25$

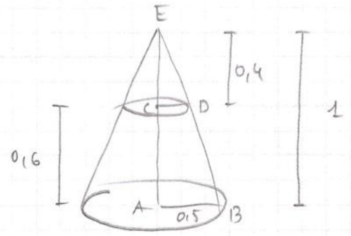
$$V_{\text{cone sombreado}} = \frac{1}{3} \pi \times 0,25^2 \times 0,5 \simeq 0,0327 \, m^3$$

3.2



$$V_{\text{líquido}} = 1 \times 1 \times 0,6 - V_{\text{cone sombreado}}$$
$$= 0,6 - 0,24504427$$
$$= 0,354955773 \, m^3$$
$$= 354956 \, cm^3$$

$i)$



Os triângulos $[ABE]$ e $[CDE]$ são semelhantes, então:

$$\frac{\overline{AB}}{\overline{CD}} = \frac{\overline{AE}}{\overline{CE}} \Leftrightarrow$$

$$\Leftrightarrow \frac{0,5}{R} = \frac{1}{0,4}$$

$$\Leftrightarrow R = 0,5 \times 0,4 = 0,2$$

$$V_{\text{cone sombreado}} = \frac{1}{3}\pi \times 0,5^2 \times 1 - \frac{1}{3} \times \pi \times 0,2^2 \times 0,4 = 0,245044227\ m^3$$

3.3.1

i) $D_f = D_g = [0, 1]$

ii) Recipiente A cheio: $V_{\text{líquido}} = 1^3 - \frac{1}{3} \times 0,5^2 \times \pi \times 1$

$$= 1 - \frac{1}{3} \times \frac{1}{4}\pi$$

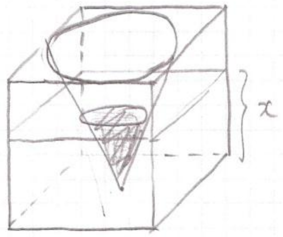
$$= 1 - \frac{\pi}{12} = \frac{12-\pi}{12}$$

iii) Recipiente B cheio: $V_{\text{líquido}} = 1^3 - \frac{1}{3}\, 0,5^2 \times \pi \times 1$

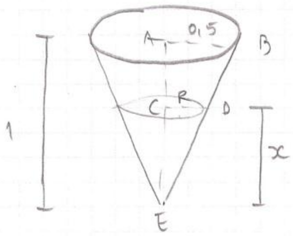
$$= \frac{12-\pi}{12}$$

Logo: $D'_f = D'_g = \left[\underbrace{0}_{\text{volume mínimo}}, \underbrace{\frac{12-\pi}{12}}_{\text{volume máximo}}\right]$ as funções são ambas crescentes em $[0, 1]$

3.3.2



$f(x) = V_{\text{líquido}} = 1 \times 1 \times x - V_{\text{cone sombreado}}$



Os triângulos [ABE] e [CDE] são semelhantes, então

$$\frac{\overline{AB}}{\overline{CD}} = \frac{\overline{AE}}{\overline{CE}} \Leftrightarrow$$

$$\Leftrightarrow \frac{0,5}{R} = \frac{1}{x} \Leftrightarrow$$

$$\Leftrightarrow R = 0,5x$$

Logo $V_{\text{cone sombreado}} = \frac{1}{3} \times \pi \times (0,5x)^2 \times x$

$(0,5)^2 = 0,25 = \frac{1}{4}$ $\quad = \frac{1}{3} \times \pi \times \frac{1}{4} x^2 \times x$

$= \frac{\pi}{12} x^3$

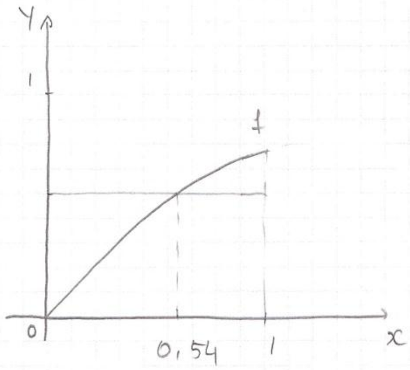
Logo $f(x) = x - \frac{\pi}{12} x^3$ //

3.3.3 500 litros $= 0,5\ m^3$

Definir $y_1 = f(x)$ Janela: $[0, 1] \times [0, 1]$

$y_2 = 0,5$

Obtemos: →



$x \simeq 0{,}54$ m $= 54$ cm

3.3.4 Justificação nas soluções.

4. $f(x) = x^3 - \frac{5}{2}x^2 + \frac{1}{2}x + 2$

4.1 O resto da divisão do polinómio $f(x)$ por $x + \frac{1}{2}$ é $f\left(-\frac{1}{2}\right)$

i) $x + \frac{1}{2} = 0$
$\Leftrightarrow x = -\frac{1}{2}$

$$f\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^3 - \frac{5}{2}\times\left(-\frac{1}{2}\right)^2 + \frac{1}{2}\times\left(-\frac{1}{2}\right) + 2$$

$$= -\frac{1}{8} - \frac{5}{2}\times\frac{1}{4} - \frac{1}{4} + 2$$

$$= -\frac{1}{8} - \frac{5}{8} - \underset{(2)}{\frac{1}{4}} + \underset{(8)}{2} = \frac{-1-5-2+16}{8} =$$

$$= \frac{8}{8} = 1$$

4.2

$$A_{[OABC]} = A_{[OAB]} - A_{[OCB]}$$

O ponto I é o ponto de intersecção da recta AC com o eixo das ordenadas

$$= \frac{\overline{OB} \times \overline{AI}}{2} - \frac{\overline{BO} \times \overline{IC}}{2} =$$

$$= \frac{\overline{OB} \times \overline{AI} - \overline{OB} \times \overline{IC}}{2}$$

$$= \frac{\overline{OB} \times (\overline{AI} - \overline{IC})}{2}$$

$$= \frac{\overline{OB} \times \overline{AC}}{2} \quad //$$

NOTA: $\overline{AI} - \overline{IC} = \overline{AC}$

Nas soluções está feito de outra forma.

4.3

i) $B(0, f(0)) = (0, 2)$ então $\overline{OB} = 2$

$$f(0) = 0^3 - \frac{5}{2} \times 0^2 + \frac{1}{2} \times 0 + 2 = 2$$

ii) Os pontos D, C e A têm ordenada 1.

então $f(x) = 1 \Leftrightarrow x^3 - \frac{5}{2}x^2 + \frac{1}{2}x + 2 = 1$

$$\Leftrightarrow x^3 - \frac{5}{2}x^2 + \frac{1}{2}x + 2 - 1 = 0 \Leftrightarrow$$

Uma das soluções desta equação é $-\frac{1}{2}$

pois, $f(-\frac{1}{2}) = 1 \Leftrightarrow f(-\frac{1}{2}) - 1 = 0$, ou seja, $-\frac{1}{2}$ é raiz do polinómio $x^3 - \frac{5}{2}x^2 + \frac{1}{2}x + 2 - 1 =$

$$= x^3 - \frac{5}{2}x^2 + \frac{1}{2}x + 1$$

$\Leftrightarrow \quad x^3 - \frac{5}{2}x^2 + \frac{1}{2}x + 1 = 0$

$\Leftrightarrow \quad (x+\frac{1}{2})(x^2-3x+2) = 0 \quad \Leftrightarrow \quad x+\frac{1}{2}=0 \lor x^2-3x+2=0$

Regra de Ruffini

$\Leftrightarrow x = -\frac{1}{2} \quad \lor \quad x = 1 \lor x = 2$

Logo $D(-\frac{1}{2}, 1)$, $C(1,1)$ e $A(2,1)$

então $\overline{AC} = 1$

$$\begin{array}{c|cccc} & 1 & -\frac{5}{2} & \frac{1}{2} & 1 \\ -\frac{1}{2} & & -\frac{1}{2} & \frac{3}{2} & -1 \\ \hline & 1 & -3 & 2 & \boxed{0} \end{array}$$

Logo: $x^3 - \frac{5}{2}x^2 + \frac{1}{2}x + 1 = (x+\frac{1}{2})(x^2-3x+2)$

iii) $A_{[OABC]} = \frac{\overline{OB} \times \overline{AC}}{2} = \frac{\cancel{2} \times 1}{\cancel{2}} = 1$ //

5.

5.1 i) $B(1, f(1)) = (1,2)$

$f(1) = \frac{1}{3}\times 1^3 - \frac{5}{3}\times 1^2 + \frac{2}{3}\times 1 + \frac{8}{3}$

$= \frac{1}{3} - \frac{5}{3} + \frac{2}{3} + \frac{8}{3} = 2$

$h(x)$ é uma função afim, logo $h(x) = mx + b$

Como a recta que representa o seu gráfico é paralela à B. Q. Ímpares vem $m = 1$, Logo $h(x) = x + b$

$(y = x + b)$, sub. $(1,2)$ vem $2 = 1 + b \Leftrightarrow b = 1$

Logo $h(x) = x + 1$ //

5.2 i) $A(x, 0)$ e A pertence ao gráfico de h

assim fazendo $h(x) = 0 \Leftrightarrow x+1=0 \Leftrightarrow x=-1$

Logo $A(-1, 0)$

ii) g é uma função quadrática com zeros nos pontos de abcissa -1 e 0, logo:

$$g(x) = a(x-0)(x+1)$$

Como $B(1, 2)$ pertence ao gráfico de g vem:

$$g(1) = 0 \Leftrightarrow a(1-0)(1+1) = 2$$

$$\Leftrightarrow a \times 1 \times 2 = 2$$

$$\Leftrightarrow a \times 2 = 2 \Leftrightarrow a = \frac{2}{2} = 1$$

Logo $g(x) = 1(x-0)(x+1) = x(x+1) = x^2 + x$

5.3 $f(x) > 0 \Leftrightarrow \frac{1}{3}x^3 - \frac{5}{3}x^2 + \frac{2}{3}x + \frac{8}{3} > 0$

$$\Leftrightarrow (x+1)\left(\frac{1}{3}x^2 - 2x + \frac{8}{3}\right) > 0$$

Regra de Ruffini.

	$\frac{1}{3}$	$-\frac{5}{3}$	$\frac{2}{3}$	$\frac{8}{3}$
-1		$-\frac{1}{3}$	2	$-\frac{8}{3}$
	$\frac{1}{3}$	-2	$\frac{8}{3}$	0

C.A.

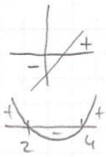
$(x+1)\left(\frac{1}{3}x^2 - 2x + \frac{8}{3}\right) = 0$

$\Leftrightarrow x+1=0 \vee \frac{1}{3}x^2 - 2x + \frac{8}{3} = 0$

$\Leftrightarrow x=-1 \vee x=2 \vee x=4$

Logo $f(x) = (x+1)\left(\frac{1}{3}x^2 - 2x + \frac{8}{3}\right)$

-1 é zero de f



	$-\infty$	-1		2		4	$+\infty$
$x-1$	$-$	0	$+$	$+$	$+$	$+$	$+$
$\frac{1}{3}x^2-2x+\frac{8}{3}$	$+$	$+$	$+$	0	$-$	0	$+$
$f(x)$	$-$	0	$+$	0	$-$	0	$+$

$\therefore f(x) > 0 \Leftrightarrow x \in \left]-1, 2\right[\cup \left]4, +\infty\right[$ //

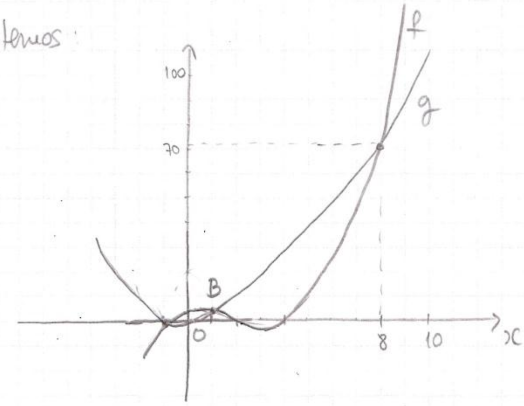
5.4 Definir:

$Y_1 = f(x)$

$Y_2 = g(x)$

Janela: $[-5, 10] \times [-5, 100]$

Obtemos:

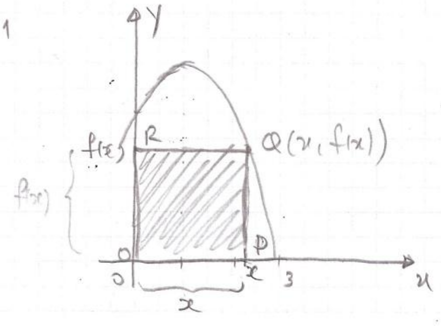


Logo $x = 8$

6. $D_f = [0, 3]$ e $f(x) = 4 - (x-1)^2 =$

$= -(x-1)^2 + 4 \rightarrow V(1, 4)$

6.1



$[OPQR]$ é quadrado se $f(x) = x \Leftrightarrow$

$\Leftrightarrow 4 - (x-1)^2 = x$

$\Leftrightarrow 4 - (x^2 - 2x + 1) = x$ →

$\Leftrightarrow 4 - x^2 + 2x - 1 = x$

$\Leftrightarrow -x^2 + x + 3 = 0$

$\Leftrightarrow x = \dfrac{-1 \pm \sqrt{1^2 - 4 \times (-1) \times 3}}{2 \times (-1)}$

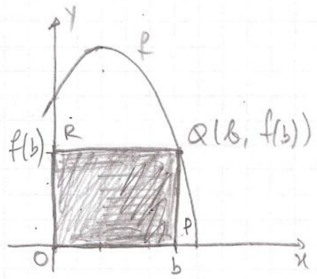
$\Leftrightarrow x = \dfrac{-1 \pm \sqrt{1+12}}{-2} \Leftrightarrow x = \dfrac{-1 \pm \sqrt{13}}{-2}$

$\Leftrightarrow x = \dfrac{-1-\sqrt{13}}{-2} \vee x = \dfrac{-1+\sqrt{13}}{-2}$

$\Leftrightarrow x = \dfrac{1+\sqrt{13}}{2} \vee x = \underbrace{\dfrac{1-\sqrt{13}}{2}}_{<0}$ X

Logo $Q\left(\dfrac{1+\sqrt{13}}{2}, \dfrac{1+\sqrt{13}}{2}\right)$, abcissa é igual à ordenada $[f(x) = x]$

6.2



$A_{[OPQR]} = \overline{OP} \times \overline{QP}$

$= b \times (-b^2 + 2b + 3)$

$= -b^3 + 2b^2 + 3b$ //

$b \in]0,3[$

$\overline{QP} = f(b) =$

$= 4 - (b-1)^2$

$= 4 - (b^2 - 2b + 1)$

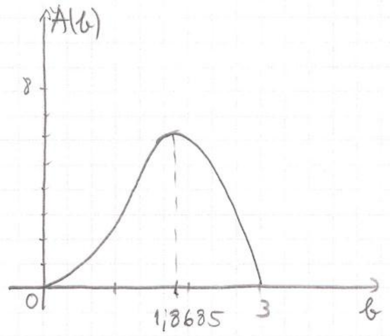
$= 4 - b^2 + 2b - 1 =$

$= -b^2 + 2b + 3$

6.3 b é a abcissa de Q e $Q(b, f(b))$

Definir: $Y_1 = A(b)$ Janela: $[0,3] \times [0,8]$

obtemos:

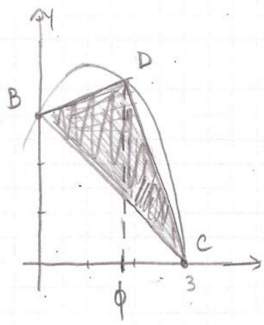


Logo $b \simeq 1{,}8685$ e $f(1{,}8685) = 4 - (1{,}8685 - 1)^2$

$= 3{,}2457$

Logo $Q(1{,}87;\ 3{,}25)$ valores aproximados às centésimas.

6.4



$\phi = \frac{1+\sqrt{5}}{2}$

i) $B(0, f(0)) = (0,3)$

$f(0) = 4 - (0-1)^2 = 3$

ii) $C(x,0) = (3,0)$

$f(x) = 0 \Leftrightarrow 4 - (x-1)^2 = 0$

$\Leftrightarrow 4 - (x^2 - 2x + 1) = 0$

$\Leftrightarrow -x^2 + 2x + 3 = 0$

$\Leftrightarrow x = -1 \vee x = 3$

iii) $f(\phi) = 4 - \left(\frac{1+\sqrt{5}}{2} - 1\right)^2 = 4 - \left(\frac{1+\sqrt{5}-2}{2}\right)^2$

$= 4 - \left(\frac{\sqrt{5}-1}{2}\right)^2$

$= 4 - \frac{(\sqrt{5})^2 - 2\sqrt{5} + 1}{4} =$

$= \underset{(4)}{4} - \frac{5 - 2\sqrt{5} + 1}{4}$

$= \frac{16 - 5 + 2\sqrt{5} - 1}{4} = \frac{10 + 2\sqrt{5}}{4} \underset{(:2)}{=} \frac{5 + \sqrt{5}}{2}$

Logo $D\left(\frac{1+\sqrt{5}}{2}, \frac{5+\sqrt{5}}{2}\right)$

iv) $[BCD]$ é rectângulo em D $\Leftrightarrow \overline{BC}^2 = \overline{BD}^2 + \overline{CD}^2$

$\Leftrightarrow (\sqrt{18})^2 = (\sqrt{3})^2 + (\sqrt{15})^2$

$\Leftrightarrow 18 = 3 + 15 \Leftrightarrow$

$\Leftrightarrow 18 = 18$

$\therefore$ O Triângulo $[BCD]$ é rectângulo em D.

CA.

. $\overline{BC} = \|\overrightarrow{BC}\| = \sqrt{3^2 + (-3)^2} =$ $\qquad$ $\overrightarrow{BC} = C - B = (3,0) - (0,3) = (3,-3)$

$= \sqrt{18}$

. $\overline{BD} = \|\overrightarrow{BD}\| = \sqrt{\left(\frac{1+\sqrt{5}}{2}\right)^2 + \left(\frac{\sqrt{5}-1}{2}\right)^2}$ $\qquad$ $\overrightarrow{BD} = D - B = \left(\frac{1+\sqrt{5}}{2}, \frac{5+\sqrt{5}}{2}\right) - (0,3)$

$= \sqrt{\frac{1+\cancel{2\sqrt{5}}+5}{4} + \frac{5-\cancel{2\sqrt{5}}+1}{4}} =$ $\qquad$ $= \left(\frac{1+\sqrt{5}}{2}, \frac{5+\sqrt{5}}{2} - \underset{(3)}{3}\right)$

$= \sqrt{\frac{12}{4}} = \sqrt{3}$ // $\qquad$ $= \left(\frac{1+\sqrt{5}}{2}, \frac{\sqrt{5}-1}{2}\right)$

$\overline{CD} = \|\overrightarrow{CD}\| =$

$= \sqrt{\left(\frac{-5+\sqrt{5}}{2}\right)^2 + \left(\frac{5+\sqrt{5}}{2}\right)^2}$

$= \sqrt{\frac{25-\cancel{10\sqrt{5}}+5}{4} + \frac{25+\cancel{10\sqrt{5}}+5}{4}}$

$= \sqrt{\frac{60}{4}} = \sqrt{15}$

$\overrightarrow{CD} = D - C$

$= \left(\frac{1+\sqrt{5}}{2}, \frac{5+\sqrt{5}}{2}\right) - (3,0)$

$= \left(\frac{1+\sqrt{5}}{2} - \underset{(2)}{3}, \frac{5+\sqrt{5}}{2}\right)$

$= \left(\frac{-5+\sqrt{5}}{2}, \frac{5+\sqrt{5}}{2}\right)$