

## FS-FBMC: an alternative scheme for filter bank based multicarrier transmission

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**Abstract** - An alternative scheme, named frequency spreading filter bank multicarrier (FS-FBMC), is introduced for multicarrier transmission. It is based on the FFT and closely follows the principle of OFDM, while preserving the key advantages of FBMC, namely the absence of guard time and the spectral separation of the sub-channels. It is optimal in the sense that it can provide perfect equalization of the transmission channel without the need for an additional delay and it maximizes the signal-to-noise ratio in the receiver. It generalizes the classical FFT-PPN approach, which, in fact, is a technique to remove some redundancy in the computations.

### I. INTRODUCTION

The switch-over from analog to digital television is a unique opportunity for cognitive radio because new spectrum is becoming available in the UHF-VHF bands, the so-called TV white spaces [1]. However, it is a challenging application for cognitive radio due to the high level of protection required for the TV and HF microphone signals, which must be combined with maximum spectral efficiency. In such a context, the conventional OFDM modulation has some limitations and, instead, a filter bank based multicarrier (FBMC) modulation seems particularly appropriate.

An in-depth study of FBMC and its relationship to OFDM has been carried out recently [2]. Among the issues which have been left open is the following: while perfect channel equalization can be achieved with OFDM, can it be achieved with FBMC and under what conditions? The present paper provides an answer and, at the same time, it offers an alternative approach for the implementation of the FBMC concept, exploiting frequency domain options. In fact, the proposed approach considerably simplifies and clarifies the FBMC concept and its relationship to the OFDM concept.

A digital filter can be implemented in the frequency domain, using the samples of its frequency response, obtained by the discrete Fourier Transform of the time domain coefficients. The approach is attractive when the number of non-zero samples in the filter frequency response is small. The design technique which permits the control of these non-zero samples is the so-called frequency sampling technique [3].

In FBMC transmission, the filter bank prototype filter can be designed, with the help of the frequency sampling technique, to minimize the number of non-zero samples [4]. Then, a frequency domain implementation of the filter banks can be

contemplated. The approach is named “frequency spreading” because the input data flow carried by a sub-channel is “spread” over a number of carriers equal to the number of non-zero samples of the prototype filter in the frequency domain. The paper is organized as follows. To begin with, the principle of frequency spreading in the transmitter is presented. Then, the despreading and equalization operations are detailed in section 3. The computational complexity issue is briefly discussed in section 4. To conclude, some aspects of the OFDM-FBMC comparison are highlighted.

### II. FREQUENCY SPREADING IN THE TRANSMITTER

When the frequency sampling technique is applied to the design of the prototype filter of a filter bank for multicarrier transmission, the number of non-zero samples in the frequency response is given by  $P=2K-1$ , where  $K$  is the overlapping factor, i.e. the number of multicarrier symbols which overlap in the time domain. The case  $K=4$  is illustrated in Fig.1, which shows the pulse frequency response of the prototype filter [5]. The pulse amplitudes are  $H_0 = 1$ ;  $H_1 = 0.971960 = H_{-1}$ ;  $H_2 = \sqrt{2}/2 = H_{-2}$ ;  $H_3 = 0.235147 = H_{-3}$  and they satisfy the equation

$$\frac{1}{K} \sum_{k=-K+1}^{K-1} |H_k|^2 = 1 \quad (1)$$

With this filter, when a data symbol is applied to a sub-channel, in fact, it is a set of 7 frequency components which are fed to the transmission channel. This suggests a direct approach to derive the transmitted signal, namely an inverse FFT combined with an overlap-and-sum scheme. If the system has  $M$  sub-channels, the size of the iFFT is  $L = KM$ .

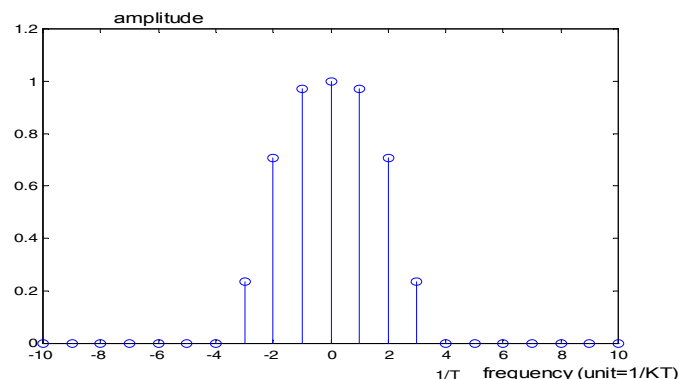


Fig.1. Pulse frequency response of the prototype filter

The iFFT-based transmitter is sketched in Fig.2, for  $K=4$ . Of course, it is necessary to abide by the rules of FBMC transmission. The first rule is the orthogonality of neighbouring sub-channels. Therefore, if the sub-channels with indices “ $i$ ” and “ $i+2$ ” are assumed real, the real data symbols  $d_i(nM)$  and  $d_{i+2}(nM)$  are multiplied by the coefficients  $H_k$  ( $1 \leq k \leq 3$ ) and fed to the 7 inputs of the iFFT with indices:  $iK-3, iK-2, \dots, iK+3$  and  $(i+2)K-3, (i+2)K-2, \dots, (i+2)K+3$ , respectively,

as shown in Fig.2. Obviously, the data symbols  $d_i(nM)$  and  $d_{i+2}(nM)$  do not overlap. Now, orthogonality of neighbouring sub-channels is ensured by making the symbol  $d_{i+1}(nM)$  imaginary. It is interesting to notice that, because of the Nyquist property of the prototype filter, all the carriers are loaded with the same power, if the data symbols have the same magnitude.

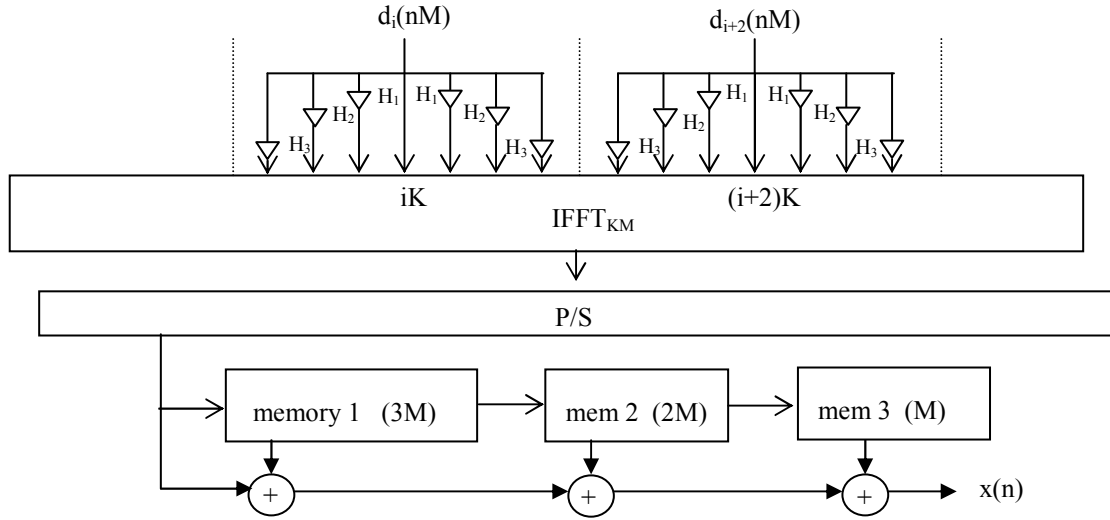


Fig.2. iFFT-based FBMC transmitter

The output of the iFFT is fed to a parallel-to-serial converter, which delivers the time sample stream for transmission.

Since the rate of the multicarrier symbols is  $1/M$ ,  $K$  iFFT output blocks overlap in the time domain. Thus,  $K$  iFFT output blocks have to be stored and summed to generate the transmitted stream. The operation is illustrated in Fig.3 for  $K=4$ , with the 4 blocks of  $4M$  samples which contribute to the summation that produces the block of  $M$  samples applied to the transmission channel.

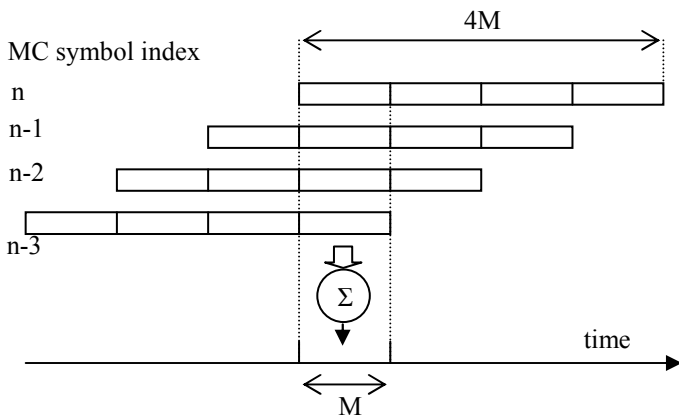


Fig.3. Derivation of the transmitter output stream

In fact, it is sufficient to save a total of  $(3+2+1)M = 6M$  samples and the process is represented in Fig.2.

The maximal spectral efficiency is reached with FBMC when Offset QAM modulation is used [6]. Then, the operations represented in Fig.2 are repeated with a delay of  $M/2$  samples and the 2 output streams obtained are added before application to the transmission channel.

### III. FREQUENCY DESPREADING IN THE RECEIVER

In the receiver, assuming perfect frequency and time synchronization, after serial-to-parallel conversion, the received signal is fed to an FFT of size  $L = KM$ , as shown in Fig.4 for  $K=4$ . The FFT outputs with indices  $iK-3, \dots, iK+3$  are weighted by the coefficients  $H_k$  and a summation yields the data symbols  $d_i(nM)$ , actually multiplied by 4, the sum of the squared coefficients.

The counterpart of the overlap-and-sum operation of the transmitter is a sliding window in the time domain at the receive side. The input of the FFT consists of 4 blocks of  $M$  samples which are shifted at the multicarrier symbol rate. It can be readily verified that the Nyquist criterion is satisfied for the sub-channels, as well as the orthogonality property of neighbouring sub-channels. In back to back connection, the delay of the system is the time it takes to fill the memory of the FFT in the receiver, namely  $KM$  samples or  $K$  multicarrier symbols.

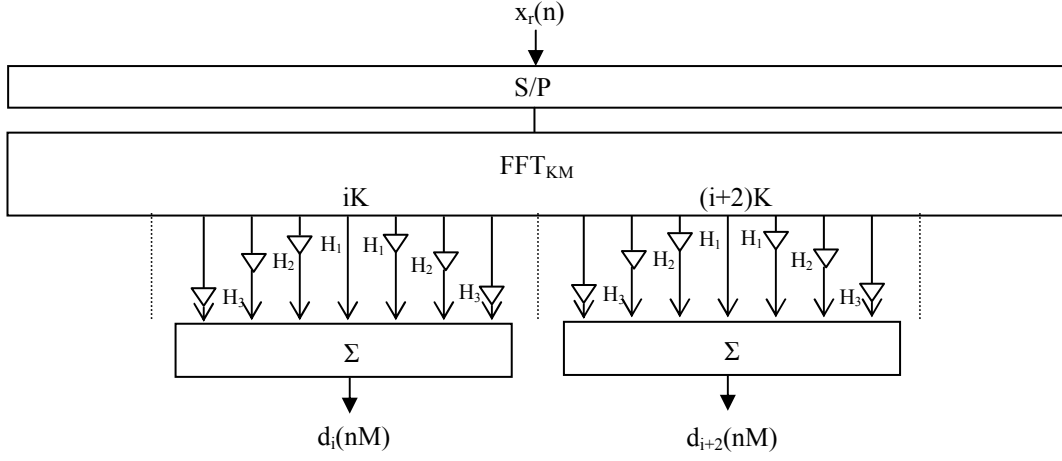


Fig.4. FFT-based FBMC receiver

Now, in the presence of the transmission channel, equalization has to be introduced. If the channel has been measured with the frequency spacing  $1/KM$ , assuming no noise and perfect time and frequency alignment, the equalizer coefficients are derived from the channel frequency response  $C(i)$  by

$$EQ(i) = \frac{1}{C(i)} ; 0 \leq i \leq KM - 1 \quad (2)$$

The equalizer is shown in Fig.5, for sub-channel “i”. With this equalizer, all the spectral components of the signal have been corrected and, if the channel is perfectly estimated, the signal is perfectly equalized. Another important feature is that no additional delay is required, in contrast with time domain sub-channel equalization.

A potential gain in performance is worth pointing out. In the presence of channel noise, if the level of noise on each frequency component can be measured or estimated, the principle of maximum ratio combining can be applied to maximize the signal-to-noise ratio at the output of the sub-channels.

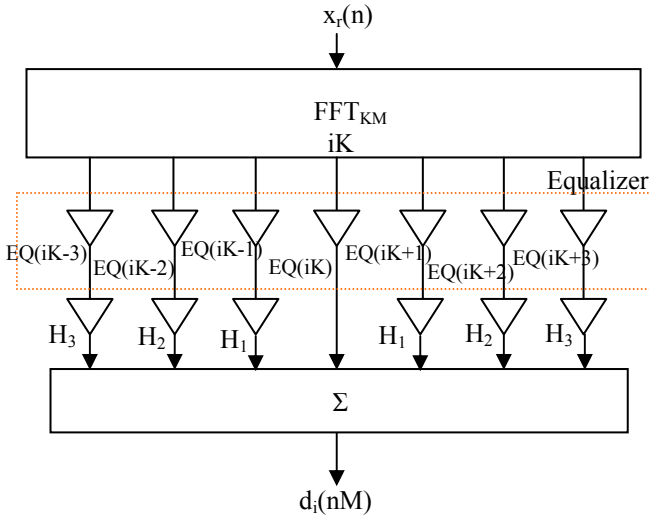


Fig.5. Frequency domain equalizer for sub-channel “i”

It is worth pointing out that the measurement of the transmission channel with spacing  $1/KM$  is obtained with the help of a pseudo-random input data sequence, because of the overlapping of the neighbouring sub-channels and the complementarity of the filter coefficients. For example, for  $K=4$ , if  $d_i=1$  and  $d_{i+1}=j$ , after multiplication by the coefficients and summation, the samples at the input of the iFFT in the transmitter are

$$x_e(4i) = 1 ; x_e(4i+1) = H_1 + jH_3 ;$$

$$x_e(4i+2) = (1+j)H_2 ; x_e(4i+3) = H_3 + jH_1$$

and they all have unit magnitude.

#### IV. COMPUTATIONAL COMPLEXITY

The counterpart for the nice properties mentioned above is the computational complexity. The block diagrams of the transmitter and receiver are shown in Fig.6 and Fig.7 respectively.

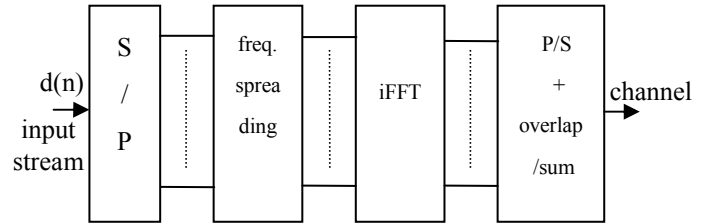


Fig.6. Principle of FS-FBMC transmitter

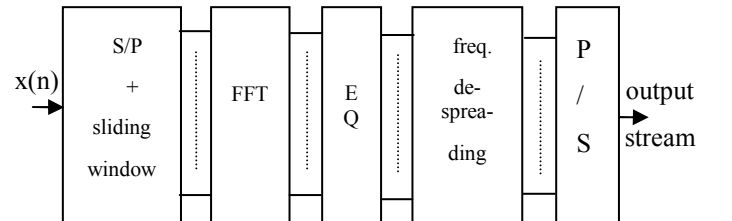


Fig.7. Principle of FS-FBMC receiver

The computational complexity of the scheme lies in the FFT of size  $L=KM$ . Some savings in the computation of the FFT can be achieved, with the help of the so-called FFT-pruning technique, exploiting the specific environment, namely the combination of the iFFT and the overlap-and-sum operation in the transmitter and the combination of the sliding window and the FFT in the receiver. However, a significant amount of redundancy in the computations remains.

In fact, the redundancy in the computations is removed through time domain processing, using the polyphase network-FFT technique. This scheme requires an FFT of size  $M$ , and  $KM$  multiplications and the storage of  $KM$  samples for the polyphase network (PPN) [7]. However, sub-channel equalization has to be carried out in the time domain, which introduces additional memories and delay.

Finally, 2 equivalent options are available to implement FBMC, as shown in Fig.8.

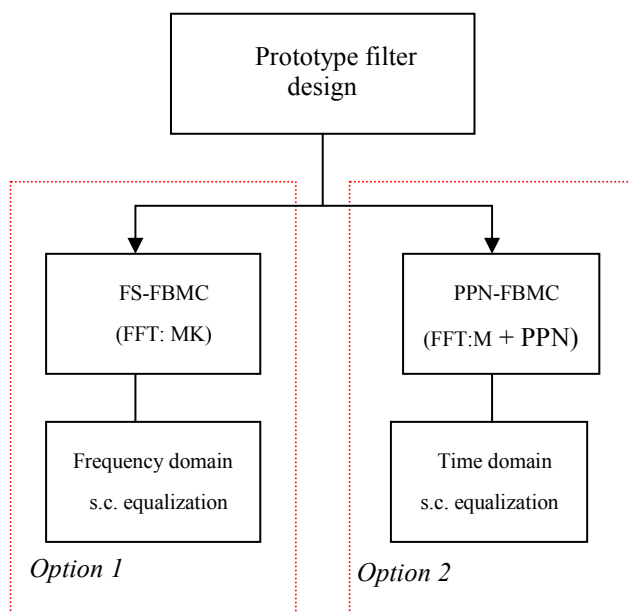


Fig.8. The 2 options for the implementation of FBMC

The choice between these two options depends on the system specifications and the constraints in computation capacity.

The impact of OQAM modulation is worth pointing out, because it doubles the processing rate. For example, in the receiver, the FFT time window slides by  $M/2$  samples instead of  $M$  samples between two computations.

## V. CONCLUSION

The FS-FBMC concept is a simple approach to introduce and characterize the FBMC technique. The fact that the system consists only of an FFT, completed by some simple processing at the input and output, clarifies the technique and makes its implementation straightforward. The

comparison OFDM-FBMC is also clarified, the main difference being the FFT sizes. In particular, the high resolution spectrum analysis capability of FBMC comes from the larger size of its FFT, with respect to OFDM.

On the theoretical side, the FS-FBMC scheme reveals the two conditions which are necessary to achieve perfect sub-channel equalization, in the absence of noise,

- measurement of the transmission channel with frequency spacing  $1/KM$ ,
- number of equalizer coefficients equal to  $2K-1$ .

This result has an impact on the implementation of time domain equalizers in FFT-PPN based systems: the number of taps should not exceed  $2K-1$ .

Flexibility is another interesting feature of FS-FBMC. Given a particular FFT, various schemes can be implemented. For example, a system with overlapping factor  $K=4$  can be reconfigured into a system with overlapping factor  $K=2$  and twice the number of sub-channels.

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## References

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